CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Comparing Strategies for Proportion Problems

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Comparing Strategies for Proportion Problems

MATHEMATICAL GOALS

This lesson unit is intended to help you assess whether students recognize relationships of direct proportion and how well they solve problems that involve proportional reasoning. In particular, it is intended to help you identify those students who:

- Use inappropriate additive strategies in scaling problems, which have a multiplicative structure.
- Rely on piecemeal and inefficient strategies such as doubling, halving, and decomposition and have not developed a single multiplier strategy for solving proportionality problems.
- See multiplication as making numbers bigger and division as making numbers smaller.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, 4, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on a task designed to reveal their current levels of understanding and difficulties. You review their work, writing questions to help them to improve.
- During the lesson, students first work in pairs or threes on the same task. Then working in the same small groups, they analyze work produced by other students on the task.
- In a whole-class discussion, students compare and evaluate the methods they have seen and used.
- Finally, students review their initial, individual response, using their learning to complete a new similar task.

MATERIALS REQUIRED

- Each individual student will need a mini-whiteboard, pen, and eraser, a calculator and a copy of the tasks *A Sense of Scale* and *A Sense of Scale (revisited)*.
- Each small group of students will need a large sheet of poster paper and a copy of each of the *Sample Responses to Discuss*.

TIME NEEDED

15 minutes before the lesson, a 60-minute lesson, and 15 minutes in a follow-up lesson. These timings are approximate. Exact timings will depend on the needs of your students.

BEFORE THE LESSON

Assessment task: *A Sense of Scale* (15 minutes)

Have the students complete this task in class or for homework a few days before the formative assessment lesson. This will give you a chance to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the *A Sense of Scale* task and a calculator.

> *Read through the questions and try to answer them as carefully as you can.*
>
> *Explain how you figured out your answers and record all your calculations.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Explain to students that they should not be concerned if they cannot complete everything in the task. In the next lesson they will work on this material, which should help them to make progress.

Students who sit together often produce similar answers and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the next lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

A Sense of Scale

1. Here is a recipe for making 4 pancakes:

6 tablespoons flour
¼ pint milk
¼ pint water
1 pinch salt
1 egg

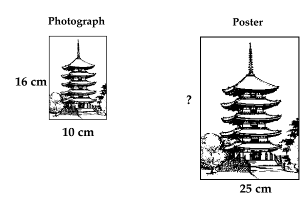
You want to make 10 pancakes.

a. How much flour do you need?

b. How much milk do you need?

2. Calculate the prices of the paint cans.
The prices are proportional to the amount of paint in the can.

3. The photograph is enlarged to make a poster.
The photograph is 10cm wide and 16cm high.



a. The poster is 25cm wide.
How high is the poster?

b. The building on the poster is 30cm tall.
Is it possible to figure out how tall the building is on the photograph?
If you think it is possible, show how. If you think it is not, explain why.

Assessing students' responses

Carefully read through students' responses to the task. Make notes about what the work shows you about their current levels of understanding and different solution strategies. This will forewarn you of issues that may arise during the lesson.

We suggest that you do not score students' work. The research shows that this will be counterproductive as it will encourage students to compare scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on page T-4.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues:	Suggested questions and prompts:
Uses mental or jotted strategies For example: The student has (correctly or incorrectly) calculated solutions but written little.	• Can you explain in more detail how you figured out your solution? • Help your reader to understand your solution.
Uses informal strategies For example: The student answers every question using a different calculation method. Or: The student has used doubling and halving with addition. Elion and Faith's *Sample Responses to Discuss* use these types of strategies.	• Can you think of a method that could be used for any quantity, e.g. 13 pancakes, a width of 22.5 cm? • Can you find a really efficient way of solving the problem?
Identifies the problem structure as additive rather than multiplicative (Q1, Q3) For example: The student adds the same number each time, rather than finding the scale factor and multiplying by the same amount each time. Gavin's solution, in the *Sample Responses to Discuss*, is an example of this misconception.	• (Q1) How much flour is needed for one pancake? How can you use this in your solution? • (Q3) Enlargement changes the size but not the shape of the rectangle. Draw the two rectangles from your solution. Are they similar? What is the effect of enlargement on side length?
Chooses inappropriate arithmetic operations For example: The student chooses to divide rather than multiply, perhaps thinking division makes things smaller, multiplication makes them bigger.	• Write some sentences to explain how to calculate the answer. • What size of answer do you expect? Why? Use a calculator to check your estimate.
Uses unit rate method For example: The student calculates the number of ounces of flour per pancake and then multiplies by the total number of pancakes.	• What is the scale factor? Can you use this number in your other calculations?
Uses method of cross multiplying proportions For example: The student specifies the proportional relationship between known and unknown quantities and cross-multiplies. E.g. $\frac{15}{1} = \frac{x}{2.5}$ (correct) or $\frac{1}{15} = \frac{x}{2.5}$ (incorrect) (Q4).	• (Q2) Which of these numbers are quantities of paint? Which are prices? What is the relationship between the two quantities of paint? What is the relationship between this price and that quantity of paint? • Can you explain why your method works? • Can you find a different way of calculating this answer?
Draws the outline of the poster accurately (Q3)	• How did you figure out the height of the poster? • How does drawing a poster accurately help?
Answers all problems correctly and efficiently	• Find at least two different, correct methods for solving these problems. Which do you prefer? Why? • Think about the three problems you have answered. Write down how they are different and how they are the same.

