CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Interpreting Multiplication and Division

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

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Interpreting Multiplication and Division

MATHEMATICAL GOALS

This lesson unit is designed to help students to interpret the meaning of multiplication and division. Many students have a very limited understanding of these operations and only recognise them in terms of 'times' and 'share'. They find it hard to give any meaning to calculations that involve non-integers. This is one reason why they have difficulty when choosing the correct operation to perform when solving word problems.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6.NS: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, 5, and 6:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

INTRODUCTION

The unit is structured in the following way:

- Before the lesson, students work individually on a task designed to reveal their current levels of understanding. You review their scripts and write questions to help them improve their work.
- In the lesson, students work collaboratively matching cards that contain symbols, descriptions of the structure of the situations and word problems. This is done in two phases. After each phase there is a whole-class discussion reflecting on the structures being learned.
- In a follow-up lesson, students use their learning and your questions to review their initial answers and to complete another similar task.

MATERIALS REQUIRED

- Each student will need copies of the assessment tasks: *Multiplication and Division* and *Multiplication and Division (revisited)*, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need a copy of the *Card Set: Calculations* (cut up), *Card Set: Words and Diagrams* (cut up), *Card Set: Pizza Problems* (cut up), a large sheet of paper for making a poster, a glue stick, some felt-tipped pens, and some blank paper.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson, a 90-minute lesson (or two 50-minute lessons), and 15 minutes in a follow-up lesson. Timings are approximate. Exact timings will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Multiplication and Division* (15 minutes)

Have students complete this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Multiplication and Division

Complete the gaps in the table. The first row has been done for you.
The diagram should show the structure of the problem.
The calculation should show a single multiplication or division.

	Problem	Explain how to do the problem, and draw a diagram to help	Calculation	Answer
1	I buy three boxes of yogurt. Each box contains four yogurts. How many yogurts do I buy altogether?	Calculate three groups of four. 	3 x 4 or 4 x 3	12
2	Two pizzas are shared equally among five people. How much does each person get?			
3	Max cuts a cake into three equal slices. He eats one half of a piece. What fraction of the cake does he eat?			
4	*Make up this problem yourself!*		$3 \div \frac{1}{2}$	
5	*Make up this problem yourself!*		$\frac{1}{2} \div \frac{1}{4}$	

Explain what you would like students to do.

Read this task carefully.

Spend a few minutes answering the questions on the sheet.

Fill in all the blank boxes.

In the second column, explain how to do the problem in words and draw a diagram to show this.

In the third column show a calculation to solve the problem.

In the final column show the numerical answer.

In questions 4 and 5 you have to make up the problems to match the calculations.

Do not be too concerned if you cannot finish everything. [Tomorrow] we will have a lesson on these ideas, which should help you to make further progress.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and any difficulties they encounter.

We suggest that you do not score students' work. The research shows that this will be counter-productive as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions, and highlight appropriate questions for each student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to students in the follow-up lesson.

Common issues	Suggested questions and prompts
Always divides larger numbers by smaller ones For example: The student reverses the division and writes 5 ÷ 2 instead of 2 ÷ 5 (Q2).	• Can you ever divide a smaller number by a larger one? Show me an example. • What kind of answer do you get if you do this?
Always gives smaller answers following a division For example: Student answers $1\frac{1}{2}$ to $3 \div \frac{1}{2}$ (Q4). Or: Student answers $\frac{1}{8}$ to $\frac{1}{2} \div \frac{1}{4}$ (Q5).	• How would you say these aloud: $3 \div \frac{1}{2}$? $\frac{1}{2} \div \frac{1}{4}$? Write down exactly what you would say in words. • Can you do this without using the words 'divide' or 'share'? [E.g. 'How many halves go into three?']
Does not identify 'of' with multiply For example: Student writes: $\frac{1}{3} \div \frac{1}{2}$ for one half of one third (Q3).	• How would you write 'one half of one fifth' as a calculation? • Now work this out on a calculator. • Is your answer what you expected? Why?
Uses multiple calculations For example: The student writes 1 ÷ 3 ÷ 2 (Q3).	• Can you write 1 ÷ 3 ÷ 2 as a one-step calculation?
Has difficulty expressing the structure using a diagram	• Can you show me what this calculation means using a diagram? [Supply any appropriate calculation.] • Your diagram does not have to be very artistic or accurate!
Completes the task correctly The student needs an extension task.	• Make up some questions that would need some more difficult calculations. For example, can you make up a question where you would need to solve: $20 \div 2\frac{1}{4}$?

