PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Evaluating Statements: *Consecutive Sums*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Evaluating Statements: *Consecutive Sums*

MATHEMATICAL GOALS

This lesson unit is intended to help students to:

- State and test mathematical conjectures.
- Understand and use alternative methods of proof.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 3, 7, and 8:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6.EE: Apply and extend previous understandings of arithmetic to algebraic expressions.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the *Consecutive Sums* task individually. You review their responses and formulate questions for students to consider in order to improve their work.
- At the start of the lesson, students think individually about their responses to the questions set.
- Next, students consider a conjecture about the sums of *four* consecutive whole numbers before looking in pairs at some sample work in which other students attempt to justify the conjecture. They seek to improve the arguments given and compare the different approaches used.
- In the same small groups, students then work together to classify other conjectures about consecutive sums according to whether they are always, sometimes or never true.
- In a whole-class discussion, students discuss their categorization of the conjectures.
- Finally, students reflect individually on the different approaches seen and used.

MATERIALS REQUIRED

- Each individual student will need some plain paper to work on, a copy of the *Consecutive Sums* task, a mini-whiteboard, pen and eraser and a copy of the *How Did You Work?* questionnaire.
- Each small group of students will need copies of the four *Sample Responses to Discuss*, cut-up copies of *Sample Conjectures to Discuss,* (cut-up copies of *Sample Conjectures to Discuss (Extension)* and *Blank Cards* may be needed for those who complete the main task), a sheet of poster paper, a marker and a glue stick.
- Provide calculators for students who wish to use them.
- There is a projector resource to help to introduce activities and support whole-class discussions.

TIME NEEDED

20 minutes before the lesson and a 120-minute lesson (or two 60-minute lessons). Timings given are approximate. Exact timings will depend on the needs of your class. An optional follow-up lesson could be used to develop the task further, with students creating and proving further conjectures in relation to the consecutive sums problem.

BEFORE THE LESSON

Introducing the task: *Consecutive Sums* (20 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to identify the kinds of difficulties they have with it. You should then be able to target your help more effectively in the subsequent lesson.

Consecutive Sums

> The number 18 can be made by adding ***three*** *consecutive whole numbers*:
>
> 5 + 6 + 7 = 18
>
> Which other numbers can be made by adding three consecutive whole numbers?

1. Experiment with some numbers and try to state a conjecture about which numbers can be made.

2. Try to prove your conjecture.

3. If you have time, try to state and prove another conjecture about sums of consecutive whole numbers.

Give each student a copy of the assessment task *Consecutive Sums*. Introduce the task briefly, making sure that students understand the problem and the vocabulary used, namely 'consecutive' and 'conjecture'.

In this task we are going to be looking at the results we get when we sum ***positive*** *whole numbers (not including zero).*

What do we mean by 'consecutive' whole numbers? [A sequence of whole numbers with a common difference of one.]

What is a 'conjecture'? [A statement that you think ***might*** *be true but which you are not yet absolutely sure of. It can be tested to see whether or not it is true.]*

Can you give an example of a conjecture? [If students do not say anything, offer an example conjecture such as 'At least one person in this room had pasta for dinner yesterday' and then ask students to make up another.]

What do you understand by 'prove' in mathematics? [An argument showing why a statement is certainly true. Statements that concern infinitely many possible cases cannot be proved by listing a finite number of confirming cases.]

It is important that, as far as possible, students are allowed to answer the questions without assistance. If students are struggling to get started, ask questions that help them understand what is required, but make sure you do not do the task for them.