SUGGESTED LESSON OUTLINE

Improving individual solutions (5 minutes)

Give each student his or her script and a mini-whiteboard, pen, and eraser. Remind students of their work on *A Sense of Scale*.

Recall the problems you were working on in the last lesson. Today we are going to work together to improve that work.

First I have some questions about your individual solutions. I would like you to read through the questions I have written and spend a few minutes on your own improving your work. Use your mini-whiteboards.

If you have not added questions to students' work, display your list of questions on the board now.

Collaborative activity: producing small-group solutions (15 minutes)

Organize students into groups of two or three.

Give each group a large sheet of paper for making a poster to show their solutions.

Your task now is to come up with a really good, efficient method for solving each problem. Work together on one problem at a time.

Take turns to explain your method to others in the group. Listen carefully to each other. Ask questions if you don't understand or agree.

If you discuss more than one method, ***together*** *decide which method is best. Then, on the poster, write a complete solution using that method, explaining your reasoning.*

Before you move on to the next problem, make sure every person in your group understands and can explain the group's method.

Slide P-1 of the projector resource summarizes these instructions.

While students work in small groups you have two tasks: to note their different approaches to the task and to support student reasoning:

Note different approaches to the task

Listen and watch students carefully. Note different approaches to the task and any incorrect solutions. You will be able to use this information in the whole-class discussion.

- Do students incorrectly treat the problems as having an additive structure? For example, the student might argue that the number of pancakes increases by 6, so the number of tablespoons of flour will also increase by 6.
- Do students use doubling and halving with addition?
- Do students calculate unit rates? If so, which rate do they use (e.g. ounces of flour per pancake, or pancakes per ounce of flour)?
- Do any students use cross-multiplication with a proportion involving three known quantities and one missing quantity? If so, do they correctly organize the quantities to show their inter-relationships correctly? Can they explain why the method works?
- Do any students use multiplication by a scale factor? Are they successful? Can they explain why the method works?
- Are students checking that their answers are correct and that their explanations make sense?

Support student reasoning

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking. The questions in the *Common issues* table may be helpful.

To support students really struggling with a particular part of the task, you might hand out one or two of the *Sample Responses to Discuss*. If the whole-class is struggling with the same issue you could write a couple of relevant questions on the board, or hold a short whole-class discussion.

You may find students use relatively inefficient methods. For example, they may prefer doubling and halving with addition. This works well when dealing with fairly 'simple' numbers such as integers, but is hard to generalize to more 'difficult' numbers such as those with three decimal places.

Ask questions to develop students' thinking about their methods:

Why do you prefer this method?

Show me how to calculate the amount of flour needed for one pancake. How can you use this information to solve the problem?

Does your method work for calculating all the amounts and quantities? If not, can you think of one?

Can you think of a method to calculate any amounts including 'difficult' numbers such as 1.73?

How do you calculate a scale factor? How can you use the scale factor to solve the problem?

What is the unit rate? How can you use this to solve the problem?

In which problems do you prefer using scale factors/unit rates? Why?

Students might use an efficient strategy such as multiplying by a scale factor or setting up a proportion and cross-multiplying with little understanding of why their method works.

