PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

# Interpreting and Using Data: *Setting Taxi Fares*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

# Interpreting and Using Data: *Setting Taxi Fares*

## MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to select and use mathematical ideas to solve a problem and then compare and critique alternative approaches. The lesson presents students with a distance-time scatter plot representing journeys made by a taxi cab. They use this to decide upon a suitable rate at which the driver should charge passengers.

## COMMON CORE STATE STANDARDS

This lesson relates to **all** the *Mathematical Practices* in the *Common Core State Standards for Mathematics*: with a particular emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.EE: Understand the connections between proportional relationships, lines, and linear equations.

8.SP: Investigate patterns of association in bivariate data.

## INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the *Taxi Fares* task individually. You then assess their responses and formulate questions that will prompt students to review their work.
- At the start of the lesson, students think individually about their responses to the questions set.
- Next, students work in small groups to combine their thinking and work together to produce a collaborative solution to the *Taxi Fares* task, in the form of a poster.
- Working in the same small groups, students evaluate, comment on and complete sample responses. In a whole-class discussion students compare and evaluate the methods they have seen and used. In a follow-up lesson, students reflect on their work and what they have learned.

## MATERIALS REQUIRED

- Each student will need a copy of the assessment task *Taxi Fares* and some plain or squared paper to work on. They will also need a copy of the questionnaire *How Did You Work?* Calculators and graph paper should be available on request.
- Each small group will need a large sheet of paper and copies of *Sample Responses to Discuss* and one or two post-it notes. There is a projector resource to support whole-class discussions.

## TIME NEEDED

20 minutes before the lesson for the assessment task, a 90-minute lesson (or two shorter ones), and 15 minutes in a follow-up lesson (or for homework). Timings are approximate.

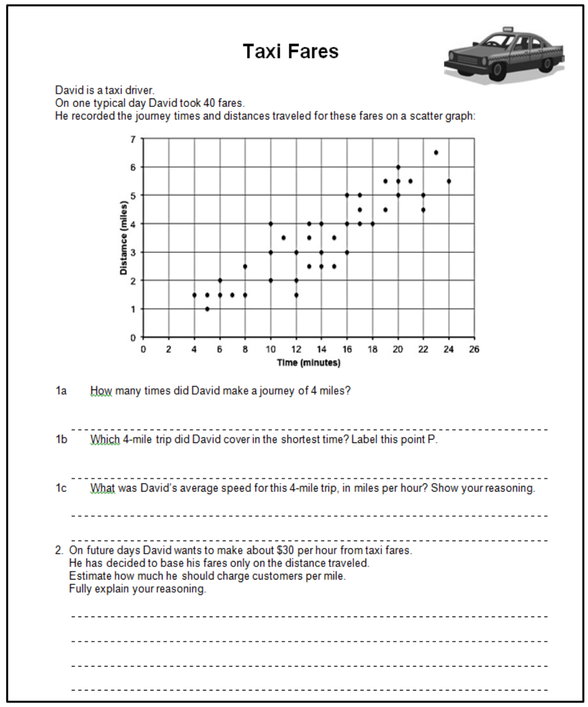
## BEFORE THE LESSON

### Assessment task: *Taxi Fares* (20 minutes)

Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the assessment task *Taxi Fares* and introduce it. Check that students understand the context:

> *Has anyone hired a taxi recently? How far did you go? How long did the journey take?*
>
> *The scatter plot shows the distances and times of 40 fares.*
> *What was the furthest David had to travel? [6.5 miles.]*
> *How long did this journey take? [23 minutes.]*
>
> *How would you calculate his average speed for this journey?*
>
> *Why is it the **average** speed?*



Taxi Fares

David is a taxi driver.
On one typical day David took 40 fares.
He recorded the journey times and distances traveled for these fares on a scatter graph:



1a How many times did David make a journey of 4 miles?

1b Which 4-mile trip did David cover in the shortest time? Label this point P.

1c What was David's average speed for this 4-mile trip, in miles per hour? Show your reasoning.

2. On future days David wants to make about $30 per hour from taxi fares.
He has decided to base his fares only on the distance traveled.
Estimate how much he should charge customers per mile.
Fully explain your reasoning.