Students who sit together often produce similar solutions, so when they compare their work, they have little to discuss. For this reason we suggest that when students do the task individually, you ask them to move to different seats. Then, at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this may produce more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem-solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues:	**Suggested questions and prompts:**
Does not state a conjecture For example: The student lists various sums of three consecutive whole numbers but does not state a conjecture about *which* numbers can be made (Q1).	• Do you notice anything about your results? • Experiment some more and see if you find any patterns.
States a conjecture that is contradicted by one or more of the results For example: The student says that the totals are always even, despite having 2 + 3 + 4 = 9 listed as a result (Q1).	• Can you test your conjecture on the examples that you have produced? Does it always work?
States a false conjecture For example: The student states that all whole numbers can be made by adding three consecutive numbers.	• Which whole numbers have you tried? • Why do you think that your conjecture is true for **all** whole numbers?
States a conjecture but does not test it For example: The student states that the totals are always multiples of three but does not attempt to justify this.	• Are you sure of your conjecture? Why? • Can you test your conjecture on some examples? • Can you see why it happens? • Can you explain why this is?
Tests a conjecture unsystematically For example: The student shows that 30 and 54 can be made as the sum of three consecutive whole numbers.	• Do you think that your conjecture is true for other numbers? How could you check?
Provides an incomplete proof For example: The student attempts to justify their conjecture by listing a few examples, such as showing that 6, 9, 12, etc. can be made.	• You have made a lot of multiples of 3. Do you think that you can make them all? • Are all numbers that **aren't** multiples of 3 **impossible**? How do you know?
No further conjectures are given For example: The student explains why the sum of three consecutive whole numbers must be a multiple of 3 but does not go further.	• What other things do you notice? • What do you think would happen if you summed four numbers rather than three? Can you explore this?
Uses whole numbers that are not consecutive For example: When investigating whether the sum of two whole numbers is even, examines non-consecutive numbers, e.g. 9 + 5 = 14 - even, 5 + 5 = 10 - even (Q3).	• Is this a conjecture about the sum of **consecutive** whole numbers? • What do we mean by 'consecutive' whole numbers?

SUGGESTED LESSON OUTLINE

Reviewing individual solutions to the task (10 minutes)

Display Slide P-1 of the projector resource, which states the problem:

Consecutive Sums
The number 18 can be made by adding ***three*** *consecutive whole numbers*:
5 + 6 + 7 = 18
Which other numbers can be made by adding three consecutive whole numbers?

Recall what we were working on previously. Explain to me what the task was about.

Give students a chance to describe their understanding of the task in their own words.

If students are still unclear about the terms 'consecutive' and 'conjecture', you could review this vocabulary again here.

Return the students' work on the *Consecutive Sums* task and if you have not added questions to students' work, write a short list of your most common questions on the board. Students can then select a few questions appropriate to their own work.

I looked at your work and I have some questions for you.

I would like you to think, on your own, about my questions and how your work could be improved.

Students may want to jot down their ideas as they consider how to improve their work. They can either write directly on their original work using a different colored pen or could use another piece of paper.

Whole-class introduction (10 minutes)

Once students have had a chance to familiarize themselves again with the task and thought individually about their responses to the questions posed, spend some time collating students' ideas for conjectures about the sum of three consecutive whole numbers.

What conjectures did you make about adding three consecutive whole numbers?

If students' descriptions of their conjectures are unclear, ask them to explain what they mean. Write students' conjectures on the board, right or wrong. Avoid discussing whether the conjectures are right or wrong at this stage. Hopefully there will be some disagreement about this, which provides a reason for the rest of the lesson.

Did anyone else make that conjecture?

What other conjectures did you have?

Check to see whether any students extended their thinking beyond the sum of three consecutive whole numbers:

Did anyone make a conjecture about the sum of more/less than three consecutive whole numbers?

If students have considered the sum of more or less than three consecutive whole numbers, write the conjectures on the board also.

Explain that in today's lesson they will be looking at a number of conjectures about sums of consecutive whole numbers.

Today we are going to be testing conjectures about consecutive sums. The aim is to find ways of being sure about whether or not the conjectures are true.

Individual work: introducing the conjecture (5 minutes)

Give each student a piece of plain paper to work on and display Slide P-2 of the projector resource:

Consecutive Sums 2

Conjecture:

The sum of ***four*** consecutive whole numbers is always even.

5 + 6 + 7 + 8 = 26

Here is a conjecture about the sum of four consecutive numbers.

Spend a few minutes on your own thinking about this conjecture.

Investigate whether or not it is true, making any notes/calculations you need to help you on your piece of paper.

Show your work clearly as you will be sharing it with someone else in the class in a few minutes.

Students should be encouraged to think systematically when investigating this conjecture. If they are struggling, they may want to start by experimenting with some numbers.

Some students may have stated this conjecture on the final question of the assessment task (and so it may have been written on the board in the whole-class introduction). If this is the case, acknowledge the work these students have already done and encourage them to check whether or not their work can be improved further.

