PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

# Finding the Shortest Route: *A Schoolyard Problem*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

# Finding the Shortest Route: *A Schoolyard Problem*

## MATHEMATICAL GOALS

This lesson unit is intended to help students to:

- Select appropriate mathematics to solve a problem.
- Compare and evaluate different methods for solving a problem and make generalizations about the appropriateness of different approaches.
- Understand the Pythagorean Theorem and how it can be used to solve problems in the real world.

## COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics,* with particular emphasis on Practices 1, 3, 4, 5, 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.G: Understand and apply the Pythagorean Theorem.

## INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt *The Schoolyard Problem* individually. You review these initial attempts, formulating questions that will help students to improve their work.
- After a brief lesson introduction, students respond individually to the questions on their work.
- Then, in groups of 2 or 3, students compare their different approaches and examine and comment on some sample student responses. They identify features of these responses that may help them with their own work.
- In the same small groups, students now work together to produce a collaborative solution in the form of a poster.
- In a whole-class discussion, students explain and compare the strategies they have seen and used.
- Finally, students reflect on their work and their learning.

## MATERIALS REQUIRED

- Each student will need a copy of the task *The Schoolyard Problem*.
- Each small group of students will need a sheet of poster paper, a marker, and copies of the *Sample Responses to Discuss*.
- Provide calculators, rulers, and squared/graph paper for students who choose to use them.
- There is a projector resource to support whole-class discussion.

## TIME NEEDED

20 minutes before the lesson, a 100-minute lesson (or split into two shorter lessons) and 10 minutes in a follow-up lesson (or for homework). Timings given are approximate. Exact timings will depend on the needs of your class.

## BEFORE THE LESSON

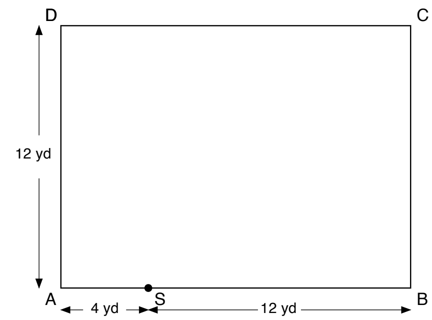
### Introducing the task: *The Schoolyard Problem* (20 minutes)

Ask the students to do this task, in class or for homework, a day or more before the lesson. This will give you an opportunity to assess their work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

**The Schoolyard Problem**

Some children are playing a game in a rectangular schoolyard ABCD that is 16 yards by 12 yards.

The diagram shows the schoolyard viewed from above.




The children start at point S, which is 4 yards along the 16-yard wall AB.
They have to run and touch each of the other three walls and then get back to S.
The first person to return to S is the winner.

What is the shortest route for them to take?

Explain how you know this is the shortest path.

Make sure that students understand the diagram and can imagine the scenario. Slide P-1 is available, showing the diagram.

You could ask:

*What games have you played in a schoolyard, perhaps when you were younger?*

*In this task, some children have invented their own game, where they start at point S and have to touch every wall and get back to where they started.*

*Each child wants to run the shortest distance possible so that they will win the game. It is your job to work out the shortest route.*

*Read the problem carefully and try to answer the questions as well as you can. Show all your working out so that I can understand your reasoning.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. If students are struggling to get started, then ask questions that help them understand what is required, but make sure you do not do the task for them.

Students who sit together often produce similar solutions so that, when they compare their work, they have little to discuss. For this reason we suggest that when students do the task individually you ask them to move to different seats. Then, at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

### Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their problem solving strategies.

We strongly suggest that you do not score students' work. Research suggests that this will be counterproductive, as it will encourage students to compare scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We recommend that you:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students at the beginning of the lesson.

| **Common issues:** | **Suggested questions and prompts:** |
|---|---|
| **Overlooks or misinterprets some constraints**<br>For example: The route does not allow a child to touch every wall or does not return to S. | • Read through the problem again. What are the rules of the game? |
| **Settles on a solution that satisfies the constraints but is inefficient**<br>For example: The student's path goes around the perimeter of the rectangle. | • How could you check that this is the shortest path? |
| **Uses measurement to get approximate distances**<br>For example: The student uses a scale drawing to estimate the length of one or more paths. | • Measuring is a good way to start. Can you think of a way of calculating the distances more precisely? |
| **Work is not supported by calculations**<br>For example: The student makes incorrect assumptions about which route is the shortest due to omitting to calculate the length of the path(s). | • How could you calculate the length of each path?<br>• Can you use math to show that you have found the shortest distance travelled? |
| **Makes incorrect assumptions**<br>For example: The student assumes that a triangular path will be shorter than a quadrilateral as there are fewer 'sides' to travel along.<br><br>Or: Assumes that triangle SCX (where X is 4 yards from A on side AD) is isosceles. | • Can you show me a path that is triangular and a path that is a quadrilateral? How do the lengths of the sides of the triangle in this path compare to the lengths of the sides of the quadrilateral?<br>• What do you notice about the lengths XC & CS? What does this tell you about triangle SCX? |
| **Does not use straight paths**<br>For example: The student draws curved paths. | • What would happen if you replaced this curve by a straight line? Would the path be longer or shorter? Why? |
| **Student completes the task** | • How can you be sure that your route is the shortest route possible?<br>• What would happen if the point S were in a different place?<br>• What would happen if the schoolyard were a different size or shape? |

## SUGGESTED LESSON OUTLINE

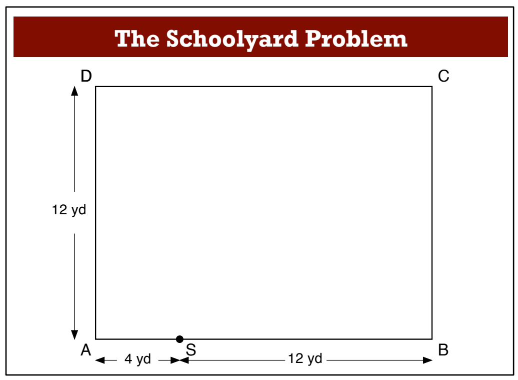
It is likely that students will have already used a variety of methods when attempting to work out the shortest path. The aim of this lesson is to allow students the opportunity to consider the pros and cons of different approaches and to explore ways of generalizing the problem by going beyond specific cases.

### Whole-class introduction (10 minutes)

Remind the students of the task:

*Try to recall 'The Schoolyard Problem'.*
*Describe to your partner what you can remember about the problem.*

Display Slide P-1 showing the diagram of the schoolyard:




*Can someone come and draw on the board what they think the shortest route might be?*
*Can someone else draw a route that they think might be shorter?*

Have several students draw their possible paths.

