Assessing students’ work

The following descriptions indicate typical levels of performance. After each description is an example of some work at this level.

Note: The work below is from the UK version of the task. Please read ‘£’ as ‘$’ – the task is otherwise identical.

Little progress

- **Representing**: Selects a method for organising the work but not for determining whether or not the lottery will raise money.
- **Analysing**: Writes down just a few pairs of numbers that may be chosen, missing most combinations.
- **Interpreting and evaluating**: Decides whether or not the lottery is a good money raiser but the reasons are not given or are incorrect.
- **Communicating and reflecting**: Does not communicate reasoning, or communicates reasoning that is unclear, incorrect and/or unrelated to the analysis.

Sample response: Tom

Tom correctly identifies 7 of the 15 possible ways of choosing two different numbers on the ticket, but his reasons for stating that the lottery is not a good money raiser are unrelated to this work.

1. There are 7 ways.

   ![Diagram showing possible number pairs]

   - \( \binom{1}{2} = 3 \)
   - \( \binom{3}{6} = 2 \)
   - \( \binom{2}{5} = 2 \)
   - \( \binom{2}{6} = 2 \)

2. The lottery will not be a good money raiser because people will pay only \( £1 \) for tickets and if they do win they will get \( £10 \), and that is too much. Because then they can keep on buying tickets and will most probably win.

Questions for Tom:

Tom could be encouraged to improve his response by asking the following questions

- *How do you know that you have found all possible number pairs? Can you find any more? Can you find them using a systematic method?*
- *Can you see how your list of possible pairs can help you decide whether or not the lottery will be a good money raiser?*
Some progress

- **Representing:** Selects a method for organising the work but not for determining whether or not the lottery will raise money.
- **Analysing:** Lists most pairs of numbers but omits some pairs or includes impossible pairs such as 2,2 or 7,4.
- **Interpreting and evaluating:** Decides whether or not the lottery is a good money raiser but the reasons given are incorrect.
- **Communicating and reflecting** Communicates reasoning clearly but this is incorrect and/or unrelated to the analysis.

Sample response: Sam

Sam has made substantial progress with the first part of the task, as he correctly lists all of the 15 possible ways of choosing two different numbers on the ticket. He does not make any connection between this work and his subsequent reasoning, which assumes that because two numbers are chosen from six then there is a "1 in 3" chance of winning.

Questions for Sam:

Sam could be encouraged to improve his response by asking the following questions:

- Can you explain to me why you think there is a 1 in 3 chance of winning?
- Can you see how your list of possible pairs can help you decide whether or not the lottery will be a good money raiser?
- What do you think is likely to happen if 300 players play the lottery?
- What would be the most likely number of winners?
- How much money would then be raised or lost?
**Substantial progress**

- **Representing:** Selects a method for organising the work and a suitable method for determining whether or not the lottery will raise money.
- **Analysing:** Systematically lists the number of ways two numbers may be chosen from six but may list some extras.
- **Interpreting and evaluating:** Includes some correct work towards determining whether or not the lottery is a good money raiser.
- **Communicating and reflecting:** Communicates some correct reasoning effectively, relating this to the analysis, but this may be incomplete or contain errors.

**Sample response: Megan**

Megan systematically correctly lists all of the 15 possible ways of choosing two different numbers on the ticket. Her reasons for stating that it is not a good money raiser are incomplete.

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
2 & 3 & 4 & 5 & 6 & 1 \\
3 & 4 & 5 & 6 & 1 & 2 \\
4 & 5 & 6 & 1 & 2 & 3 \\
5 & 6 & 1 & 2 & 3 & 4 \\
6 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\[\text{There are 15 ways of choosing 2 different numbers on the ticket.}\]

\[\text{\$1 10 \times \text{what they paid.}}\]

I think it will not be a good money raiser because there are 15 combinations so it is most likely that someone will win and whoever wins gets 10 \times more than they paid.

Example: 20 players enter lottery will make £20 out of all of these people at least 1 person has a high probability of winning so this means only half of the money will be raised and not given away. However there is a more than likely chance more than 1 person will get 1 of the 15 combinations.

**Questions for Megan:**

Megan could be encouraged to improve her reasoning by asking the following questions:

- What do you think is likely to happen if 300 players play the lottery?
- What would be the most likely number of winners?
- How much money would then be raised or lost?
Task accomplished

- **Representing:** Selects a method for organising the work and a suitable method for determining whether or not the lottery will raise money.
- **Analysing:** Systematically lists all the different ways two numbers may be chosen from six.
- **Interpreting and evaluating:** Decides correctly whether or not the lottery is a good money raiser.
- **Communicating and reflecting:** Communicates their reasoning effectively, completely and correctly.

Sample response: Hannah

Hannah correctly lists all of the 15 possible ways of choosing two different numbers on the ticket. Her reasons for stating that it is not a good money raiser are well reasoned and her work is clear and easy to follow.

```
1 2
1 3
1 5
1 6
2 3
2 4
2 5
3 5
3 6
4 5
4 6
5 6
```

Hannah states that there are fifteen ways of choosing two different numbers on the ticket.

```
1 2
1 3
1 5
1 6
2 3
2 4
2 5
3 5
3 6
4 5
4 6
5 6
```

Hannah correctly lists all of the 15 possible ways of choosing two different numbers on the ticket. Her reasons for stating that it is not a good money raiser are well reasoned and her work is clear and easy to follow.

**Questions for Hannah:**

Hannah could be encouraged to improve her response by asking the following questions:

- **Would you be more sure of making a profit if more people played the lottery? Can you explain why?**
- **Can you suggest ways in which the lottery could be made a better money raiser?**
- **What effect would it have if you increased the number of balls in the bag by one?**
- **What effect would it have if you asked people to choose three numbers instead of two?**