Collaborative activity: sharing work (5 minutes)

Once students have had a chance to work on the conjecture individually, organize the class into groups of two or three students. Grouping students who have taken different approaches may lead to more profitable discussions, so noting different student approaches during the individual work may help with this.

You each have your own, individual work.

I want you to take turns to share your work with your partner(s).

Listen carefully to each other and ask questions if you don't understand or agree.

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

Give each student a mini-whiteboard, a pen and an eraser.

Now that you have had a chance to work individually and discuss your work with someone else, in your pairs you are going to look at some more student work on this conjecture.

Give each group of students copies of *Sample Responses to Discuss*. There may not be enough time for all students to analyze all four responses, so choose approaches that offer each group some fresh ideas. Issue appropriate responses to each group.

Display Slide P-3 of the projector resource and explain how students are to work in their groups:

Sample Responses to Discuss

In your groups:

1. Try to understand what the student has done. You may want to annotate the work/do some math on your mini-whiteboard.
2. Explain the students' reasoning in your own words for each piece of work (Q1).
3. Explain why each piece of work does or does not convince you (Q2).
4. Decide which of the approaches you have looked at you prefer. Be prepared to explain your choice.

The approaches have been chosen to show numerical, algebraic and diagrammatic approaches to the problem. The four approaches on the work are described below:

Tim's approach uses properties of even and odd numbers to justify the conjecture. He notes that there are two possible orderings of even/odd and checks that both sum to an even answer. He assumes the truth of the rules: odd + even = odd and odd + odd = even.

Four consecutive numbers is either

$$\underbrace{\text{odd} + \text{even}}_{\text{odd}} + \underbrace{\text{odd} + \text{even}}_{\text{odd}} = \text{even}$$

or

$$\underbrace{\text{even} + \text{odd}}_{\text{odd}} + \underbrace{\text{even} + \text{odd}}_{\text{odd}} = \text{even}$$

So it has to be even.

Alex uses algebra to represent the four numbers, starting at $n - 1$. Students might find it easier to begin with n, but sometimes having n as the middle (or near the middle) number makes for easier simplification, especially when there are many numbers. It is also not essential to start with n and this possible misconception may surface here. Alex misses out the sum calculation. Students may find it helpful to put this step in when working through this approach.

Let the four numbers be

$$n-1, \quad n, \quad n+1, \quad n+2$$

$$\text{Sum} = \underbrace{4n}_{\text{even}} + \underbrace{2}_{\text{even}} = \text{even}$$

Rumana's three examples may be interpreted as the beginning of an exhaustive list of all possible sums of four consecutive whole numbers. She explains why the sum increases by 4 each time and assumes that, since 10 is an even number, adding any number of 4s (also even) will always result in an even answer.

Multiple of 2?

$1+2+3+4 = 10$ ✓

$2+3+4+5 = 14$ $+4$ ✓

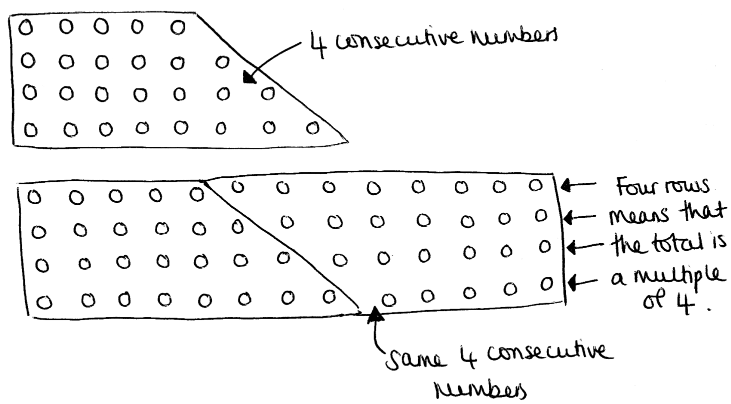
$3+4+5+6 = 18$ $+4$ ✓

Every time, each of the four numbers on the left is 1 bigger, so the sum is 4 bigger. If we start with 10 and keep adding 4s then all the answers will be even.