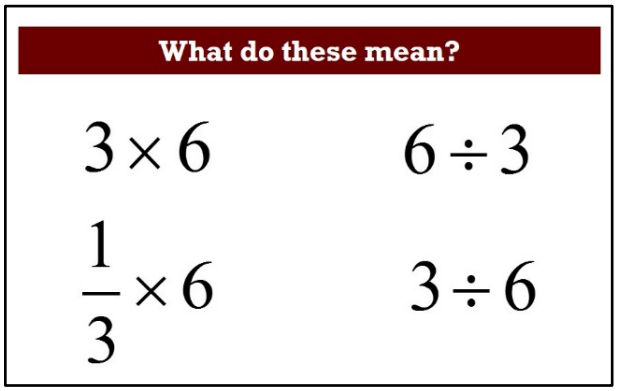
SUGGESTED LESSON OUTLINE

Do not give out the students' assessment task at the beginning of this lesson in order to sort out the difficulties they encountered. This would reduce the opportunities for students to confront their misunderstandings for themselves during the collaborative activity. If you find most of your students had real difficulty with the assessment task then this lesson unit may not be appropriate.

Whole-class introduction (20 minutes)

Throughout this introduction if students are struggling to answer the questions allow a few minutes to solve the questions individually, then ask them to discuss the question with a partner.

Give out the mini-whiteboards, pens, and erasers.
Show Slide P-1:



Think about the meaning of each of these statements.
Try to think of more than one way to describe each statement.
Can you make a drawing to show one of them?

Encourage alternative interpretations to emerge and do not correct them at this stage, even if answers are poorly expressed or incorrect. Students may use shading to indicate parts of their drawings and you could draw attention to this as a useful way to represent calculations. If students are struggling with these questions, encourage them to make up a real-life context for each calculation.

At this stage the purpose is to listen and record what students say. This should begin to engage some interest, as disagreements emerge. For example, for 3 ÷ 6 they might say:

It's threes into six. *(Incorrect.)*
It's three split into six equal parts. *(Correct.)*

Explain that some answers may be wrong. You will return to these later and reconsider them. It is not appropriate to lecture students on what is correct, as that will spoil the activities that are to follow. If possible, you could write students' interpretations on the board and leave them there to return to later.

Show the following calculation (See Slide P-2): $6 \div \frac{1}{2}$.

Ask the following questions:

What does this statement mean? [6 divided by a half.]
Can you say this in a different way?
What is the answer?
Can you make up a real-life problem that requires this calculation?

Students might begin by trying to remember the rules for dividing fractions.

They might read the calculation as 'six shared by a half'. This description may be meaningless for them, so are unable to suggest a problem.
Alternatively, they may think that the answer is 3.

How else can we read this calculation? ["How many halves are there in six?"]

Do you now know the answer? [12.]

Can you now make up a real-life problem that requires this calculation?
["If I buy six pizzas and divide them each in half, how many slices will I end up with?]

Small-group work: matching *Calculations* to *Words and Diagrams* (20 minutes)

Ask students to work in pairs or threes and give each small group a copy of *Card Set: Calculations* and *Card Set: Words and Diagrams*. These should be cut up. These are also shown on Slides P-3 and P-4. Make sure that blank paper is available in case students want to jot down ideas as they think.