Emphasize that the speed for a journey will vary over the course of the journey and that the total distance divided by the total time gives the average speed for the whole journey.

> *You are going to answer questions about some of the taxi journeys and give David some advice on how much he should charge customers per mile.*
>
> *Read the questions carefully and answer them as clearly as possible showing all your work.*

It is important that, as far as possible, students now work on the task without your assistance. Allow students the opportunity to figure out a solution for themselves. If students are struggling to get started then ask questions that help them understand what is required, but make sure you do not do the task for them. The assessment is to help you find out what help they will need.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

## Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table below. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions, and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

| Common issues | Suggested questions and prompts |
|---|---|
| **Reads the wrong axis (Q1a)**<br>For example: The student states David only made a journey of 4 miles on one occasion. | • Read the question again. Does the question tell you about the length of the journey in miles or in minutes? |
| **Misinterprets 'average speed' (Q1c)**<br>For example: The student adds up all the times for the 4-mile journeys and divides by 6. | • When you take a single taxi journey, does the cab always go at a constant speed?<br>• How can you calculate the cab's average speed for one journey? |
| **Makes a calculation error (Q1c)**<br>For example: The student divides 10 by 4, to get 2.5 and then labels this as miles per hour.<br>Or: The student gives an answer of 40 mph. | • For miles per hour what calculation do you need to make: 'distance ÷ time' or 'time ÷ distance'?<br>• What units have you used in your calculation? |
| **Uses an inappropriate method to figure out the typical speed (Q2)**<br>For example: The student assumes average speed is the mean of the shortest and longest journeys.<br>Or: The student takes a point in the 'middle' of the graph and uses that as the average speed. | • Can you think of a method to improve the accuracy of your answer?<br>• Will using all the data improve your answer? Why / Why not? |
| **Tries to work with the point values individually (Q2)**<br>For example: The student figures out David's total average speed for 40 journeys by adding together all the separate distances and times. | • Can you find a method for the problem that doesn't involve so much work? |
| **Uses incorrect method for calculating average speed (Q2)**<br>For example: The student attempts to total individual average journey speeds and then divides this total by 40. | • Suppose I cover 4 miles in 5 minutes, then another 4 miles in 10 minutes. What is my total average speed? 8 miles in 15 minutes = 32 mph? [correct] or (48 mph + 24 mph) ÷ 2 = 36 mph? |
| **Line of best fit is very inaccurate (Q2)**<br>For example: The student draws a line that passes through the 'top' and 'bottom' points. | • How have you chosen where to put the line of best fit? |
| **Student incorrectly figures out the slope of the line of best fit**<br>For example: The student divides time by distance. | • In your head, say a sentence to describe to David what the slope of the line of best fit represents.<br>• What scale is used on the graph?<br>• Check your answer makes sense. |
| **Makes mistakes with the units**<br>For example: The student figures out a 'typical' average speed of 0.25 miles per minute and then divides this figure by \$30.<br>Or: the student thinks David wants to earn $30 \times 60$ = \$1800 per minute. | • Is the size of your answer sensible?<br>• How much does David want to earn per minute? How can this answer help you?<br>• How many miles does David travel per hour? How can this answer help you? |

## SUGGESTED LESSON OUTLINE

### Reviewing individual solutions to the problem (10 minutes)

Give each student a mini-whiteboard, a pen and an eraser and return their work on the *Taxi Fares* task. If mini-whiteboards are not available, students may want to use the back of their assessment task to jot down their ideas about ways to improve their work.

If you have not added questions to individual pieces of student work, either give each student a printed version of your list of questions, with the questions that relate to their work highlighted, or write your list of questions on the board so that students can select questions from the board that are appropriate to their own work.

*Recall what we were working on previously. What was the task about?*

*I have had a look at your work and I have some questions I would like you to think about.*

*On your own, carefully read through the questions I have written. I would like you to use the questions to help you think about ways of improving your work.*

*Use your mini-whiteboards to make a note of anything you think will help to improve your work. You will be sharing these notes with a partner later on.*

This is an opportunity for students to review their own work before working collaboratively on producing a group solution. While students are reviewing their work, it may be appropriate to ask individual students questions that help them to clarify their thinking. However, the purpose of the activity is not to address misconceptions; there will be opportunities for students to deal with these collaboratively later in the lesson.