Steph uses a diagram to represent the sum of 4 consecutive numbers as a trapezoid of dots. She looks at one particular case and assumes that her argument will work for all cases.

Students may find her final argument difficult. She assumes that half of a multiple of 4 must be a multiple of 2.

$5+6+7+8$



Since twice the sum of 4 consecutive numbers is a multiple of 4, the sum of 4 consecutive numbers must be a multiple of 2 (even).
This works for all cases!

Whole-class discussion: comparing different approaches (10 minutes)

Discuss the different ways in which the students in the sample work have tried to prove this conjecture.

What is different about the methods?

Which method do you prefer and why?

What are the advantages or disadvantages of the different methods?

Students may find it difficult to discuss the advantages and disadvantages of methods. If this is the case, you could ask more focused questions, such as:

Which method do you find the easiest or quickest to understand? Why?

Which method do you feel uses the simplest or most complicated mathematics?

Which method do you find most/least convincing? Why?

Which methods do you think could be used for other similar conjectures? Why?

Students may feel that a 'proper' proof uses algebra, so this could be a good opportunity to emphasize that there is no one right method for proof. Different methods have different advantages and disadvantages. You should, however, consider the generalizability of each method.

You may want to use Slides P-4 to P-7, which contain the sample student work.

If your lesson is an hour long, this may be a good time to make the break.

Collaborative activity: making posters (30 minutes)

Give each group of students a piece of poster paper, a marker, a glue stick, and cut-up cards from *Sample Conjectures to Discuss*. If your students are finding this work difficult, give each group just two of the cards.

Students are to make a poster showing whether the conjecture on each card is always, sometimes or never true and showing an explanation of how they know.

> *Divide your poster paper into three sections and label them 'Always', 'Sometimes' and 'Never'.*
>
> *I am going to give you some new conjectures on cards. I want you to decide if they are always true, sometimes true or never true.*
>
> *Use ideas you have learned from the sample student work to help you and make any notes or calculations on your mini-whiteboards as you investigate each of the conjecture cards.*
>
> *If you think the conjecture is always true, glue it in the 'Always' section and explain how you know that it is always true.*
>
> *If you think the conjecture is sometimes true, glue it in the 'Sometimes' section and say when it is true and when it isn't true.*
>
> *If you think the conjecture on your card is never true, glue it in the 'Never' section and explain how you know that it is never true.*
>
> *You may not have time to place all of the conjecture cards and it is not essential that you do so. It is more important that everyone in your group understands fully the placement of each card and that you have explained your thinking on your poster.*

Slide P-8 summarizes these instructions:

Always, Sometimes or Never True?

1. Decide whether each conjecture is Always, Sometimes or Never True.
   - If the conjecture is **Always** or **Never** true, write down how you can be sure.
   - If the conjecture is **Sometimes** true, write down when it is true and when it is not.
2. Use ideas from the sample work you have looked at and make notes on your mini-whiteboards.
3. Agree about each conjecture before moving on.
4. Explain your thinking on your poster.

While students work in small groups, you have two tasks: to note different approaches to the task and to support student problem solving.

Note different student approaches to the task

Listen and watch students carefully and note their different approaches to the task. Do they use words, like Tim; algebra, like Alex; numerical arguments, like Rumana; or drawings, like Steph? Do they explain their conclusions clearly?

Notice too, any assumptions students make. For example, when working on Conjecture C do they assume that they can consider an odd number of numbers only, as this list of numbers will contain an actual middle number or do they extend to an even number of numbers as well?

Notice too, which conjectures students choose to tackle first. Do they have a strategy or do they choose them at random? This information will help you to focus a whole-class discussion later in the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach. Instead, ask questions that help them to clarify their thinking. The suggestions in the *Common issues* table may be helpful.

If students are stuck, encourage them to use ideas from the *Sample Responses to Discuss*:

How might Alex have justified this conjecture? Can you use algebra to do this?

If students place the cards correctly and have explained fully, encourage them to use alternative methods of justification from the *Sample Responses to Discuss*:

Can you justify it a different way?

How might Steph have justified this conjecture? Can you use a diagram?

If students need additional challenge still, you might also use the *Sample Conjectures to Discuss (Extension)* cards and/or encourage students to generate and justify their own conjectures on the two *Blank Cards*.