Show Slide P-5 and introduce the process of working on the task:

Matching Calculations and Words/Diagrams

- Take turns to match a calculation to a word/ diagram interpretation of that calculation.
- When a matching is made, explain how you can obtain the answer to the calculation using the diagram.
- Where a diagram does not exist, draw one of your own.

You might find more than one calculation that matches a word/diagram or more than one word/diagram that matches a calculation. That is fine. Put the cards together however you think they go best.

This process will encourage students to consider alternative ways of describing multiplication and division. Note that there are several alternative calculation cards that may be matched with some diagrams. This is intentional and should provide some interesting discussions among students.

While students discuss in small groups, you have two tasks: to note different student approaches to the task and to support student learning.

Note different student approaches

Which of the three representations do students find most difficult/easy to interpret: the calculations, the verbal description of the calculation or the diagram? Is this the same for each scenario or does it depend on the structure of the calculation? Can students explain the relationships between these different representations?

Support student learning

Try to support students' thinking and reasoning, rather than prompting them to use any particular methods. Ask questions to help students clarify their thinking and explain their work:

James, you have matched these two cards together. Can you explain why you think these cards match?

How does this diagram represent this calculation?

You may find the questions in the *Common issues* table useful.

Whole-class discussion: reviewing the learning (10 minutes)

Now return to the original questions on the board. Ask learners to say which they now think are correct interpretations and ask them to give reasons.

Ask further questions that encourage students to generalize what they have learned to other calculations. For example, using mini-whiteboards, you could ask students to show answers to the following:

How would you write these calculations in symbols:

- *Twenty divided by nine?*
- *How many times does fifteen go into one hundred?*
- *What is one fifth of eighty?*
- *How many thirds are there in sixty?*
- *What is one half of one fifth?*
- *How many times does a third go into 12?*

The emphasis is on getting correct calculations, even if students are not able to work them out.

Avoid going from example to example without students following what other students are saying. Instead, encourage students to comment on what other students have just said:

Did you understand what Qaylah just said?

Can you put it into your own words?

Do you agree with her? Why?

Extending the lesson over two days

If you are taking two days to complete the unit then you may want to end the first lesson here. Ensure that students keep a record of the matches they have made and then at the start of the second day, give students time to review their matched cards before asking them to add in the word problems.

Small-group work: matching *Calculations*, *Interpretations*, and *Problems* (20 minutes)

Students remain in the same small groups. Give each group a copy of *Card Set: Pizza Problems* (cut up). These are also shown on projector Slide P-6. They will also need a large sheet of paper for making the poster, a glue stick, and some felt-tipped pens.

Display Slide P-7 of the projector resource and explain how students are to work together:

Working Together

- I want each group to make a poster to show each pizza problem card matched to an appropriate calculation.
- The calculation should show one correct way of solving the problem.
- Next to each pairing, explain why the cards match, using diagrams and/or words.
- You should also write an answer to the problem.

As students work together, watch carefully. If some struggle, ask them to try to explain what the calculations mean in words, as in the previous activity. Also, encourage students to draw their own diagrams to show the problem in a different way.

It is not necessary for every group to match up every problem. If you are short of time, you could ask students to choose four problems from *Card Set: Pizza Problems* and just match these ones.

Whole-class discussion (20 minutes)

Ask a representative from each group to choose a problem and calculation that they are sure go together and to explain why they go together to the rest of the class. Then ask the rest of the class if they agree or think they can improve on the explanation.

You might also ask a group that is unsure to talk about a problem or calculation that they cannot match.

Finally ask the class some further questions based on the pizza context.

*Show me **a calculation** that matches each of these problems:*

- *If I share 2 pizzas equally between 4 people, how much pizza will each get? [2 ÷ 4.]*
- *There are eight pizzas at a party. One third are spicy. How many are spicy? [(1 ÷ 3) × 8]*

Show me a pizza problem that would result in each of the following calculations:

$2 \div 8$

$\frac{1}{8} \div 2$

$\frac{1}{2} \times \frac{1}{8}$

Follow-up lesson: reviewing the assessment task (15 minutes)

Give each student their original papers from the assessment task, *Multiplication and Division*. If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

First look at your original responses and the questions [on the board/written on your script.] Answer these questions and revise your response.