*Which one of these do you think might be the shortest route? Why?*

Avoid getting into measurements or calculations at this stage. See what students' intuitive feelings are about the kinds of routes they think might be best. Hopefully there will be some disagreement.

*We need to be sure about which route is the shortest. Today we are going to compare different approaches for determining the shortest route.*

### Reviewing individual solutions to the task (15 minutes)

Return the students' work on *The Schoolyard Problem*.

If you have not added questions to their work, write a short list of your most common questions on the board. Students can then select a few questions appropriate to their own work.

*I have looked at your work and I have some questions for you.*

*I would like you to think, on your own, about my questions and how your work could be improved.*

Students may want to jot down their ideas as they consider how to improve their work. They can write directly on their original work using a different colored pen or could use another piece of paper.

### Collaborative activity: comparing approaches (15 minutes)

Organize students into groups of two or three. You could deliberately put together students who have taken different approaches.

*You each have your own solution and have been thinking about how you might improve this.*

*I want you to share your work with your partner(s).*

*Take turns to explain how you approached the task and how you think you could now improve your solution. Make some notes of your ideas for improving your work, giving reasons.*

Slide P-2 summarizes these instructions:

**Collaborative Task**

- Share your work with your partner(s).
- Take turns to explain how you approached the task.
- Talk about how you think you could now improve your solution.
- Make some notes of ideas for improving your solution including reasons why.

Give students time to look at each other's work and to explain their methods to one other. Encourage them to compare approaches and identify their advantages and disadvantages.

### Extending the lesson over two days

If you are taking two days to complete the unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of their previous work before moving on to the next collaborative activity.

## Collaborative analysis of *Sample Responses to Discuss* (25 minutes)

Give each group of students a copy of each of the *Sample Responses to Discuss*. It may not be appropriate, or there may not be enough time, for all students to analyze all three sample responses. Each response highlights different features and so, depending on the approaches that students have taken, it may be appropriate to issue different sample responses to different groups.

*In your groups you are now going to look at some student work on the task. Notice in which ways this work is similar to yours and in which ways it is different. Compare the different approaches and think about the advantages and disadvantages of each approach.*

*There are some questions for you to answer as you look at the work. You may want to add annotations to the work to make it easier to follow or do some math on a blank piece of paper.*

Slide P-3 summarizes these instructions:

**Sample Responses to Discuss**

- Look at each piece of student work.
- Answer the questions on the sheet.
- Compare each piece of work to your own work on the task.
- Consider the advantages and disadvantages of each approach.

(Note that, when photocopying, inaccuracies may creep in, and each large square on Malika's grid may no longer be exactly one centimeter. If this is the case, forewarn your students that checking the lengths by measurement will not be useful.

Some students may have looked at triangular paths only or may not have used the Pythagorean Theorem to calculate the lengths of sections of their routes. The sample work has been found useful to introduce new ideas and approaches, where these have not been considered by students. As students examine the sample responses, encourage them to evaluate the relative merits of the different approaches:

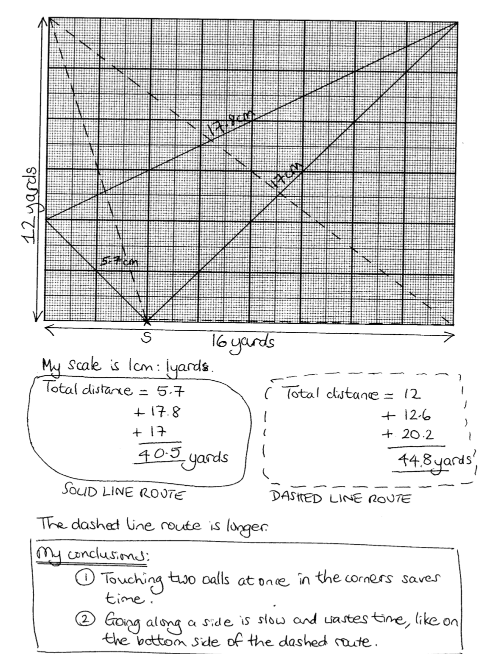
> *What are the advantages of measuring routes on a scale drawing? What are the disadvantages of this approach? [Quick and efficient, but inaccurate]*
>
> *What are the advantages and disadvantages of using the Pythagorean theorem? [Time consuming, but accurate].*
>
> *Both Malika and Robert look at triangular routes. In what ways are their approaches different? [Malika used measurement. Robert relied on algebraic reasoning]*
>
> *When might measuring lengths on a scale drawing be a better approach than using the Pythagorean Theorem? [Early on in the problem, when there are larger differences in path lengths]*
>
> *When might using the Pythagorean Theorem be a better approach? [Later on, when discriminating between path lengths that are nearly equal]*
>
> *What do you notice about the routes Malika, Paul and Robert have considered?*
>
> *How does Malika's/Paul's/Robert's work compare with your own work on the task?*
>
> *In what way has looking at Malika's/Paul's/Robert's work helped you with your own solution?*

**Malika's** routes involve running to one or more of the corners. This may be helpful in suggesting to students the possibility of using a corner, where two sides can quickly be touched.

She has drawn the schoolyard to scale and measured her distances in centimeters rather than calculating the lengths. This can be a good way for students to begin and allows them to find out the rough length of particular routes, but it is not completely accurate. With some very close routes, it will be impossible for students to be sure by measuring which is shorter.

Malika's conclusions are not fully warranted by her two isolated examples. Even if she has done other examples on another piece of paper, she still has not demonstrated that the optimal route will include children running to a corner and not running along part of a side. Her conclusions could be useful, however, as conjectures that could lead her on to further work.

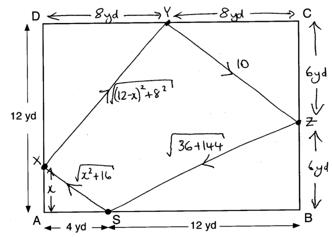
**Sample Response to Discuss: Malika**



**Paul** simplifies the problem by fixing two of the positions on the perimeter (Y and Z), turning the situation into a one-variable problem. Two of the legs of his route can be calculated precisely using the Pythagorean Theorem, but the other two depend on the position of the point X, which Paul describes using the length *x* from A along side AD. This gives him expressions for the lengths of the remaining two legs of the route. Substituting different values of *x* into these expressions by hand would be tedious, so he takes advantage of technology by using a spreadsheet. (Alternatively, he could have used the 'table' function on a graphing calculator.)