Why does your method work?

This number you're multiplying by - the scale factor - where does that come from? How does it connect these quantities/amounts?

Check that each member of the group understands and can explain each answer. If you find a student is struggling to respond to your questions, return to the group a few minutes later and check they have worked together on understanding.

If any students finish early, ask them to find a different way of solving one of the problems and to compare the efficiency of their different methods.

You may at this point want to hold a brief whole-class discussion. Focus on the variety of methods students used, any interesting ways of working and incorrect methods you have noticed. Encourage students to compare and evaluate different methods and to think about which method can be applied to any amount or quantity. Slides P-2 to P-4 of the projector resource may be helpful with this discussion.

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

Once the students have had time to tackle all the questions together, give each group a copy of all three *Sample Responses to Discuss*. This task gives students an opportunity to evaluate a variety of approaches to the scaling problems.

Here are some Sample Responses to the problems you've been working on, written by students in another class. I want you to review their work.

Choose one sample solution to work on together. Read through the work carefully. Note any errors in the student's solution and answer the questions on the sheet. Try to focus on reviewing the math issues, not effort or how neatly the work is written.

Make sure every person in your group understands and can explain your answers before moving on to the next sample solution.

During the small-group work, support the students as before. Note the explanations students struggle most to understand. Note similarities and differences between the *Sample Responses* and those the students took in the small-group work.

Whole-class discussion: comparing different approaches (20 minutes)

Focus the whole-class discussion on developing the key idea of this lesson: these problems all share the same structure and could all be solved in the same way.

Use your knowledge of the students' individual and group work to call on a wide range of students for contributions. You may want to draw on the questions in the *Common issues* table to support your own questioning.

To support this discussion there are Slides of each of the *Sample Responses to Discuss* (P-5 to P-7).

Although these problems all look very different, they have something in common.
What do you think is the same about all these problems? [They involve working with a proportional relationship between two quantities and can all be solved using the same methods.]

How has Eilon figured out the solution?

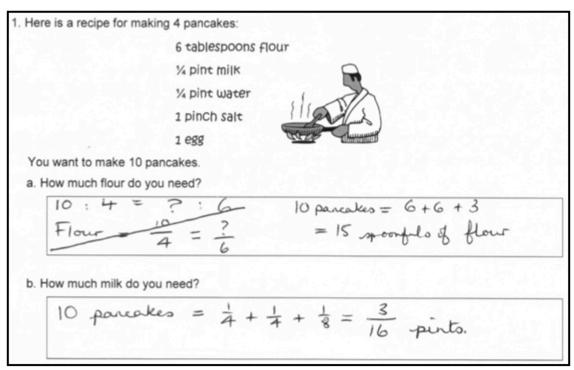
What mistakes has Faith made?

Did any of you use that method? A different method?

Which method did you use for this problem? Why did you prefer that method?

Did anyone use a method that can be applied to every single problem?

The following analysis of the *Sample Responses* may be helpful in supporting your questioning:


1. Here is a recipe for making 4 pancakes:
6 tablespoons flour
¼ pint milk
¼ pint water
1 pinch salt
1 egg
You want to make 10 pancakes.
a. How much flour do you need?
10 : 4 = ? : 6
~~Flour $\frac{10}{4} = \frac{?}{6}$~~
10 pancakes = 6 + 6 + 3
= 15 spoonfuls of flour
b. How much milk do you need?
10 pancakes = $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{3}{16}$ pints.

Eilon first attempts to use ratios, but abandons this method for an informal additive strategy. He has made a common error in his fractions calculation, adding the numerators to each other, then adding the denominators to each other, rather than finding a common denominator first and adding the numerators.

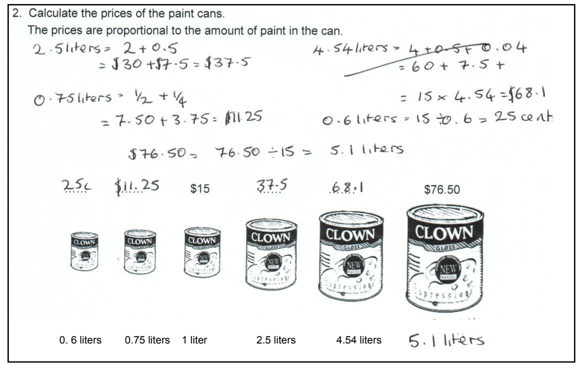
What method do you think Eilon was trying to use with a proportion?