Then give each student a copy of the review task, *Multiplication and Division (revisited)*.

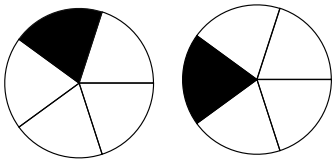
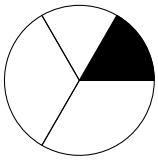
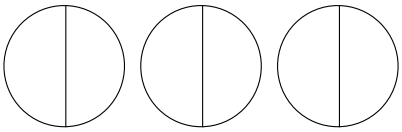
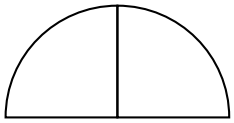
Now look at the new task sheet, Multiplication and Division (revisited). Can you use what you have learned to answer these questions?

Some teachers give this as homework.

SOLUTIONS

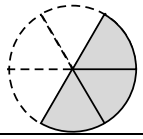
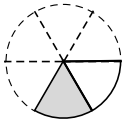
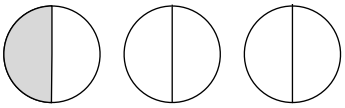
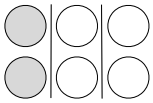
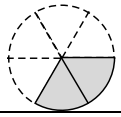
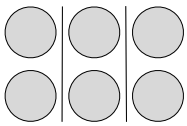
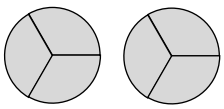
Assessment task: *Multiplication and Division*

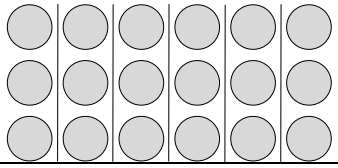
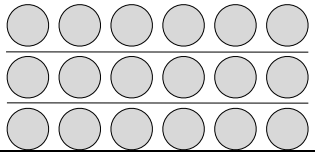
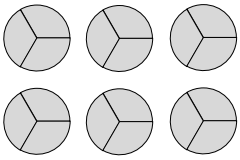
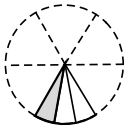
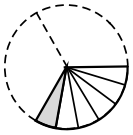
The shaded parts were given. The responses shown below are not the only ones possible.

	Problem	**Explain how to do the problem, and draw a diagram to help.**	**Calculation**	**Answer**
2	Two pizzas are shared equally among five people. How much does each person get?	What is two divided by 5? 	$2 \div 5$ or $\frac{1}{5} \times 2$ or $2 \times \frac{1}{5}$	$\frac{2}{5}$
3	Max cuts a cake into three equal slices. He eats one half of a slice. What fraction of the cake does he eat?	What is one half of one third? 	$\frac{1}{2} \times \frac{1}{3}$ or $\frac{1}{3} \times \frac{1}{2}$ or $\frac{1}{3} \div 2$	$\frac{1}{6}$
4	*Make up this problem yourself!* E.g. Three pizzas are each divided in half. How many slices are there?	How many halves are there in 3? 	$3 \div \frac{1}{2}$	6
5	*Make up this problem yourself!* E.g. A glass holds half a pint. A cup holds a quarter of a pint. How many cups can I fill from the glass?	How many quarters are there in one half? 	$\frac{1}{2} \div \frac{1}{4}$	2

Main lesson task

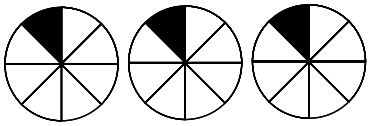
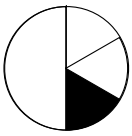
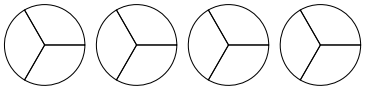
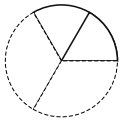
In many cases, more than one calculation can be matched with a diagram; only the most obvious answers are given in the table below. The possible equivalence of different calculations or different 'words and diagrams' cards is a useful discussion point. Students sometimes match cards based on the answer rather than on the calculation, and this may be something to draw out.