### Collaborative small-group work: making posters (20 minutes)

Organize the class into groups of two or three students. You may like to group students that have used different methods when completing the task, so that they have something to discuss.

*Today you are going to work together on the Taxi Fares task to produce a joint solution that is better than your individual work.*

Before students work on the task, they need to discuss what they have learned from reviewing their individual solutions. Explain to students that this activity will help them to decide which approach to collaboratively pursue when they work together on a joint solution to Q2 of the task in the next activity (they should discuss their responses to Q1 here too.)

Show and explain to students Slide P-1 of the projector resource:

**Sharing Individual Solutions**

1. Take turns to share with your partner(s) your own individual solution to the task and the notes you made on your mini-whiteboard about how you might improve your work.
2. Listen carefully to each other, asking questions if you don't understand.
3. Notice any similarities or differences between the methods described.

Once students have had a chance to discuss their work, hand out to each group a sheet of poster paper. Display Slide P-2 of the projector resource and explain to students how to work collaboratively:

**Joint Solution: Making Posters**

1. In your group agree on the best method for completing Q2 of the problem.
2. Produce a poster that shows a joint solution to Q2 of the *Taxi Fares* task, that is better than your individual work.
3. State on your poster any assumptions you have made.
4. Give clear reasons for your choice of method.

Emphasize to students that, providing they justify their decisions, they can improve one of their individual methods, combine methods or use a completely different method.

While students are working in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

### Note different student approaches

In particular, note whether students' original methods are the same or different. If they are different, how do they decide which method to use for their joint solution? What are their reasons for the choice of method? Are students aware of any assumptions they have made? Do they justify these assumptions? How do they organize their work? Are they concerned whether their answer makes sense? Do their answers contain an inappropriate number of decimal places?

You may also want to look for non-standard approaches, or ones that include a common misconception, or ones that are either especially efficient or especially inefficient. You may decide to use some of these when conducting the whole class discussion later in the lesson.

### Support student problem solving

If students are struggling to produce a joint solution to the task, try to resist making suggestions that move them towards a particular approach. Instead, ask questions to help students clarify their thinking, encouraging them to identify by comparing their individual solutions, the strengths and weaknesses of each method. Can any of these methods be improved to produce a group solution that is better than the original individual response? Can they think of any other approaches to try?

*What have you done that you both [all] agree on? Why have you chosen this method?*

*What decisions have you already made? Why did you make those decisions?*

*What else do you need to find out?*

*What do you now know that you didn't know before?*

If students are still struggling to interpret the scatter plot:

*Show me the longest journey David traveled.*

*Show me the journey with the slowest average speed. How do you know?*

*Show me two journeys with differing distances, but took the same time.*
*Which of the two journeys had the faster average speed?*

*Show me two journeys that had the same average speed. How do you know?*

If several students in the class are struggling you might want to write a relevant question on the board or hold a brief whole-class discussion. You may also want to use some of the questions in the *Common issues* table to support your own questioning.

### ***Sharing Posters*** **(15 minutes)**

Once students have finished their posters, ask them to share their work by visiting another group. This gives all students the opportunity to engage with another group's reasoning, as well as voicing their own thinking.

Give each group one or two post-it notes.

Show Slide P-3 and explain how students are to share their work and the purpose of the activity:

**Sharing Posters**

1. One person from each group get up and visit a different group.
2. If you are staying with your poster, explain your work to the visitor, giving reasons for your choice of method.
3. If you are the visitor, look carefully at the work, asking clarifying questions to help you to understand the method used.
4. Discuss whether or not the method described on the poster is similar to the visitor's method.
5. The visitor is to write on a post-it note suggestions on how the work could be improved.

As students are sharing their work you might like to offer some or all of the following prompts:

*What is right on the poster?*

*What questions do you have about the math on the poster (because you are not sure about or do not understand what they have written)?*

*What do you think this group needs to fix on their poster?*

These questions may be particularly helpful to students who are used to analyzing each other's work in this way.