Whole-class discussion (15 minutes)

Discuss the different ways in which students have determined whether the conjectures are always, sometimes or never true.

Did anyone find a conjecture that they thought was always true? Which one? Why did you think it was always true?

Did anyone else prove that conjecture? Did you do it the same way or in a different way?

What about one that was sometimes true? What about one that was never true?

Who came up with their own conjecture? Where did it go on the poster? Why?

Slides P-9 and P-10, showing the different conjectures, are available to assist this discussion.

Individual reflection (15 minutes)

It is important that students understand that there are different methods of proof and that they have different benefits and drawbacks. One method is not intrinsically better than another.

What different methods have we seen and used today?

Give each student a copy of the *How Did You Work?* questionnaire.

Think carefully about your work on this task and the advantages and disadvantages of the different methods you have seen and used. On your own, answer the review questions as carefully as you can.

Your preferred method may be different now to what it was earlier in the lesson. Give a reason for the approach you like the best.

Some teachers give this task as homework.

Follow-up lesson (optional)

The consecutive sums task is very rich. Some classes have spent over a week of lessons working on it. Students could explore which numbers can be made as a sum of consecutive whole numbers and which cannot. They could also investigate which numbers can be made in more than one way. If you have enough time, students could gain a lot from engaging with this problem over a more extended period.

SOLUTIONS

Assessment task: *Consecutive Sums*

There are many correct conjectures that students might make about the sum of three consecutive whole numbers. Here is a selection:

- The sum is always a multiple of three.
- You cannot make any number that **is not** a multiple of three.
- The sum is three times the middle number.
- The sum is even/odd if the lowest (or highest) number is odd/even.
- The sum is even/odd if the middle number is even/odd.
- There is no highest possible number.

Proofs may be constructed using words, diagrams or algebra. For example: Let the three consecutive numbers be n-1, n, n+1. Their sum is equal to $3n$, a multiple of 3.

Poster task

Various methods of proof are possible. Here, only one is given in each case:

<table>
<tr><th>Always</th><th>Sometimes</th><th>Never</th></tr>
<tr><td>B
The sum of five consecutive whole numbers is divisible by 5.
Let the five numbers be: $n - 2$, $n - 1$, n, $n + 1$, and $n + 2$.
The sum is $5n$, which is always a multiple of 5.</td><td></td><td>A
The sum of four consecutive whole numbers is divisible by 4.
Let the four numbers be: $n - 1$, n, $n + 1$, and $n + 2$.
The sum is $4n + 2$, which is always half way between two multiples of four. So the sum is even but never a multiple of 4.</td></tr>
<tr><td colspan="2">C
To find the sum of consecutive whole numbers, find the middle number and then multiply it by how many numbers there are.
For an odd number of numbers, the middle number is the median and the mean, so the total must be this number multiplied by how many numbers there are.
For an even number of numbers, whether the conjecture is always true or not depends on how students interpret the phrase ‘middle number’. If they say that there is no middle number, then the rule in the conjecture cannot be followed. However, if they interpret ‘middle number’ as the mean of the two middle numbers, then the conjecture is still true, for the same reason.</td><td>D
The sum of five consecutive whole numbers is divisible by the sum of the first and last numbers.
Let the five numbers be: $n - 2$, $n - 1$, n, $n + 1$, and $n + 2$. The sum is $5n$, whereas the sum of the first and the last numbers is $2n$.
$5n/2n = 5/2$, which is not a whole number, so the conjecture is never true.</td></tr>
</table>

Always	Sometimes	Never
E **The sum of two consecutive whole numbers is an odd number.** Since odd and even numbers alternate, when you choose two consecutive numbers you always get one of each. Since odd + even = odd, this conjecture is always true.	**G** **If a number can be written as the sum of consecutive whole numbers in only one way, the number is prime.** This is true when the unique sum contains just two consecutive numbers (e.g. 7=3+4; 13= 6+7), but is not true where the unique sum uses more than two consecutive numbers (e.g. 6=1+2+3). The converse statement, that all prime numbers (excluding 2) can be written as the sum of consecutive whole numbers in only one way, is true.	
F **The sum of six consecutive whole numbers is divisible by the sum of the middle two numbers.** Let the six numbers be: $n-2$, $n-1$, n, $n+1$, $n+2$, $n+3$. The sum is $6n+3$. The sum of the two middle numbers is $2n+1$ and since $6n+3=3(2n+1)$, this conjecture is always true.		
H **Powers of 2 cannot be written as sums of consecutive whole numbers.** You certainly cannot make them with an odd number r of consecutive whole numbers, since if you could r would be a factor of the number, and powers of 2 have no odd factors (apart from 1). So let's suppose that we can make a power of 2 by summing an even number of consecutive whole numbers. Then, sum = average number × number of numbers = (½ × sum of two middle numbers) × number of numbers = sum of two middle numbers × (½× number of numbers) = odd number × whole number		