Calculations	Words and Diagrams	Pizza Problems
C3, C8, C9 and C14 $3 \times \frac{1}{6}$ $\frac{1}{6} \times 3$ $\frac{1}{6} \div \frac{1}{3}$ $3 \div 6$ These all have the answer 1/2	W1 What is three groups of one sixth? 	P10 A whole pizza is divided into six equal slices. I eat three of these. What fraction of the whole pizza do I eat?
	W12 What fraction of one third is one sixth? 	P11 At a restaurant, each person is given one third of a whole pizza. I eat one sixth of a whole pizza. What fraction of my slice do I eat?
	W8 Three divided into six equal parts. How much is each part? 	P8 Three pizzas are divided equally among six people. What fraction of a whole pizza does each person get?
C1, C4, C6 and C11 $\frac{1}{3} \times 6$ $6 \times \frac{1}{3}$ $6 \div 3$ $\frac{1}{3} \div \frac{1}{6}$ These all have the answer 2	W2 What is one third of six? 	P6 There are six pizzas. One third are vegetarian. How many are vegetarian?
	W3 How many one-sixths are there in one third? 	P7 I cut a pizza into six equal slices. I then eat a third of the whole pizza. How many slices do I eat?
	W7 Six divided into three equal groups. How many in each group? 	P4 Six pizzas are placed into three boxes. The same number go into each box. How many is this?
	W6 Six groups of one third. How much altogether? 	P5 Six people are each given one third of a pizza to eat. How many whole pizzas are they given altogether?

C5, C7 and C12 3×6 6×3 $6 \div \frac{1}{3}$ These all have the answer 18.	W4 Six groups of three. How many altogether? 	P2 Six boxes of pizza are delivered to a party. Each box contains three pizzas. How many pizzas are delivered altogether?
	W5 Three groups of six. How many altogether? 	P1 Three boxes of pizza are delivered to a party. Each box contains six pizzas. How many pizzas are delivered altogether?
	W10 Six divided into thirds. How many slices? 	P3 Six boxes, each containing one pizza, are delivered to a party. I divide each pizza into three equal slices. How many slices are there altogether?
C13, C2, C10 $\frac{1}{6} \times \frac{1}{3}$ $\frac{1}{3} \times \frac{1}{6}$ $\frac{1}{3} \div 6$ These all have the answer 1/18.	W11 How much is one third of one sixth? 	P9 I am given one sixth of a pizza. I only eat one third of this slice. What fraction of the whole pizza do I eat?
	W9 How much is one sixth of one third? 	P12 I cut a pizza into three equal slices. I then eat a sixth of one slice. What fraction of the whole pizza do I eat?

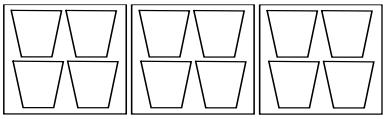
Assessment task: ***Multiplication and Division (revisited)***

The shaded parts were given. The responses shown below are not the only ones possible.