#### Extending the lesson over two days

If you are taking two days to complete this lesson unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of the task they have been working on before moving on to the collaborative analysis of sample responses.

### **Collaborative analysis of *Sample Responses to Discuss* (25 minutes)**

Once students have had sufficient time to discuss their joint solutions, distribute copies of the *Sample Responses to Discuss* to each group. These are sample responses to Q2 of the *Taxi Fares* task.

Display and explain to students Slide P-4 of the projector resource:

## Sample Responses to Discuss

1. Read each piece of sample student work carefully.
2. Try to understand what they have done. You may want to add annotations to the work to make it easier to follow.
3. Take turns explaining your thinking to your partner.
4. Listen carefully and ask clarifying questions.
5. When your group has reached its conclusions, write your answers to the questions underneath the work.
6. Finally, compare the sample responses.
   - What are the strengths and weaknesses of each?
   - Which do you prefer?
   - Justify your answers

Emphasize to students that the purpose of the activity is not to check the accuracy of the arithmetic as neither method contains errors, but to understand and evaluate a variety of possible approaches to the task and to notice any differences and/or similarities with their own work:

*The arithmetic is right. Don't spend time recalculating.*

*What is each student trying to do?*

*How can it be improved?*

The task encourages students to be flexible in their approach to the problem and recognize relationships within mathematics and among different points of view, and so deepen their mathematical understandings.

**Nick** adds together the times for the 40 journeys and the distances for the same journeys. He correctly figures out the total earnings for the 40 journeys.

Nick's work is accurate, but time consuming. The task only asks for an estimate and there is no need for the accuracy Nick has provided.

**Nick's** work is incomplete. He now needs to figure out the amount David should charge per mile [$282/140.5 miles = $2 per mile (approximately)].

Distance = 1.5+1.5+1+1.5+2+1.5+1.5+2.5+2+3+4
+3.5+1.5+2+3+2.5+4+3.5+2.5+3+4+2.5+3.5
+3+4+5+4+4.5+5+4+4.5+5.5+5+5.5+6+5.5+
+4.5 +5+6.5+.5.5 = 140.5 miles

Time = 4+6+5+5+6+7+8+8+10+10+10+11+12+12+12+13
+13+13+14+14+14+15+15+16+16+16+17+17+17+18+19
+19+20 +20+20+21+22+22+23+24 = 564 minutes

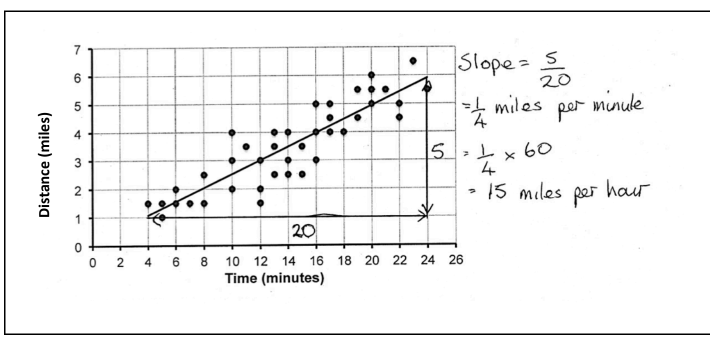
Each minute : 30÷60 = $0.5
In 564 minutes 564×0.5 = $282 (140.5 miles)

Note that students are likely to make errors when using **Nick's** method; errors when extracting the figures from the graph and arithmetic errors.

**Lydia** has drawn a line of best fit.

She has figured out the typical representative speed of a journey in miles per hour in an appropriate way.

**Lydia's** work is incomplete. She now needs to figure out the amount David should charge per mile.



## Whole-class discussion: comparing different approaches (20 minutes)

Hold a whole-class discussion to consider the different approaches used within the sample responses. Look at each response in turn and ask students to comment on their strengths and weaknesses. Throughout the discussion encourage students to discuss whether there is a need for an accurate answer, given that the data cannot provide an accurate prediction of future journeys. If you have time, you may also want to include in the discussion some of your own students' responses to the problem.

It may be helpful to display Slides P-5 to P-6 during this discussion.