His solution is more systematic than Malika's, since he tries every integer value of *x* from 1 to 12. However, his conclusion that $x = 4$ is the minimum, although correct, is not warranted by the calculations that he has done, since he does not know that there is not a better **non-integer** value. He also needs to explore the effect of moving points Y and Z to other positions.

### Sample Response to Discuss: Paul




I fixed the positions of Y and Z so they were at the middles of their sides. But X can be anywhere on the left side.

So total length of journey

$$= 10 + \sqrt{180} + \sqrt{x^2+16} + \sqrt{(12-x)^2+8^2}$$

I used a spreadsheet to calculate the values:

| x | total length of journey |
|---|---|
| 1 | 41.140984 |
| 2 | 40.69479229 |
| 3 | 40.45800244 |
| 4 | 40.38697061 |
| 5 | 40.44967792 |
| 6 | 40.62751042 |
| 7 | 40.91264675 |
| 8 | 41.30495168 |
| 9 | 41.80926941 |
| 10 | 42.43294873 |
| 11 | 43.18336552 |
| 12 | 44.06551851 |

← $x = 4$ is the smallest distance

**Robert** has compared two triangular routes both of which take advantage of the corners of the schoolyard, in allowing a child to touch two walls from the same position.

Whilst the values of *a*, *b* and *d* could be calculated using, for example, the Pythagorean Theorem, Robert has generalized the problem and found a result that would stand true for any size of rectangular schoolyard.

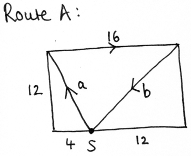
When comparing the magnitudes of '4 + *b*' and '*d*', Robert uses the fact that in any triangle the length of each side is less than the sum of the other two sides and so can conclude that the length of the diagonal must be less than '4 + *b*'.

Whilst determining which of the two routes A and B is shortest, Robert does not check that this route is the shortest of all possible routes. Whilst it seems clear that he has chosen to look at routes that make use of the corners of the schoolyard, he does not justify his choice of these particular two routes or why he has chosen not to look at other routes involving the corners, for example, the route outlined in his shaded triangle diagram, which is indeed a shorter route than route B.

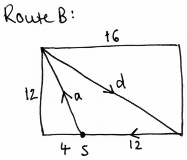
### Sample Response to Discuss: Robert

Let's compare two routes:

Route A:




Route B:




Which route is shortest?

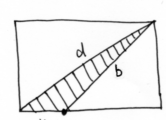
A = a+16+b　　B = a + d + 12

"a+12" is common to both, so can ignore that.

The question is

4+b versus d

Now d is the diagonal of the rectangle, so we can draw it the other way:




If you think about the shaded triangle,

d < 4+b

so route B is shorter than route A.

## Collaborative activity: making posters (20 minutes)

Give each group a piece of poster paper and a marker pen.

Ask students to have another go at the task but, this time, combining their ideas and using what they have learned by analyzing sample student responses to produce a joint solution in the form of a poster.

> *Together, in your group, produce a poster that shows a joint solution to the task that is better than your initial individual responses.*
>
> *Give clear reasons for your choice of method.*

Slide P-4 summarizes these instructions:

**Producing a Joint Solution**

- Together, in your group, produce a poster that shows a joint solution to the task, which is better than your individual responses.
- Give clear reasons for your choice of method.
- Justify how you know you have found the shortest route.

While students work in small groups, you have two tasks: to note different approaches to the task and to support student problem solving.

### Note different student approaches to the task

Listen and watch students carefully and note different student approaches to the task. Do they measure distances? Do they use the Pythagorean Theorem? Do they make use of triangle and/or quadrilateral properties? Do they use more than one method within their solution approach? What sorts of routes do they try? Do they describe their method and solution effectively? Do they show that their chosen route is the shortest? If so, how? Do they make any generalizations and if so what are they?

### Support student problem solving

Try not to make suggestions that move students towards a particular solution. Instead, ask questions that help students to clarify their thinking. You could ask them to consider these questions:

> *In your groups, what have you done individually that you agree on?*
>
> *Do all of your calculations make sense?*
>
> *How confident are you of your conclusions? Why?*
>
> *In what ways have the sample responses helped you to clarify your thinking? How can you use what you have learned to help you to produce a solution that is better than what you produced before?*

The purpose of these questions is to help students to track and review their problem solving strategies. They should be encouraged to give reasons for the choices they make.

### Whole-class discussion: comparing different solution methods (15 minutes)

Discuss the different routes that students have used and any calculations that they have done.

*What were you thinking as you were trying to find the shortest route?*

*What was the best route that you found?*

*Do you think that it is the shortest route possible? Why / Why not?*

*Did anyone find a shorter route?*

*How did you measure your routes?*

*Did anyone use an approach like Malika's/Paul's/Robert's?*

*Did anyone use a different approach? What approach did you take? Why?*

*Did anything surprise you?*

You may want to use Slides P-5 to P-10, which contain the sample work.

### Follow-up lesson: individual reflection (10 minutes)

Give each student a copy of the questionnaire *How Did You Work?*

*Think carefully about your work on this task and the different methods you have seen and used.*

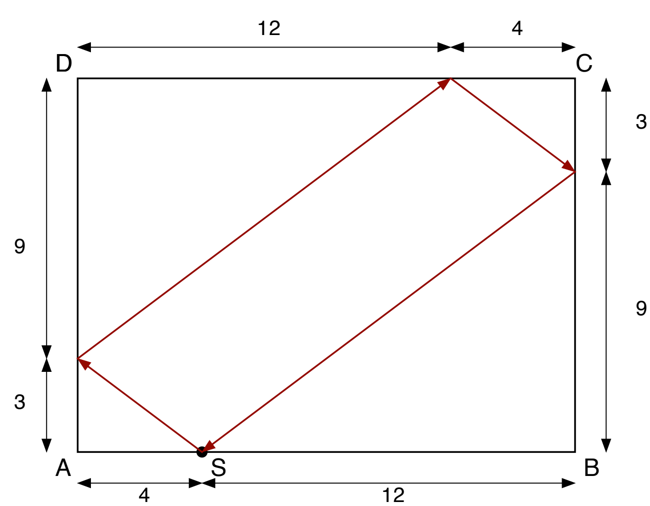
*On your own, answer the review questions as carefully as you can.*

Some teachers give this as a homework task.

## SOLUTIONS

It is likely that not all students will find the shortest route. The purpose of this task is not primarily to obtain the right answer, but to discuss and compare alternative routes and methods.