Why do you think Eilon abandoned the work he started with proportion?

Did any of you use that method? Can you show us how to complete the work correctly?

What does $\frac{10}{4}$ represent? [The scale factor – the relationship between the number of pancakes and the scaled number of pancakes.]

Faith uses a different strategy for each paint can. She uses the informal additive strategy correctly for the 'easier' amounts, 0.75 liters and 2.5 liters. She then tries, unsuccessfully, to use this same method to calculate the price for 4.54 liters of paint. She then changes strategy and successfully uses a multiplicative strategy to figure out the price.



To calculate the price for 0.6 liters of paint, Faith has chosen to divide rather than multiply. This may be a symptom of the misconception that **division always makes numbers smaller and multiplying always makes numbers larger**. As the can is small, Faith assumes the answer is in cents.

To calculate the amount of paint in the largest can, Faith correctly divides $76.50 by 15.

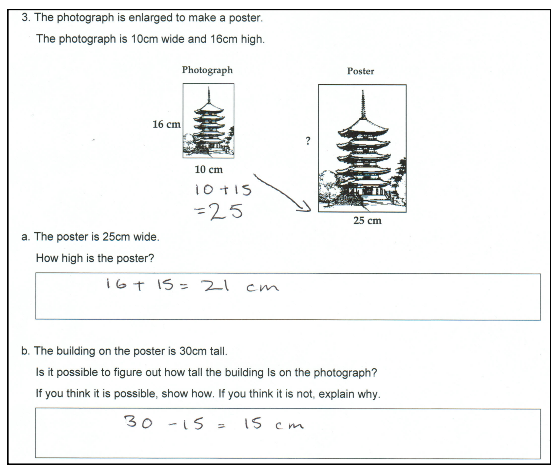
Faith could improve her answer by writing sums of money correctly, with two decimal places not one. She has also incorrectly used the equal sign.

Why has Faith deleted some of her work?

Why does Faith divide 76.50 by 15?

Is there one method that Faith could use for every single problem? [She could multiply each quantity by the same scale factor.]

Gavin incorrectly uses an additive strategy. He is not considering proportion, but using the difference between known lengths to calculate unknown ones. This is a common error in ratio problems.



What is the scale factor of enlargement?

How could Gavin use the scale factor to calculate the lengths?

Students may find it difficult to solve the second, reverse question. Some students are not convinced that lengths within pictures scale by the same factor.

Do you think the pagoda scales by the same factor as the side lengths?

Suppose I take the height of the pagoda in the poster and multiply it by the scale factor. Is that how you solve this problem? What's my deliberate error?

Follow-up lesson: *A Sense of Scale (revisited)* (15 minutes)

Give the students back their original scripts from the assessment task *A Sense of Scale,* along with a copy of the task *A Sense of Scale (revisited)*.

If you did not add questions to individual pieces of work, write your list of questions on the board again now.

Look at your original response and read through my questions. Answer these questions and revise your response.

Now, using what you have learned, try to answer the questions on the new task, A Sense of Scale (revisited).

Explain how you figure out your answers and record all your calculations.

Some teachers give this for homework.

SOLUTIONS

A Sense of Scale

These questions do not require a succinct or formal method and may elicit effective but inefficient use of repeated addition for multiplication or strategies involving doubling and halving with addition. Some methods are described in Question 1. They could be applied to the other two problems.

1. **Method A**

For 4 pancakes you need 6 tablespoons of flour and $\frac{1}{4}$ of a pint of milk.

So for 2 pancakes, you need 3 tablespoons of flour and $\frac{1}{8}$ pints of milk.

So for 10 pancakes, you need 3 + 3 + 3 + 3 + 3 =15 tablespoons of flour

and $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$ of a pint of milk.

Method B

The number of pancakes increases from 4 to 10. This gives a scale factor of $\frac{10}{4} = 2.5$.

The quantity of flour for ten pancakes: $2.5 \times 6 = 15$ tablespoons.

The quantity of milk for ten pancakes: $2.5 \times 0.25 = 0.625$ or $\frac{5}{8}$ pints.