*What did Nick / Lydia do?*
*When calculating what David should charge customers per mile, was there a difference between Nick and Lydia's answers? Why was this?*

*How many decimal places do you think Nick and Lydia's answers should be rounded to? [Lydia's estimate needs only to be to one or two decimal places].*

Once you have discussed each piece of work, ask students to compare and evaluate the different methods.

*Which piece of work did you find easiest/most difficult to understand? Why was that?*

*Which method do you prefer? Please explain.*

*Which method would work best if David recorded the data for a week of journeys rather than a day?*

*How is Nick's/Lydia's work similar or different to what you did?*

*Which method would you use if you wanted to avoid calculation errors?*

*Which method would work best if David had recorded the data on a computer?*

*Did analyzing the responses enable anyone to see ways in which they could improve their own work? Please explain.*

These questions do not have definitive answers; they are meant to encourage students to think about the issues involved when working with real-world situations and to use the power of comparison to draw out the strengths and weaknesses of each approach.

## Follow-up lesson (or possible homework): individual reflection (15 minutes)

Once students have had a chance to discuss the sample responses as a whole class, distribute the questionnaire *How Did You Work?* Ask students to spend a couple of minutes, individually, answering the questions:

*Think carefully about your work this lesson and the different methods you have seen and used.*

*On your own, answer the review questions as carefully as you can.*

Some teachers give this as homework.

## SOLUTIONS

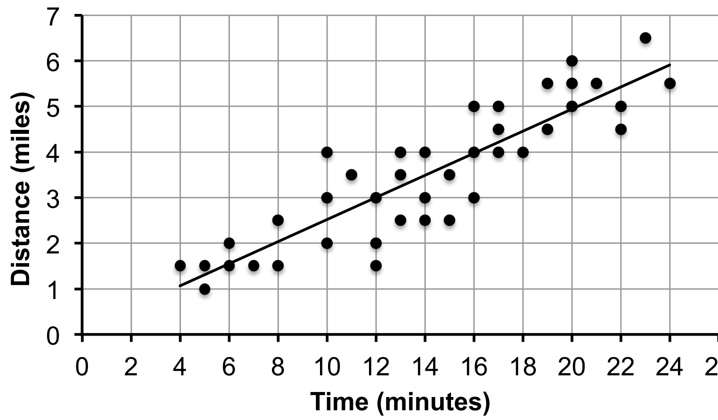
1a. David made a 4-mile journey on 6 occasions.

1b. P should be located at the point (10, 4). That is, the 4 mile journey only took ten minutes.

1c. This is an average speed of 24 miles per hour.

2. Below are two possible methods:

**Method 1: Using a line of best fit** (This is the most efficient method.)




The slope of the line is approximately 0.25. David travels about 0.25 mile per minute, or about 15 miles per hour. David wants to earn $30 per hour. Therefore he should charge 30 ÷ 15 = **$2** per mile.

**Note:** If students use this method, there is likely to be a range of acceptable answers, depending on where they place the line of best fit.

**Method 2: Using the figures for each journey**

| Time (minutes) | Distance (miles) |
|---|---|
| 4 | 1.5 |
| 5 | 1 |
| 5 | 1.5 |
| 6 | 1.5 |
| 6 | 2 |
| 7 | 1.5 |
| 8 | 1.5 |
| 8 | 2.5 |
| 10 | 2 |
| 10 | 3 |
| 10 | 4 |
| 11 | 3.5 |
| 12 | 1.5 |
| 12 | 2 |
| 12 | 3 |
| 13 | 2.5 |
| 13 | 4 |
| 13 | 3.5 |
| 14 | 2.5 |
| 14 | 3 |
| 14 | 4 |
| 15 | 2.5 |

| | | |
|---|---|---|
| | 15 | 3.5 |
| | 16 | 3 |
| | 16 | 4 |
| | 16 | 5 |
| | 17 | 4 |
| | 17 | 4.5 |
| | 17 | 5 |
| | 18 | 4 |
| | 19 | 4.5 |
| | 19 | 5.5 |
| | 20 | 5 |
| | 20 | 5.5 |
| | 20 | 6 |
| | 21 | 5.5 |
| | 22 | 4.5 |
| | 22 | 5 |
| | 23 | 6.5 |
| | 24 | 5.5 |
| **Totals** | **564** | **140.5** |