	Problem	**Explain how to do the problem, and draw a diagram to help.**	**Calculation**	**Answer**
2	Three pizzas are shared equally among eight people. How much does each person get?	Three divided by eight 	$\frac{1}{8}\times 3$ or $3\times\frac{1}{8}$ or $3\div 8$	$\frac{3}{8}$
3	Max cuts a cake into two equal slices. He eats one third of a slice. What fraction of the whole cake does he eat?	One third of one half 	$\frac{1}{3}\times\frac{1}{2}$ or $\frac{1}{2}\times\frac{1}{3}$ or $\frac{1}{2}\div 3$	$\frac{1}{6}$
4	*Make up this problem yourself!* E.g. Four pizzas are each divided into thirds. How many slices are there? A jug holds four pints. A glass holds one third of a pint. How many glasses can I fill from the jug?	How many thirds in four? 	$4\div\frac{1}{3}$	12
5	*Make up this problem yourself!* E.g. A glass holds one third of a pint. A small cup holds a sixth of a pint. How many cups can I fill from the glass?	How many sixths are there in one third? 	$\frac{1}{3}\div\frac{1}{6}$	2

Multiplication and Division

Complete the gaps in the table. The first row has been done for you.
The diagram should show the structure of the problem.
The calculation should show a single multiplication or division.

	Problem	**Explain how to do the problem, and draw a diagram to help**	**Calculation**	**Answer**
1	I buy three boxes of yogurt. Each box contains four yogurts. How many yogurts do I buy altogether?	Calculate three groups of four. 	3 x 4 or 4 x 3	12
2	Two pizzas are shared equally among five people. How much does each person get?			
3	Max cuts a cake into three equal slices. He eats one half of a piece. What fraction of the cake does he eat?			
4	*Make up this problem yourself!*		$3 \div \frac{1}{2}$	
5	*Make up this problem yourself!*		$\frac{1}{2}, \frac{1}{4}$	

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Card Set: Calculations

C1 $\frac{1}{3} \times 6$	C2 $\frac{1}{3} \times \frac{1}{6}$	C3 $3 \times \frac{1}{6}$
C4 $6 \div 3$	C5 3×6	C6 $\frac{1}{3} \div \frac{1}{6}$
C7 $6 \div \frac{1}{3}$	C8 $\frac{1}{6} \times 3$	C9 $\frac{1}{6} \div \frac{1}{3}$
C10 $\frac{1}{3} \div 6$	C11 $6 \times \frac{1}{3}$	C12 6×3
C13 $\frac{1}{6} \times \frac{1}{3}$	C14 $3 \div 6$	C15

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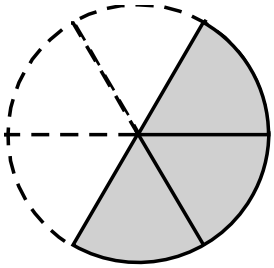
Card Set: Words and Diagrams

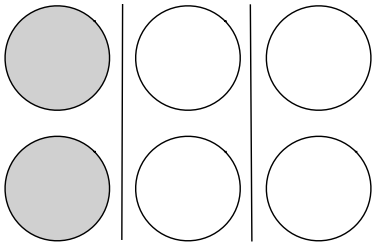
W1

What is three groups of one sixth?



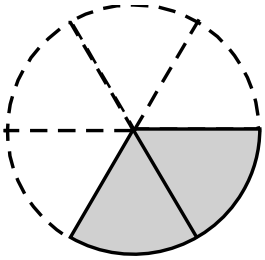
W2

What is one third of six?



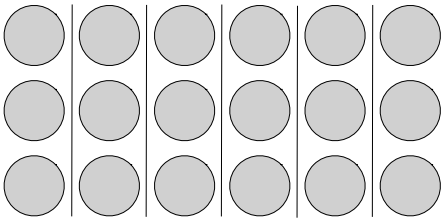
W3

How many one-sixths are there in one third?



W4

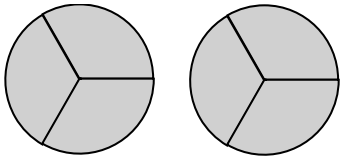
Six groups of three.
How many altogether?



W5

W6

Six groups of one third.
How much altogether?