Method C

The quantity of flour per pancake is $6 \div 4 = 1.5$ tablespoons.

The quantity of milk per pancake is $\frac{1}{4} \div 4 = \frac{1}{16}$ pints.

For 10 pancakes, you will need $10 \times 1.5 = 15$ tablespoons of flour and $\frac{1}{16} \times 10 = \frac{5}{8}$ pints of milk.

Method D

The ratio of pancakes to flour: $4:6$. This ratio stays constant as you change the number of pancakes. Scaling the number of pancakes: $10 = 2.5 \times 4$.

The amount of flour is to be scaled by the same factor to keep the ratio constant: $x = 2.5 \times 6 = 15$.

For 10 pancakes you need 15 tablespoons of flour.

Using the same form of reasoning, you need $0.625 = 2.5 \times 0.25$ pints of milk for ten pancakes.

2. Again, students may use a range of strategies to solve this problem correctly.

Using the unit rate of $15 per liter is a powerful, economical, and simple strategy:

1 liter costs $15.
0.6 liters costs $15 × 0.6 = $9
0.75 liters costs $15 × 0.75 = $11.25.
2.5 liters costs $15 × 2.5 = $37.50.
4.54 liters costs $15 × 4.54 = $68.10.

Students may find it more difficult to reverse the unit rate to find the volume of paint. The can costing \$76.50 contains $76.50 \div 15 = 5.1$ liters.

3. Again students may use a range of methods, including doubling and halving with addition.

   Using the unit rate is an efficient strategy.

   Scale factor: $25 \div 10 = 2.5$.
   Height of poster: $16 \times 2.5 = 40$ cm.
   Height of building in the photograph is $30 \div 2.5 = 12$ cm.

A Sense of Scale (revisited)

The questions in this task are structurally similar to those in the first task. Only the numbers have changed. As with the first assessment task, what is most important here are the methods students choose to use and whether they can implement those methods effectively.

Compare students' responses on the two versions of the task.
Have any students changed the methods they use?
Have any improved in use of chosen methods?
We provide correct numerical answers for convenience.

1. The scale factor is $\frac{28}{8} = 3.5$.

   a. 14 cups of flour.

   b. $1\frac{3}{4}$ cups of milk.

2. The unit rate is \$12 per liter.

   0.25 liters cost \$3.00.
   0.7 liters cost \$8.40.
   2.5 liters cost \$30.00.
   3.52 liters cost \$42.24.

   A can that costs \$57.60 contains 4.8 liters of paint.

3. The scale factor is 4.5

   a. The height of the poster is 54 cm.

   b. The height of building in the photograph is 8 cm.

A Sense of Scale

1. Here is a recipe for making 4 pancakes:

6 tablespoons flour

¼ pint milk

¼ pint water

1 pinch salt

1 egg

You want to make 10 pancakes.

a. How much flour do you need?

b. How much milk do you need?

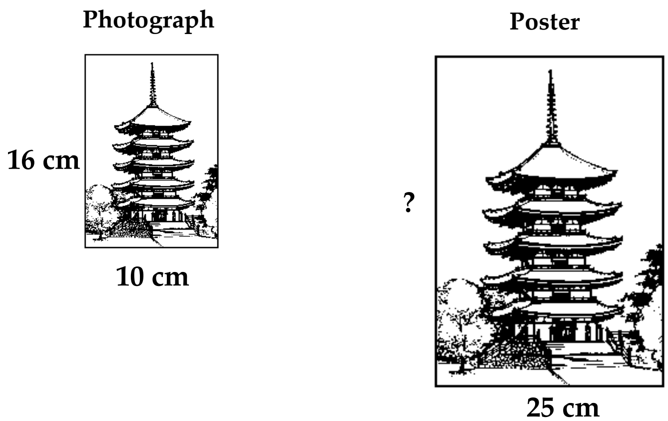
2. Calculate the prices of the paint cans.
The prices are proportional to the amount of paint in the can.

!!!!!!!!

……	……	$15	……	…….	$76.50
0.6 liters	0.75 liters	1 liter	2.5 liters	4.54 liters	......

3. The photograph is enlarged to make a poster.

The photograph is 10cm wide and 16cm high.



a. The poster is 25cm wide.

How high is the poster?

b. The building on the poster is 30cm tall.

Is it possible to figure out how tall the building is on the photograph?