However, the shortest route is as follows:

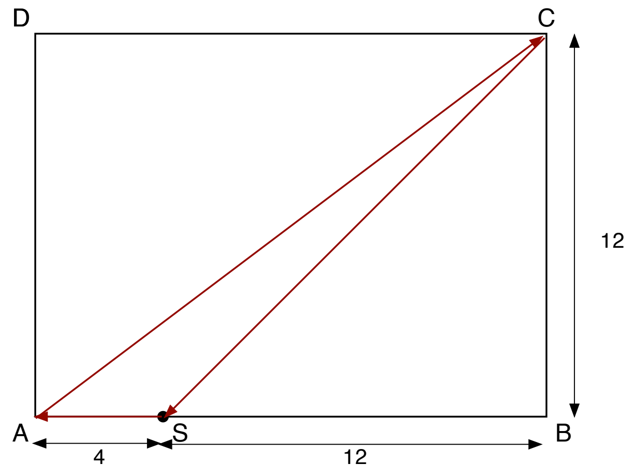



giving a total distance of 40 yards. (Of course, it would work just as well going counterclockwise around the parallelogram, rather than clockwise).

Interestingly the length of this route, 40 yards, is the minimum length possible for any position of S. If students have extended the problem to consider the position of S being moved to different positions along side AB, for example, it is likely that they will have considered what happens if S is moved to the corner of the schoolyard (to A or B) and realized that the solution is easy and intuitive – travel twice across the diagonal that passes through that corner.

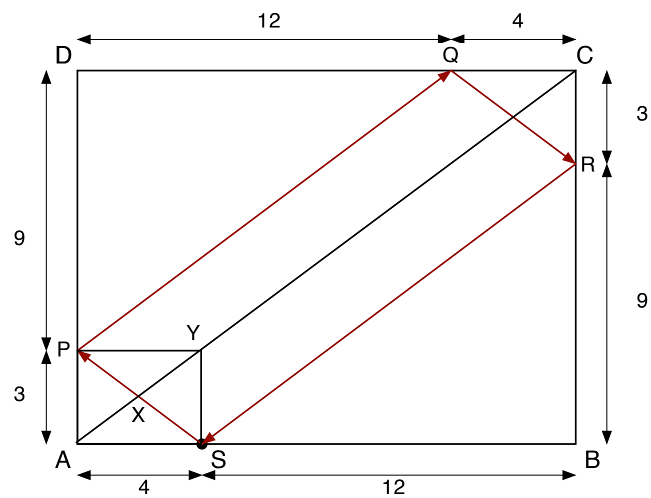
Can they see why the position of S does not affect the shortest possible distance?

Some students may conclude that the shortest route is the following, giving a total distance of 40.97 yards (2 d.p.).




Whilst this is slightly further than the optimum route of 40 yards, students might argue that when actually playing the game, it would be incredibly difficult to judge distances along the sides of the schoolyard and so in reality, being able to run to the corner of the schoolyard may actually result in a child returning to S quicker (especially as it is only about 1 yard further than the optimum solution).

In one classroom, a student tried many routes then conjectured the parallelogram. He justified this as the shortest route by the following ingenious reasoning. (This needs a few additional steps before becoming a full proof.)




*If we could start just anywhere, then it is clear that the shortest path that touches all four walls is to run along the diagonal AC and back.*

*Now draw the parallelogram PQRS as shown, with PQ and SR parallel to the diagonal AC.*

*Draw the rectangle PASY.*

*SP = AY (=QR) (Diagonals of a rectangle are equal).*

*PQ = YC (=RS) (Opposite sides of parallelograms PQCY, YCRS are equal)*

*So, SP + PQ = AY + YC = AC*

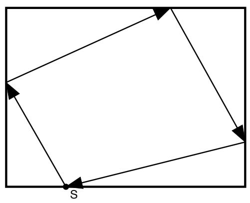
*So, Total path length = SP + PQ + QR + RS = 2(AY + YC) = twice the diagonal.*

*Since twice the diagonal is the shortest path if we could start anywhere, SPQR must also be a shortest path.*

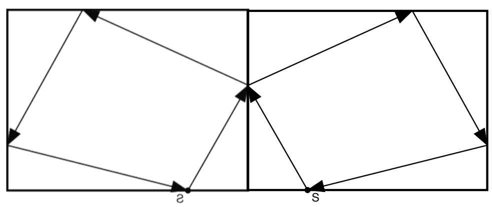
For an optimum triangular route and to allow all four walls to be touched, the triangular route must include the diagonal of the rectangle. As the sum of the lengths of the other two sides of the triangle will always be greater than the length of the diagonal, the perimeter of the triangle will always be greater than the perimeter of the parallelogram route (twice the length of the diagonal).

For some students it will be intuitive that all other quadrilaterals will have a perimeter that is greater than the perimeter of the parallelogram.

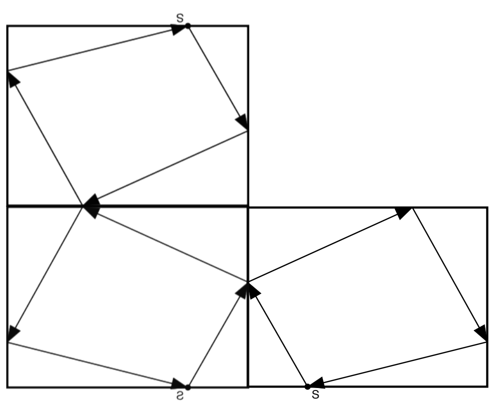
Where this is not the case, suppose that there is an optimal route, like the following:



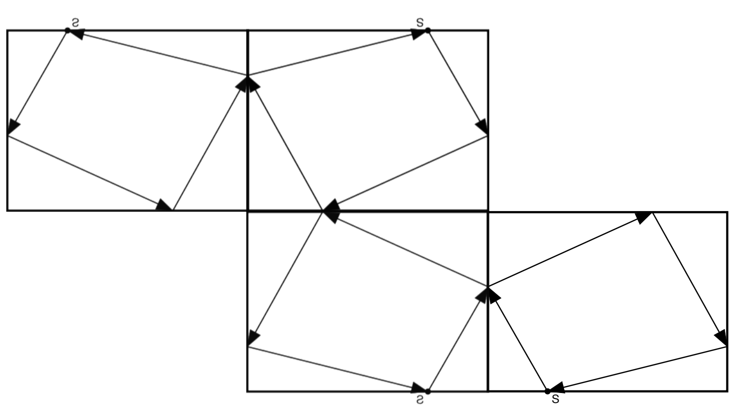

We can imagine taking this diagram and reflecting it in the left-hand vertical side, to obtain:



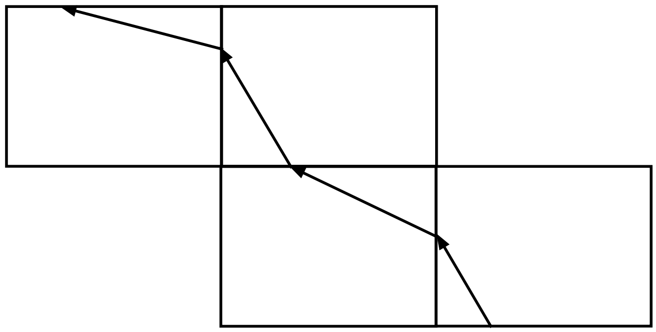

If we now reflect this new rectangle in its top horizontal side, then we obtain



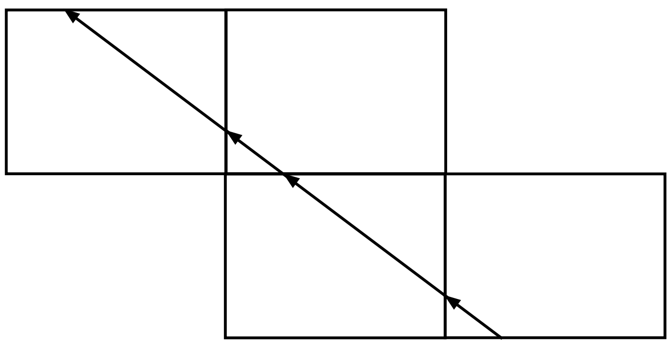

Finally, we reflect the newest rectangle in its left-hand vertical side:



If we focus on a different part of the path in each rectangle, we obtain:



These arrows show the four stages in the movement around the schoolyard, but 'opened out'. We can see from this drawing that this is not the optimal route, because there is a shorter path between the starting and finishing points, namely a straight line:



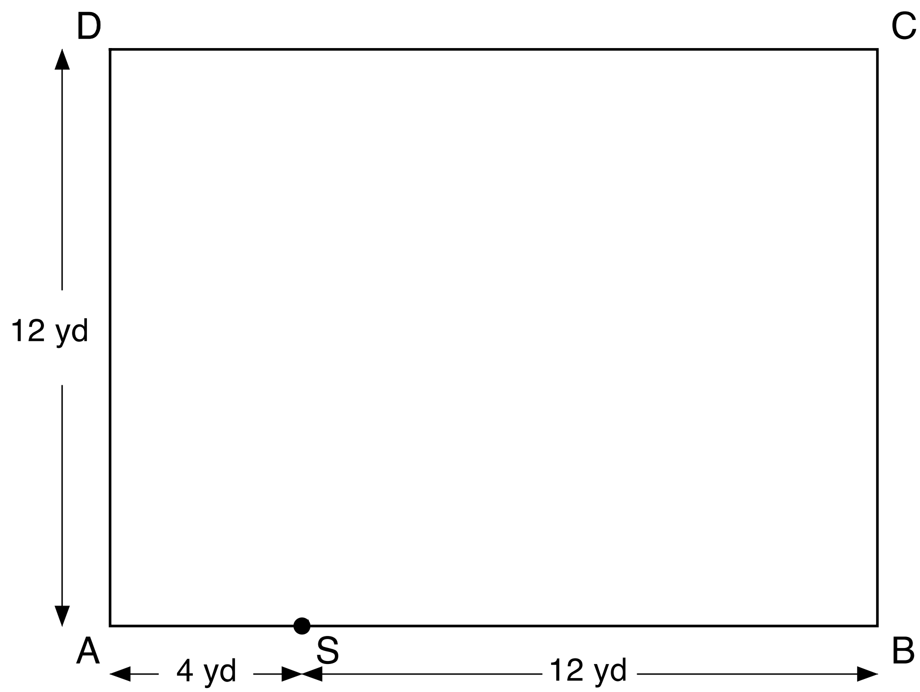
The entire line segment shown goes 32 yards to the left and 24 yards up, and has a slope of ¾. (This is equivalent to saying that the line segment goes twice the width of the playground left and twice the height of the playground up. So its length is twice the diagonal length of the playground.)

We can now use similar triangles to see that the position to head for on the wall AD in the first step is the point 3 yards along from A. This gives the parallelogram shown previously and so shows that this parallelogram has the smallest perimeter of any other possible quadrilateral.

# The Schoolyard Problem

Some children are playing a game in a rectangular schoolyard ABCD that is 16 yards by 12 yards.

The diagram shows the schoolyard viewed from above.

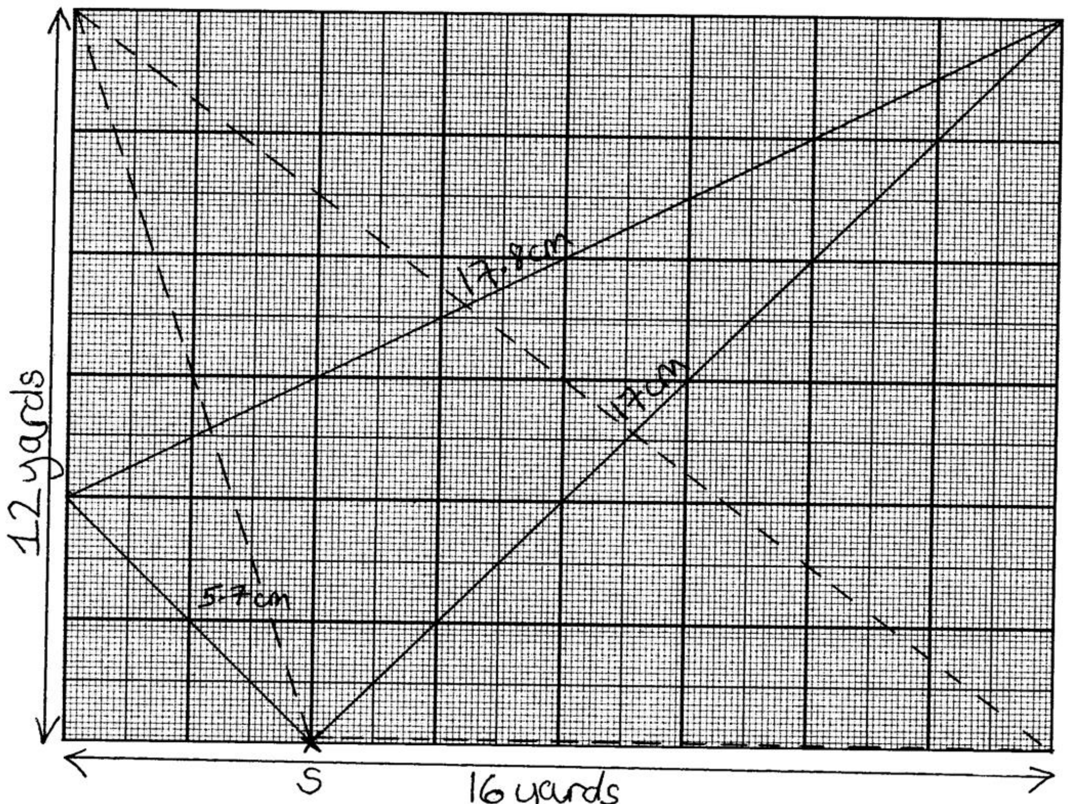



The children start at point S, which is 4 yards along the 16-yard wall AB.
They have to run and touch each of the other three walls and then get back to S.
The first person to return to S is the winner.