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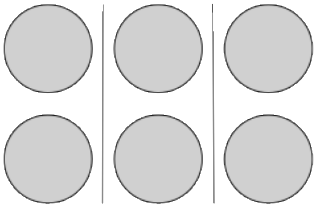
Card Set: Words and Diagrams (continued)

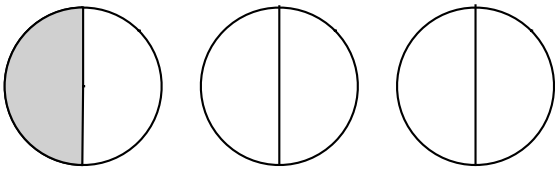
W7

Six divided into three equal groups. How many in each group?



W8

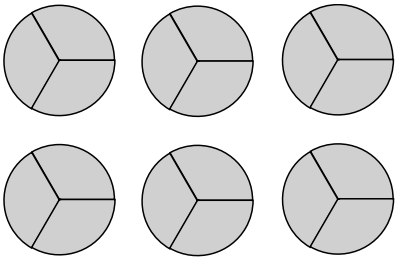
Three divided into six equal parts. How much is each part?



W9

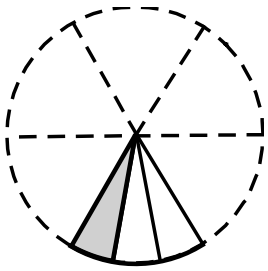
W10

Six divided into thirds.
How many slices?



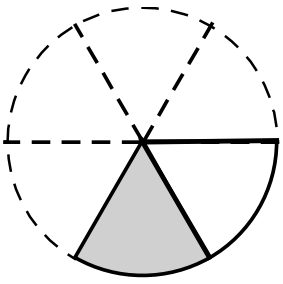
W11

How much is one third of one sixth?



W12

What fraction of one third is one sixth?



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Card Set: Pizza Problems

P1 Three boxes of pizza are delivered to a party. Each box contains six pizzas. How many pizzas are delivered altogether?	P2 Six boxes of pizza are delivered to a party. Each box contains three pizzas. How many pizzas are delivered altogether?
P3 Six boxes, each containing one pizza, are delivered to a party. I divide each pizza into three equal slices. How many slices are there altogether?	P4 Six pizzas are placed in three boxes. The same number go into each box. How many is this?
P5 Six people are each given one third of a pizza to eat. How many whole pizzas are they given altogether?	P6 There are six pizzas. One third are vegetarian. How many are vegetarian?
P7 I cut a pizza into six equal slices. I then eat a third of the whole pizza. How many slices do I eat?	P8 Three pizzas are divided equally among six people. What fraction of a whole pizza does each person get?
P9 I am given one sixth of a pizza. I eat only one third of this piece. What fraction of the whole pizza do I eat?	P10 A whole pizza is divided into six equal slices. I eat three of these. What fraction of the whole pizza do I eat?
P11 At a restaurant, each person is given one third of a whole pizza. I eat one sixth of a whole pizza. What fraction of my piece do I eat?	P12 I cut a pizza into three equal slices. I then eat a sixth of one piece. What fraction of the whole pizza do I eat?

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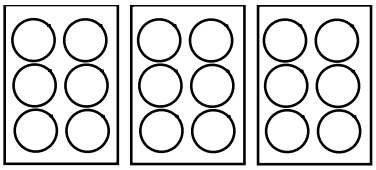
Multiplication and Division (revisited)

Complete the gaps in the table. The first row has been done for you.
The diagram should show the structure of the problem.
The calculation should show a single multiplication or division.

	Problem	**Explain how to do the problem, and draw a diagram to help.**	**Calculation**	**Answer**
1	I buy three boxes of golf balls. Each box contains six balls. How many balls do I buy altogether?	Calculate three groups of six. 	3 x 6 or 6 x 3	18
2	Three pizzas are shared equally among eight people. How much does each person get?			
3	Max cuts a cake into two equal slices. He eats one third of a piece. What fraction of the whole cake does he eat?			
4	*Make up this problem yourself!*		$4 \div \frac{1}{3}$	
5	*Make up this problem yourself!*		$\frac{1}{3}$, $\frac{1}{6}$	

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What do these mean?

