

Assessing students' work

The following descriptions indicate typical levels of performance. After each description is an example of some work at this level.

Note: The work below is from the UK version of the task. Please read '£' as '\$' – the task is otherwise identical.

Little progress

- **Representing:** Selects some of the key data to perform calculations but there may be some errors. E.g. Does not know how to handle remainders in calculations.
- **Analysing:** Performs some calculations towards finding the cost of taxis for 75 people.
- **Interpreting and evaluating:** Determines the cost of taxis for 75 people but this is incorrect due to miscalculation or misinterpretation of the calculation in the context.
- **Communicating and reflecting:** The number of small taxis and cost of transporting a party of 75 are communicated with omissions or inaccuracies.

Sample response: Sam

Sam shows some calculations towards finding the cost of taxis for 75 people, but there are errors. He has rounded down in his calculation for the number of small taxis. No work was shown for the remainder of the task.

$$\begin{array}{l}
 1. \quad 6 \times 7 = 42 \qquad \qquad \qquad 4 \overline{) 33.80} \\
 \qquad 75 = 42 + 33 \\
 \qquad 33 \div 4 = 8.25 \qquad \qquad \qquad 63x \\
 \qquad \text{He needs 8 small taxis} \qquad \qquad \qquad \underline{378} \\
 \qquad 6 \text{ large taxis} = 378 \qquad \qquad \qquad 40x \\
 \qquad 8 \text{ small taxis} = 320 \qquad \qquad \qquad \underline{8} \\
 \qquad \text{Total} = \underline{\underline{698}} \qquad \qquad \qquad 320
 \end{array}$$

Questions for Sam:

Sam could be encouraged to improve his response by asking the following questions:

- *How many people will 8 small taxis and 6 large taxis transport?
How many spare seats will there be?*
- *Can you find me a different number of large and small taxis that can still transport 75 people?*
- *Can you list some more possibilities in an organised way?*
- *Now can you calculate some costs for each of these possibilities?*

Some progress

- **Representing:** Selects key data to perform calculations of taxis required for 75 people.
- **Analysing:** Performs calculations to determine the number of small taxis needed and the cost of taxis for 75 people: 9 small taxis, £738.
- **Interpreting and evaluating:** Correctly determines both the number of small taxis needed and the cost of taxis for 75 people.
- **Communicating and reflecting:** The number of small taxis and cost of transporting a party of 75 are communicated clearly.

Sample response: Rory

Rory correctly shows that 9 small taxis with 6 large taxis are needed, with a cost of £738. He then finds the cost of using either *all* small or *all* large taxis and decides that 11 large taxis would be cheaper (£693). Thus he does not go on to look at other combinations of large and small taxis.

$$\begin{array}{l}
 1) \\
 a) \quad 6 \times 7 = 42 \\
 \quad \quad 4 \times 9 = 36 \\
 \\
 \quad \quad \quad 42 + 36 = 78 \\
 \\
 \quad \quad \quad 9 \text{ small taxis} \\
 \\
 b) \quad \text{The total cost will be } \pounds 738 \\
 \\
 \quad \quad 63 \times 6 = 378 \\
 \quad \quad 40 \times 9 = 360 \\
 \\
 \quad \quad \quad 378 + 360 = \pounds 738 \\
 \\
 2) \quad 40 \times 14 = 760 \\
 \quad \quad 63 \times 11 = 693 \\
 \\
 \quad \quad \quad \text{he should order 11 large taxis}
 \end{array}$$

Questions for Rory:

Rory could be encouraged to improve his response by asking the following questions:

- *You have said that he should order 11 large taxis. How could you check if you have reached the cheapest solution?*
- *How could you best present your reasoning and results to convince someone else that you have found the minimum cost?*

Substantial progress

- **Representing:** Selects a suitable representation (e.g. symbols or a table) to record the analysis.
- **Analysing:** Begins to systematically explore the effect of varying the numbers of small and large taxis, bearing in mind the constraints. May consider the cost per person for large and small taxis
- **Interpreting and evaluating:** Considers the context and obtains a combination of small and large taxis that gives a low but not necessarily the lowest cost..
- **Communicating and reflecting:** Communicates the conclusion and reasoning clearly.

Sample response: Ella

Ella shows that 9 small taxis with 6 large taxis are needed, with a cost of £738. She begins to find the cost of using different combinations of large and small taxis for 75 people. Her costings are correct but she does not find the cheapest combination.