What is the shortest route for them to take?

Explain how you know this is the shortest path.

# Sample Response to Discuss: Malika




My scale is 1cm : 1yards.

Total distance = 5.7
+ 17.8
+ 17
40.5 yards

SOLID LINE ROUTE

Total distance = 12
+ 12.6
+ 20.2
44.8 yards

DASHED LINE ROUTE

The dashed line route is longer.

My conclusions:

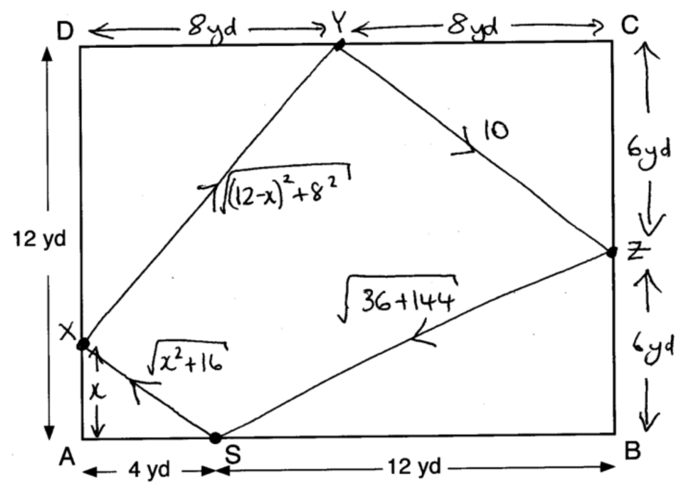
1. Touching two walls at once in the corners saves time.
2. Going along a side is slow and wastes time, like on the bottom side of the dashed route.

1. Try to describe Malika's approach.

2. What do you think of her approach? Why?

3. Do you agree with her conclusions? Why / Why not?

# Sample Response to Discuss: Paul




I fixed the positions of Y and Z so they were at the middles of their sides. But X can be anywhere on the left side.

So total length of journey

$$= 10 + \sqrt{180} + \sqrt{x^2+16} + \sqrt{(12-x)^2+8^2}$$

I used a spreadsheet to calculate the values:

| *x* | total length of journey |
|---|---|
| 1 | 41.140984 |
| 2 | 40.69479229 |
| 3 | 40.45800244 |
| 4 | 40.38697061 |
| 5 | 40.44967792 |
| 6 | 40.62751042 |
| 7 | 40.91264675 |
| 8 | 41.30495168 |
| 9 | 41.80926941 |
| 10 | 42.43294873 |
| 11 | 43.18336552 |
| 12 | 44.06551851 |

← $x = 4$ is the smallest distance

1. Try to describe Paul's approach.

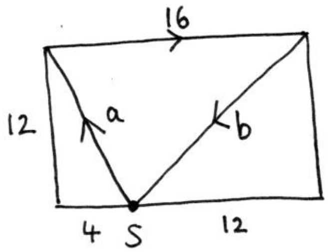
2. What do you think of his approach? Why?

3. Do you agree with his conclusions? Why / Why not?

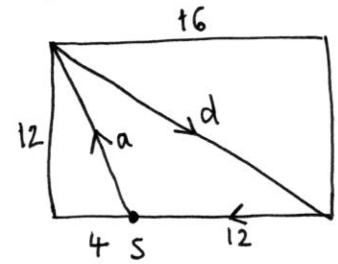
# Sample Response to Discuss: Robert

Let's compare two routes:

Route A:




Route B:




Which route is shortest?

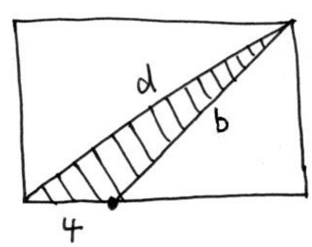
$A = a + 16 + b$     $B = a + d + 12$

"$a + 12$" is common to both, so can ignore that.

The question is

$\underline{4+b}$ versus $\underline{d}$

Now $d$ is the diagonal of the rectangle, so we can draw it the other way:




If you think about the shaded triangle,

$d < 4 + b$

so route B is shorter than route A.

1. Try to describe Robert's approach.

2. What do you think of his approach? Why?

3. Do you agree with his conclusions? Why / Why not?

# How Did You Work?

1. Compare the sample responses and your group response. What are the advantages and disadvantages of each approach?

| | Advantages | Disadvantages |
|---|---|---|
| Malika | | |
| Paul | | |
| Robert | | |
| Our group work | | |