This means that the sum has an odd factor, but that is impossible, because powers of 2 have no odd factors (apart from 1). So our supposition that we can make a power of 2 by summing an even number of consecutive whole numbers was false.		

Consecutive Sums

The number 18 can be made by adding ***three*** *consecutive whole numbers*:

$$5 + 6 + 7 = 18$$

Which other numbers can be made by adding three consecutive whole numbers?

1. Experiment with some numbers and try to state a conjecture about which numbers can be made.

2. Try to prove your conjecture.

3. If you have time, try to state and prove another conjecture about sums of consecutive whole numbers.

Sample Responses to Discuss: Tim

Four consecutive numbers is either

$$\underbrace{\text{odd} + \text{even}}_{\text{odd}} + \underbrace{\text{odd} + \text{even}}_{\text{odd}} = \text{even}$$

or

$$\underbrace{\text{even} + \text{odd}}_{\text{odd}} + \underbrace{\text{even} + \text{odd}}_{\text{odd}} = \text{even}$$

So it has to be even.

1. Try to explain Tim's reasoning in your own words.

2. Does it convince you? Why / Why not?

Sample Responses to Discuss: Alex

Let the four numbers be

$$n-1, \quad n, \quad n+1, \quad n+2$$

$$\text{Sum} = \underbrace{4n}_{\text{even}} + 2 \qquad = \text{even}$$

even

1. Try to explain Alex's reasoning in your own words.

2. Does it convince you? Why / Why not?

Sample Responses to Discuss: Rumana

Multiple of 2?

$1+2+3+4=10$ ✓

$2+3+4+5=14$ $+4$ ✓

$3+4+5+6=18$ $+4$ ✓

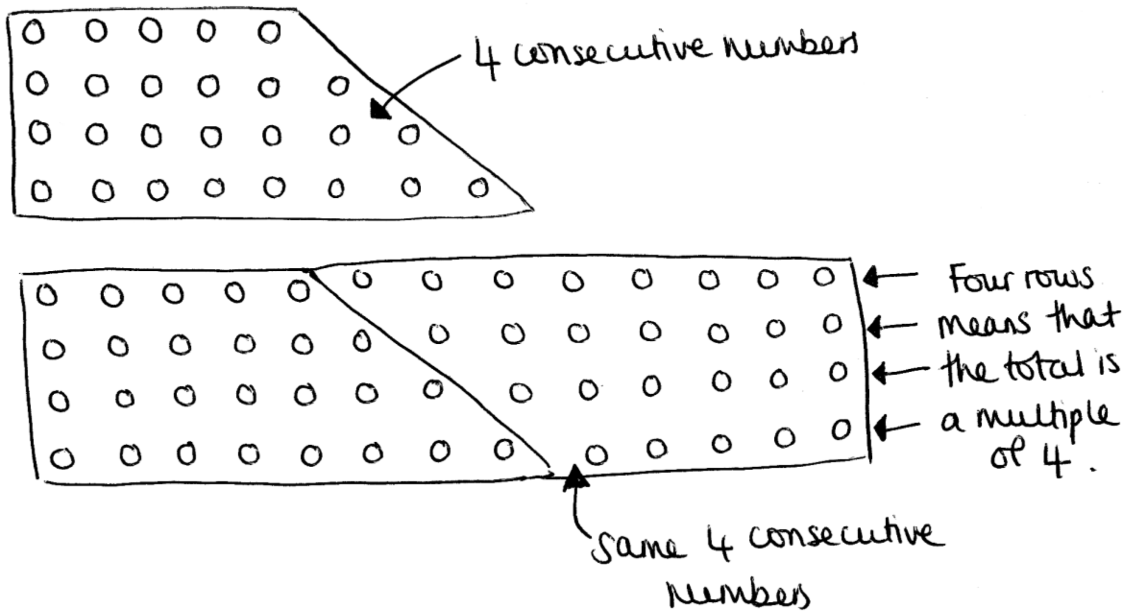
Every time, each of the four numbers on the left is 1 bigger, so the sum is 4 bigger. If we start with 10 and keep adding 4s then all the answers will be even.