If you think it is possible, show how. If you think it is not, explain why.

Sample Responses to Discuss: Eilon

1. Here is a recipe for making 4 pancakes:

6 tablespoons flour

¼ pint milk

¼ pint water

1 pinch salt

1 egg

You want to make 10 pancakes.

a. How much flour do you need?

$\cancel{10 : 4 = ? : 6}$

$\cancel{\text{Flour} = \frac{10}{4} = \frac{?}{6}}$

10 pancakes = $6 + 6 + 3$
= 15 spoonfuls of flour

b. How much milk do you need?

10 pancakes = $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{3}{16}$ pints.

Explain Eilon's method.

What mistakes has Eilon made?

At first Eilon tries to use a proportion to solve the problem. Show how you can use a proportion to calculate the amount of flour correctly.

Sample Responses to Discuss: Faith

2. Calculate the prices of the paint cans.
   The prices are proportional to the amount of paint in the can.

2.5 liters = 2 + 0.5
= $30 + $7.5 = $37.5

4.54 liters = 4 + ~~0.5 +~~ 0.04
= 60 + 7.5 +

0.75 liters = ½ + ¼
= 7.50 + 3.75 = $11.25

= 15 × 4.54 = $68.1

0.6 liters = 15 ÷ 0.6 = 25 cent

$76.50 = 76.50 ÷ 15 = 5.1 liters

25c | $11.25 | $15 | 37.5 | 68.1 | $76.50

0. 6 liters | 0.75 liters | 1 liter | 2.5 liters | 4.54 liters | 5.1 liters

Why do you think Faith used lots of different methods?

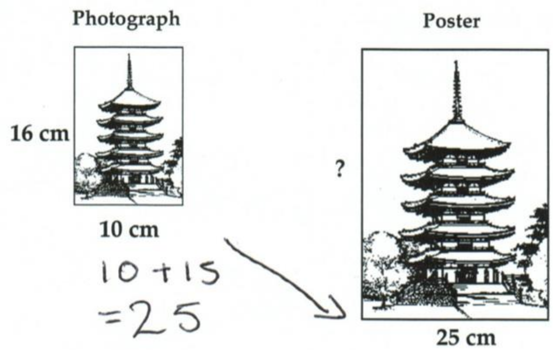
What mistakes has Faith made?

Find a method Faith could use to figure out answers to all the parts of the question.

Sample Responses to Discuss: Gavin

3. The photograph is enlarged to make a poster.

   The photograph is 10cm wide and 16cm high.



a. The poster is 25cm wide.

   How high is the poster?

16 + 15 = 21 cm

b. The building on the poster is 30cm tall.

   Is it possible to figure out how tall the building Is on the photograph?

   If you think it is possible, show how. If you think it is not, explain why.

30 - 15 = 15 cm

What mistakes has Gavin made?

Can you think a better method Gavin could use?

A Sense of Scale (revisited)

1. Here is a recipe for making 8 doughnuts:

4 cups of flour

1/2 cup of milk

3/4 cup of sugar

2 eggs

2 sticks of butter

One tablespoon of yeast

You want to make 28 doughnuts.

a. How much flour do you need?

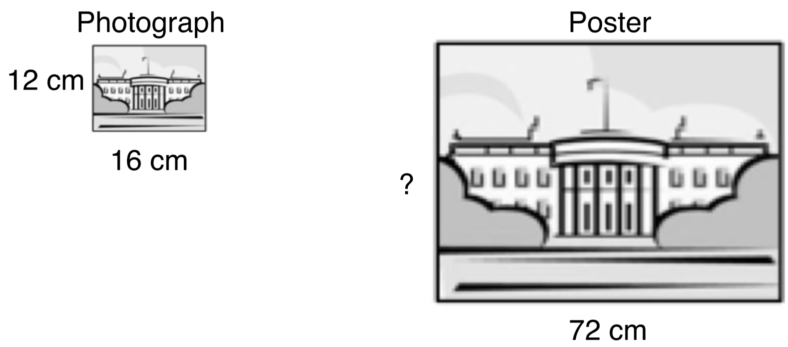
b. How much milk do you need?

2. Calculate the prices of the paint cans.
The prices are proportional to the amount of paint in the can.

!!!!!!!!!