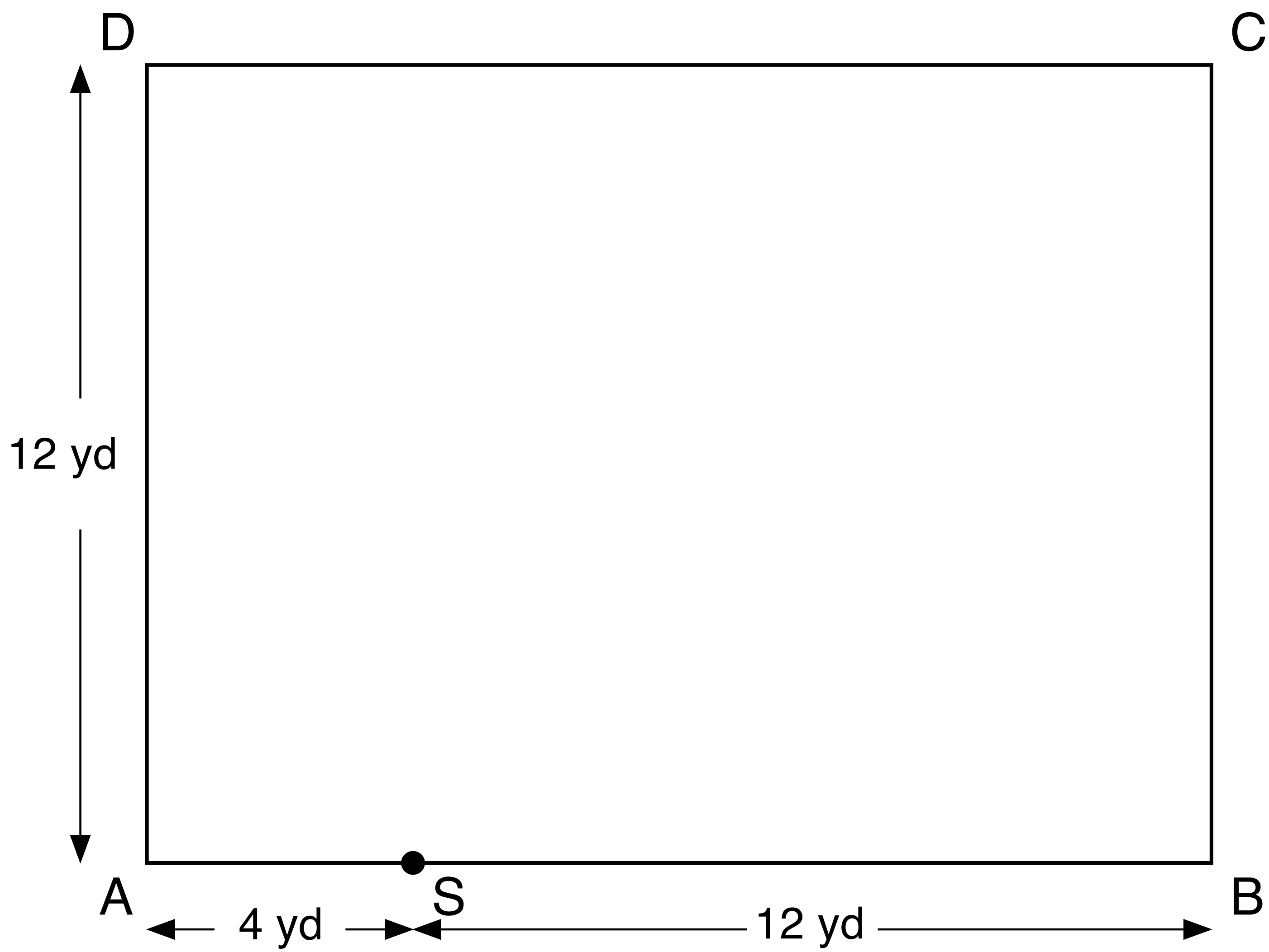
2. I made some assumptions.

   My assumptions were:

3. Now you have seen the Sample responses, suggest ways you could improve your own work:

4. What difficulties do you think a student new to this task would face?

# The Schoolyard Problem



# Collaborative Task

- Share your work with your partner(s).
- Take turns to explain how you approached the task.
- Talk about how you think you could now improve your solution.
- Make some notes on ideas for improving your solution, including reasons why.

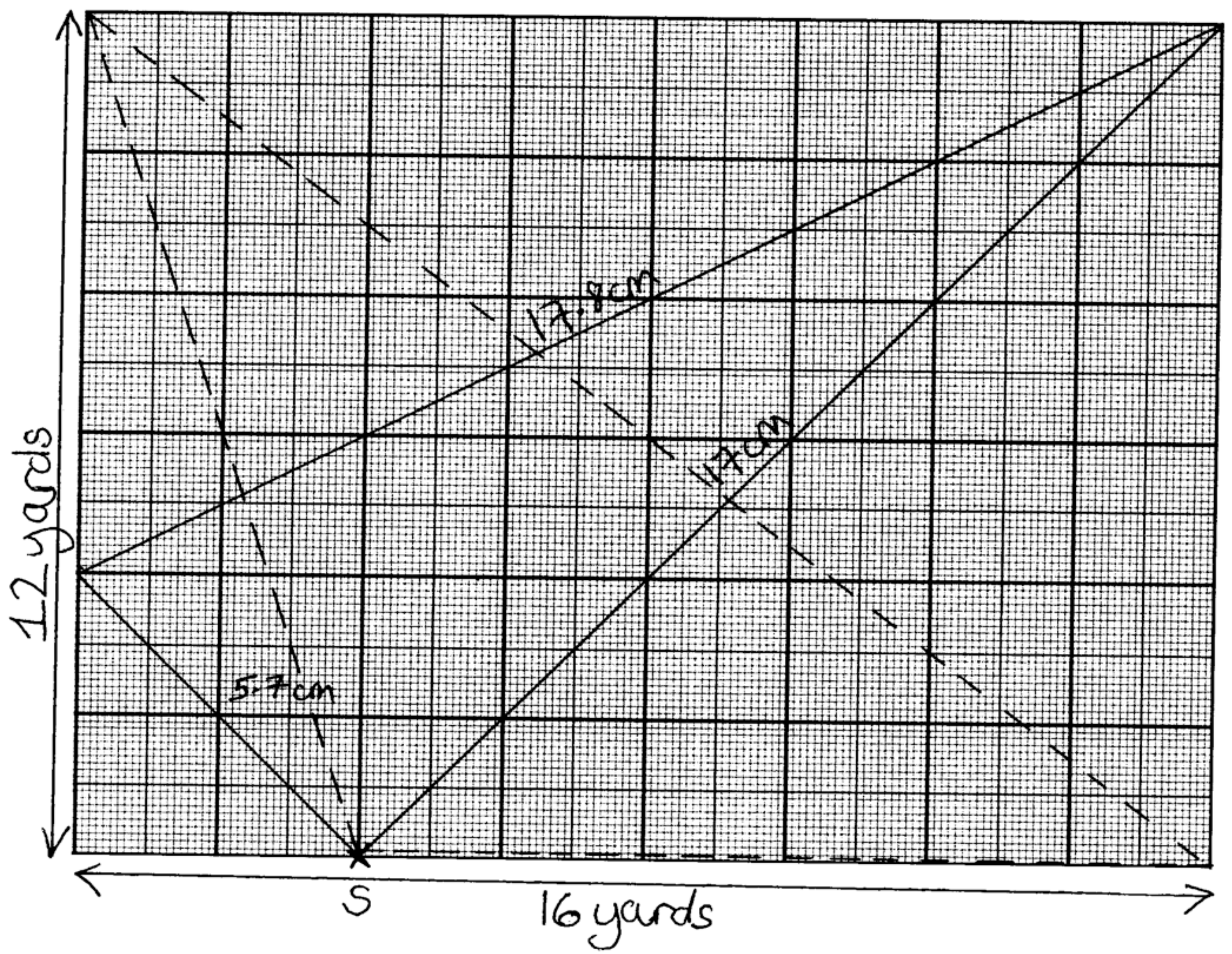
# Sample Responses to Discuss

- Look at each piece of student work.
- Answer the questions on the sheet.
- Compare each piece of work to your own work on the task.
- Consider the advantages and disadvantages of each approach.