1. Try to explain Rumana's reasoning in your own words.

2. Does it convince you? Why / Why not?

Sample Responses to Discuss: Steph

5 + 6 + 7 + 8



Since twice the sum of 4 consecutive numbers is a multiple of 4, the sum of 4 consecutive numbers must be a multiple of 2 (even).
This works for all cases!

1. Try to explain Steph's reasoning in your own words.

2. Does it convince you? Why / Why not?

Sample Conjectures to Discuss

A The sum of four consecutive whole numbers is divisible by 4.	**B** The sum of five consecutive whole numbers is divisible by 5.
C To find the sum of consecutive whole numbers, find the middle number and then multiply it by how many numbers there are.	**D** The sum of five consecutive whole numbers is divisible by the sum of the first and last numbers.
E The sum of two consecutive whole numbers is an odd number.	**F** The sum of six consecutive whole numbers is divisible by the sum of the middle two numbers.

Sample Conjectures to Discuss (Extension)

G If a number can be written as the sum of consecutive whole numbers in only one way, the number is prime.	**H** Powers of 2 cannot be written as sums of consecutive whole numbers.

Blank Cards

How Did You Work?

1. State the advantages and disadvantages of each of the following approaches to proving a conjecture about consecutive sums:

(i) Number properties (like, for example, Tim's work)

Advantages: ....

....

Disadvantages: ....

....

(ii) Algebra (like, for example, Alex's work)

Advantages: ....

....

Disadvantages: ....

....

(iii) Number patterns (like, for example, Rumana's work)

Advantages: ....

....

Disadvantages: ....

....

(iv) Diagrams (like, for example, Steph's work)

Advantages: ....

....

Disadvantages: ....

....

2. The approach I like best is (please circle) : Number properties / Algebra / Number patterns / Diagrams / Other: ______________

because: ....

....

Consecutive Sums

The number 18 can be made by adding ***three*** *consecutive whole numbers*:

5 + 6 + 7 = 18

Which other numbers can be made by adding three consecutive whole numbers?

Consecutive Sums 2

Conjecture:

The sum of ***four*** *consecutive whole numbers*

is always even.

5 + 6 + 7 + 8 = 26

Sample Responses to Discuss

In your groups:

1. Try to understand what the student has done. You may want to annotate the work/do some math on your mini-whiteboard.
2. Explain the students' reasoning in your own words for each piece of work (Q1).
3. Explain why each piece of work does or does not convince you (Q2).
4. Decide which of the approaches you have looked at you prefer. Be prepared to explain your choice.

Sample Responses to Discuss: Tim

Four consecutive numbers is either

$$\underbrace{\text{odd} + \text{even}}_{\text{odd}} + \underbrace{\text{odd} + \text{even}}_{\text{odd}} = \text{even}$$

or

$$\underbrace{\text{even} + \text{odd}}_{\text{odd}} + \underbrace{\text{even} + \text{odd}}_{\text{odd}} = \text{even}$$

So it has to be even.

Sample Responses to Discuss: Alex

Let the four numbers be

$n-1, \quad n, \quad n+1, \quad n+2$

$\text{Sum} = \underbrace{4n}_{\text{even}} + 2 \quad = \text{even}$

even

even

Sample Responses to Discuss: Rumana

Multiple of 2?