……	……	$12	……	……	$57.60
0.25 liters	0.7 liters	1 liter	2.5 liters	3.52 liters	......

3.



a. The poster is 72 cm wide.

How high is the poster?

b. The building on the poster is 36 cm tall.

Is it possible to figure out how tall the building is on the photograph?

If you think it is possible, show how. If you think it is not, explain why.

Working Together

Find a good, efficient method for solving each problem.

Work together on one problem at a time.

- Take turns to explain your method to your group.
  - Listen carefully.
  - Ask questions if you do not understand or agree.
- If you discuss more than one method, **together** decide which way you all think is best.
- Write your solution on the poster and explain it.

Before you move on to the next problem,
make sure that everyone in your group can explain the group's method.

Recipe

1. Here is a recipe for making 4 pancakes:

6 tablespoons flour

¼ pint milk

¼ pint water

1 pinch salt

1 egg

You want to make 10 pancakes.

a. How much flour do you need?

b. How much milk do you need?

Paint Prices

2. Calculate the prices of the paint cans.
   The prices are proportional to the amount of paint in the can.

……	……	$15	……	…….	$76.50

0.6 liters	0.75 liters	1 liter	2.5 liters	4.54 liters	……

Enlarging a Poster

3. The photograph is enlarged to make a poster.
   The photograph is 10cm wide and 16cm high.

Photograph

16 cm

10 cm

Poster

?

25 cm

a. The poster is 25 cm wide, how high is the poster?

b. The building on the poster is 30 cm tall.
   Is it possible to figure out how tall the building is on the photograph?
   If you think it is possible, show how. If you think it is not, explain why.

Sample Responses to Discuss: Eilon

1. Here is a recipe for making 4 pancakes:

6 tablespoons flour

¼ pint milk

¼ pint water

1 pinch salt

1 egg

You want to make 10 pancakes.

a. How much flour do you need?

$\cancel{10 : 4 = ? : 6}$

$\cancel{\text{Flour} = \frac{10}{4} = \frac{?}{6}}$

10 pancakes = 6 + 6 + 3

= 15 spoonfuls of flour

b. How much milk do you need?

10 pancakes $= \frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{3}{16}$ pints.

Sample Responses to Discuss: Faith

2. Calculate the prices of the paint cans.

   The prices are proportional to the amount of paint in the can.

2.5 liters = 2 + 0.5
= $30 + $7.5 = $37.5

~~4.54 liters = 4 + 0.5 + 0.04~~
~~= 60 + 7.5 +~~

0.75 liters = 1/2 + 1/4
= 7.50 + 3.75 = $11.25

= 15 × 4.54 = $68.1

0.6 liters = 15 × 0.6 = 25 cent

$76.50 = 76.50 ÷ 15 = 5.1 liters

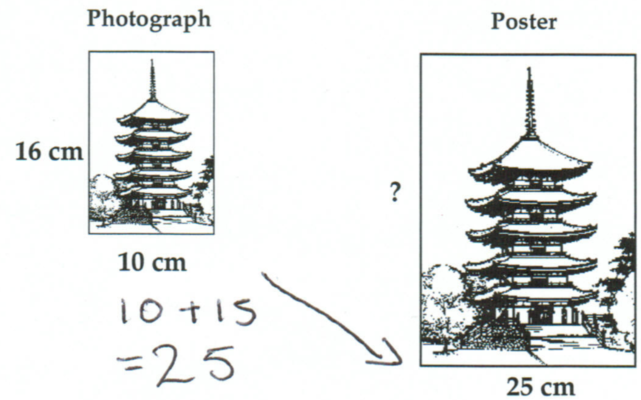
25c	$11.25	$15	37.5	68.1	$76.50

0. 6 liters	0.75 liters	1 liter	2.5 liters	4.54 liters	5.1 liters

Sample Responses to Discuss: Gavin

3. The photograph is enlarged to make a poster.

The photograph is 10cm wide and 16cm high.



10 + 15

= 25

a. The poster is 25cm wide.

How high is the poster?

16 + 15 = 21 cm

b. The building on the poster is 30cm tall.

Is it possible to figure out how tall the building Is on the photograph?

If you think it is possible, show how. If you think it is not, explain why.

30 − 15 = 15 cm

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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The full collection of Mathematics Assessment Project materials is available from
http://map.mathshell.org