# Producing a Joint Solution

- Together, in your group, produce a poster that shows a joint solution to the task, which is better than your individual responses.
- Give clear reasons for your choice of method.
- Justify how you know you have found the shortest route.

# Sample Response to Discuss: Malika (1)



# Sample Response to Discuss: Malika (2)

My scale is 1cm : 1yards.

Total distance = 5.7
+ 17.8
+ 17

40.5 yards

SOLID LINE ROUTE

Total distance = 12
+ 12.6
+ 20.2

44.8 yards

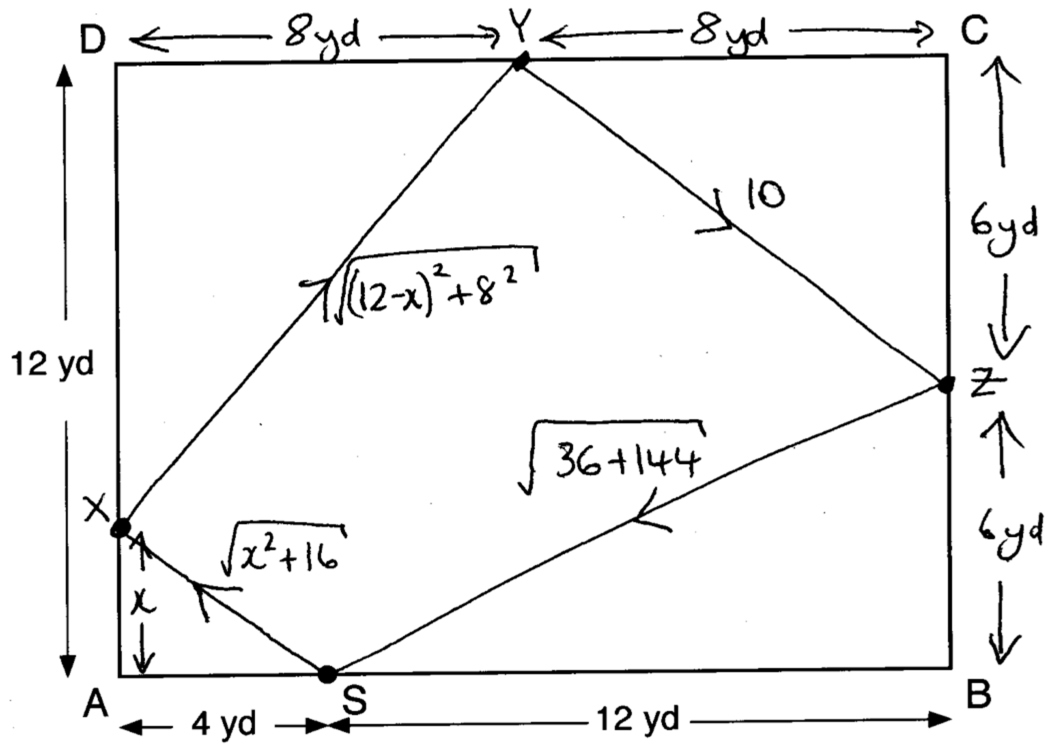
DASHED LINE ROUTE

The dashed line route is longer.

**My conclusions:**

1. Touching two walls at once in the corners saves time.
2. Going along a side is slow and wastes time, like on the bottom side of the dashed route.

# Sample Response to Discuss: Paul (1)




I fixed the positions of Y and Z so they were at the middles of their sides. But X can be anywhere on the left side.

# Sample Response to Discuss: Paul (2)

So total length of journey

$$= 10 + \sqrt{180} + \sqrt{x^2 + 16} + \sqrt{(12-x)^2 + 8^2}$$

I used a spreadsheet to calculate the values:

| *x* | total length of journey |
|---|---|
| 1 | 41.140984 |
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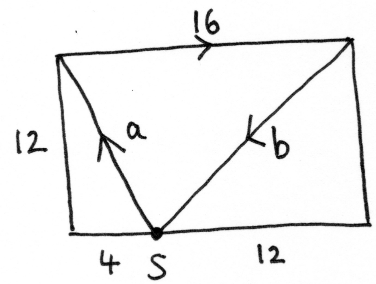
| 11 | 43.18336552 |
| 12 | 44.06551851 |

← $x = 4$ is the smallest distance

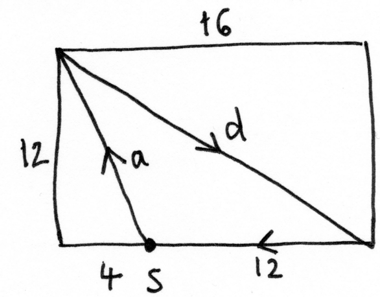
# Sample Response to Discuss: Robert (1)

Let's compare two routes:

Route A:




Route B:




Which route is shortest?

$A = a + 16 + b$ $\qquad$ $B = a + d + 12$

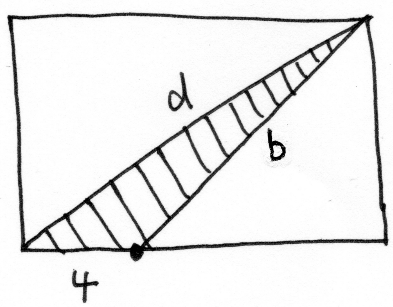
"$a + 12$" is common to both, so can ignore that.

The question is

$\underline{4 + b}$ versus $\underline{d}$

# Sample Response to Discuss: Robert (2)

Now d is the diagonal of the rectangle, so we can draw it the other way:




If you think about the shaded triangle,

$$d < 4 + b$$

so route B is shorter than route A.

Mathematics Assessment Project

# Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

**Malcolm Swan,**
**Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster,** and **Marie Joubert**
with
**Hugh Burkhardt, Rita Crust, Andy Noyes,** and **Daniel Pead**

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This project was conceived and directed for
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**Alan Schoenfeld** at the University of California, Berkeley, and
**Hugh Burkhardt, Daniel Pead,** and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org