1. a) 6 large taxis

$$\begin{array}{r}
 £63 \\
 63 \\
 63 \\
 63 \\
 63 \\
 63 \\
 \hline
 378
 \end{array}
 \quad
 \begin{array}{r}
 7 \\
 7 \\
 7 \\
 7 \\
 7 \\
 7 \\
 \hline
 42
 \end{array}
 \quad
 \begin{array}{r}
 -75 \\
 -42 \\
 \hline
 33
 \end{array}
 \quad
 33 \div 4 = 8.25$$

42 people fit in 6 large taxis

9 small taxis

$$\begin{array}{r}
 £40 \\
 40 \\
 40 \\
 40 \\
 40 \\
 40 \\
 40 \\
 40 \\
 40 \\
 \hline
 360
 \end{array}
 \quad
 £360$$

b) The total cost will be £738

$$\begin{array}{r}
 £378 \\
 + 360 \\
 \hline
 738
 \end{array}$$

2. $75 \div 7 = 10.71$

Large taxis = 11
for the whole group
 $11 \times £63 = £693$

Small taxis = 9
for the whole group
 $75 - 4 = 18 - 75$
 $19 \times £40 = £760$

7 large taxis = 49 people

7 small taxis = 26 people
 $\frac{26}{75}$

$$\begin{array}{r}
 \text{Large taxis} = 7 \times £63 = £441 \\
 \text{Small taxis} = 7 \times £40 = £280 \\
 \hline
 £721
 \end{array}$$

He could have 11 large taxis for the whole group which would cost £693,
or he could have 7 large and 7 small which would cost £721.

Questions for Ella:

Ella could be encouraged to improve her response by asking the following questions:

- You have correctly worked out the cost for 11 large taxis and for 7 small and 7 large taxis? How can you be sure you have found the cheapest solution?

Task accomplished

- **Representing:** Selects a suitable representation to record the analysis.
- **Analysing:** Systematically explores the costs of varying the numbers of small and large taxis, bearing in mind the constraints.
- **Interpreting and evaluating:** Considers the context and obtains the combination of small and large taxis that give the lowest possible cost. (9 large taxis and 3 small taxis, £687)
- **Communicating and reflecting:** Communicates the conclusion and reasoning clearly and effectively. Shows clearly and convincingly that the lowest cost has been found.

Sample response: Laura

Laura shows that 9 small taxis with 6 large taxis are needed, with a cost of £738. She finds the cost of using different combinations of large and small taxis and she correctly finds that the cheapest is £687. Her work is clear, but she could offer more explanation to show how she knows she has found the optimal solution.

$$\begin{aligned} \text{1a) } 7 \times 6 &= 42 \\ 75 - 42 &= 33 \\ 33 \div 4 &= 8.25 \end{aligned}$$

He needs 9 small taxis.

$$\begin{aligned} \text{b) } 6 \times 63 &= \pounds 378 \\ 9 \times 40 &= \pounds 360 \\ \hline &= \pounds 738 \end{aligned}$$

$$\begin{aligned} \text{2. } 75 \div 7 &= 10.71428571 \\ \text{He need 11 large taxis} \\ 11 \times 63 &= 693 \end{aligned}$$

$$\begin{aligned} 75 \div 4 &= 18.75 \\ \text{He need 19 large taxis} \\ 19 \times 40 &= 760 \end{aligned}$$

~~$$9 \times 7 = 63 \quad 75 - 63 = 12$$~~

$$\begin{aligned} 10 \times 7 &= 70 && \text{leaves 5 people} \\ &&& \text{so 2 small} \\ 10 \times 63 &= 630 \\ 2 \times 40 &= 80 \\ \hline &= 710 \end{aligned}$$

$$\begin{aligned} 9 \times 7 &= 63 && \text{leaves 12 people} \\ &&& \text{so 3 small} \end{aligned}$$

$$\begin{aligned} 9 \times 63 &= 567 \\ 3 \times 40 &= 120 \\ \hline &= \pounds 687 \end{aligned}$$

smallest

$$\begin{aligned} 8 \times 7 &= 56 && \text{leaves 19} \\ &&& \text{so 5 small} \end{aligned}$$

$$\begin{aligned} 8 \times 63 &= 504 \\ 5 \times 40 &= 200 \\ \hline &= 704 \end{aligned}$$

Lowest cost is £687 for 9 large and 3 small taxis

Questions for Laura:

Laura could be encouraged to improve her response by asking the following questions:

- How can you be absolutely sure that you have found the cheapest solution? Write down your explanation.
- Can you tidy up your results and reasoning so that they are presented as clearly and convincingly as possible?