$1+2+3+4=10$ ✓

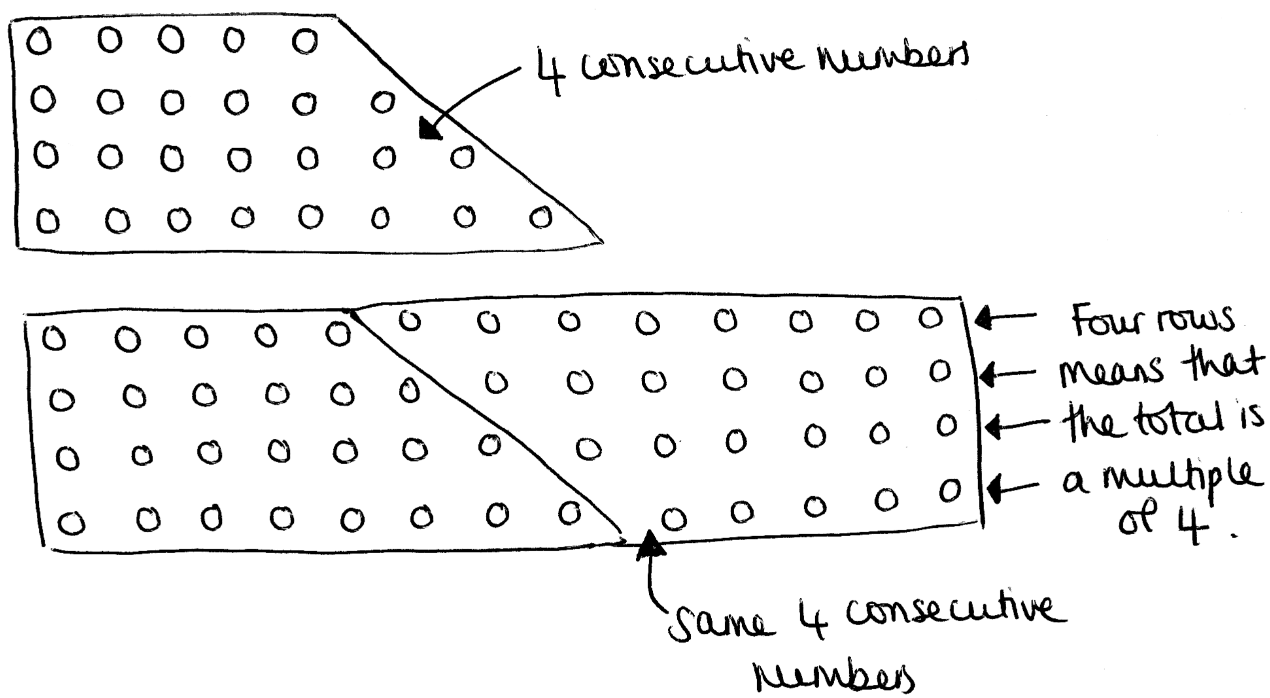
$2+3+4+5=14$ $+4$ ✓

$3+4+5+6=18$ $+4$ ✓

Every time, each of the four numbers on the left is 1 bigger, so the sum is 4 bigger. If we start with 10 and keep adding 4s then all the answers will be even.

Sample Responses to Discuss: Steph

$5 + 6 + 7 + 8$



Since twice the sum of 4 consecutive numbers is a multiple of 4, the sum of 4 consecutive numbers must be a multiple of 2 (even).
This works for all cases!

Always, Sometimes or Never True?

1. Decide whether each conjecture is Always, Sometimes or Never True.
   - If the conjecture is **Always** or **Never** true, write down how you can be sure.
   - If the conjecture is **Sometimes** true, write down when it is true and when it is not.
2. Use ideas from the sample work you have looked at and make notes on your mini-whiteboards.
3. Agree about each conjecture before moving on.
4. Explain your thinking on your poster.

Sample Conjectures to Discuss

A The sum of four consecutive whole numbers is divisible by 4.	**B** The sum of five consecutive whole numbers is divisible by 5.
C To find the sum of consecutive whole numbers, find the middle number and then multiply it by how many numbers there are.	**D** The sum of five consecutive whole numbers is divisible by the sum of the first and last numbers.
E The sum of two consecutive whole numbers is an odd number.	**F** The sum of six consecutive whole numbers is divisible by the sum of the middle two numbers.

Sample Conjectures to Discuss (Extension)

G	**H**
If a number can be written as the sum of consecutive whole numbers in only one way, the number is prime.	Powers of 2 cannot be written as sums of consecutive whole numbers.

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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