3×6

$6 \div 3$

$\frac{1}{3} \times 6$

$3 \div 6$

What does this mean?

$$6 \div \frac{1}{2}$$

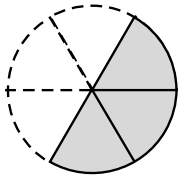
Calculations

C1 $\frac{1}{3} \times 6$	C2 $\frac{1}{3} \times \frac{1}{6}$	C3 $3 \times \frac{1}{6}$
C4 $6 \div 3$	C5 3×6	C6 $\frac{1}{3} \div \frac{1}{6}$
C7 $6 \div \frac{1}{3}$	C8 $\frac{1}{6} \times 3$	C9 $\frac{1}{6} \div \frac{1}{3}$
C10 $\frac{1}{3} \div 6$	C11 $6 \times \frac{1}{3}$	C12 6×3
C13 $\frac{1}{6} \times \frac{1}{3}$	C14 $3 \div 6$	C15

Words and Diagrams

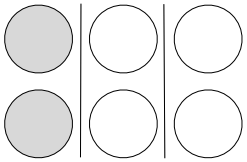
W1

What is three groups of one sixth?



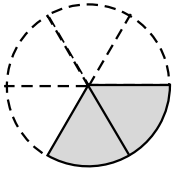
W2

What is one third of six?



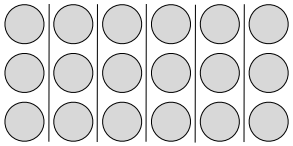
W3

How many one-sixths are there in one third?



W4

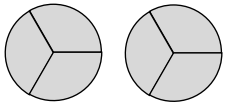
Six groups of three.
How many altogether?



W5

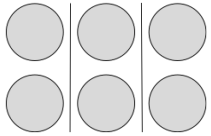
W6

Six groups of one third.
How much altogether?



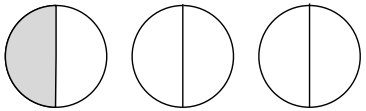
W7

Six divided into three equal groups. How many in each group?



W8

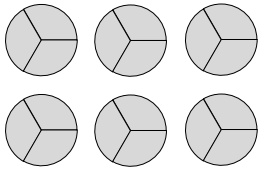
Three divided into six equal parts. How much is each part?



W9

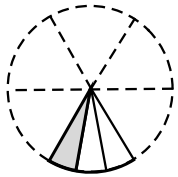
W10

Six divided into thirds.
How many slices?



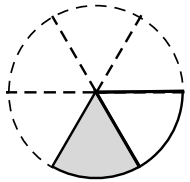
W11

How much is one third of one sixth?



W12

What fraction of one third is one sixth?



Matching Calculations and Words/Diagrams

- Take turns to match a calculation to a word/ diagram interpretation of that calculation.

- When a matching is made, explain how you can obtain the answer to the calculation using the diagram.

- Where a diagram does not exist, draw one of your own.

Pizza Problems

P1 Three boxes of pizza are delivered to a party. Each box contains six pizzas. How many pizzas are delivered altogether?	P2 Six boxes of pizza are delivered to a party. Each box contains three pizzas. How many pizzas are delivered altogether?	P3 Six boxes, each containing one pizza, are delivered to a party. I divide each pizza into three equal slices. How many slices are there altogether?
P4 Six pizzas are placed in three boxes. The same number go into each box. How many is this?	P5 Six people are each given one third of a pizza to eat. How many whole pizzas are they given altogether?	P6 There are six pizzas. One third are vegetarian. How many are vegetarian?
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Working Together

- I want each group to make a poster to show each pizza problem card matched to an appropriate calculation.
- The calculation should show one correct way of solving the problem.
- Next to each pairing, explain why the cards match, using diagrams and/or words.
- You should also write an answer to the problem.

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and **Daniel Pead**

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The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

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The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org

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