In 1 minute David wants to earn 30 ÷ 60 = \$0.50

In 564 minutes David should earn 564 × 0.5 = \$282.00

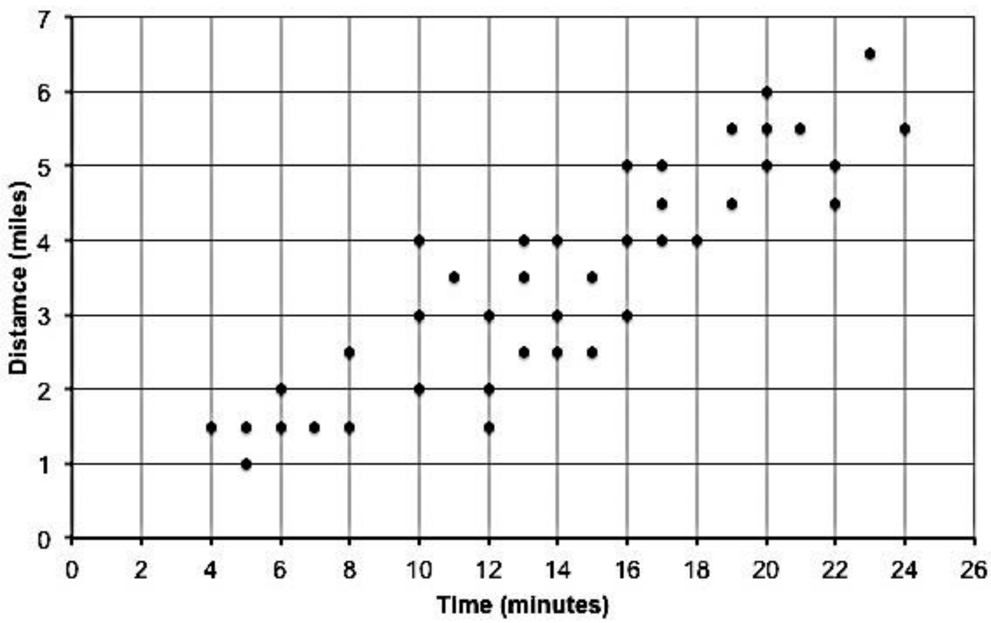
In 140.5 miles he earns \$282.00.

In 1 mile he earns 282 ÷ 140.5 = 2.00717… = **\$2.00**

**Note:** Students may assume that there is likely to be a wait time between some fares and make allowances for this when estimating how much David should charge per mile.

# Taxi Fares

David is a taxi driver.
On one typical day David took 40 fares.
He recorded the journey times and distances traveled for these fares on a scatter plot:




1a How many times did David make a journey of 4 miles?

1b Which 4-mile trip did David cover in the shortest time? Label this point P.

1c What was David's average speed for this 4-mile trip, in miles per hour? Show your reasoning.

2. On future days David wants to make about $30 per hour from taxi fares.
He has decided to base his fares only on the distance traveled.
Estimate how much he should charge customers per mile.
Fully explain your reasoning.

# Sample Responses to Discuss: *Nick*

Distance = 1.5 + 1.5 + 1 + 1.5 + 2 + 1.5 + 1.5 + 2.5 + 2 + 3 + 4
+ 3.5 + 1.5 + 2 + 3 + 2.5 + 4 + 3.5 + 2.5 + 3 + 4 + 2.5 + 3.5
+ 3 + 4 + 5 + 4 + 4.5 + 5 + 4 + 4.5 + 5.5 + 5 + 5.5 + 6 + 5.5 +
+ 4.5 + 5 + 6.5 + 5.5 = 140.5 miles

Time = 4 + 6 + 5 + 5 + 6 + 7 + 8 + 8 + 10 + 10 + 10 + 11 + 12 + 12 + 12 + 13
+ 13 + 13 + 14 + 14 + 14 + 15 + 15 + 16 + 16 + 16 + 17 + 17 + 17 + 18 + 19
+ 19 + 20 + 20 + 20 + 21 + 22 + 22 + 23 + 24 = 564 minutes

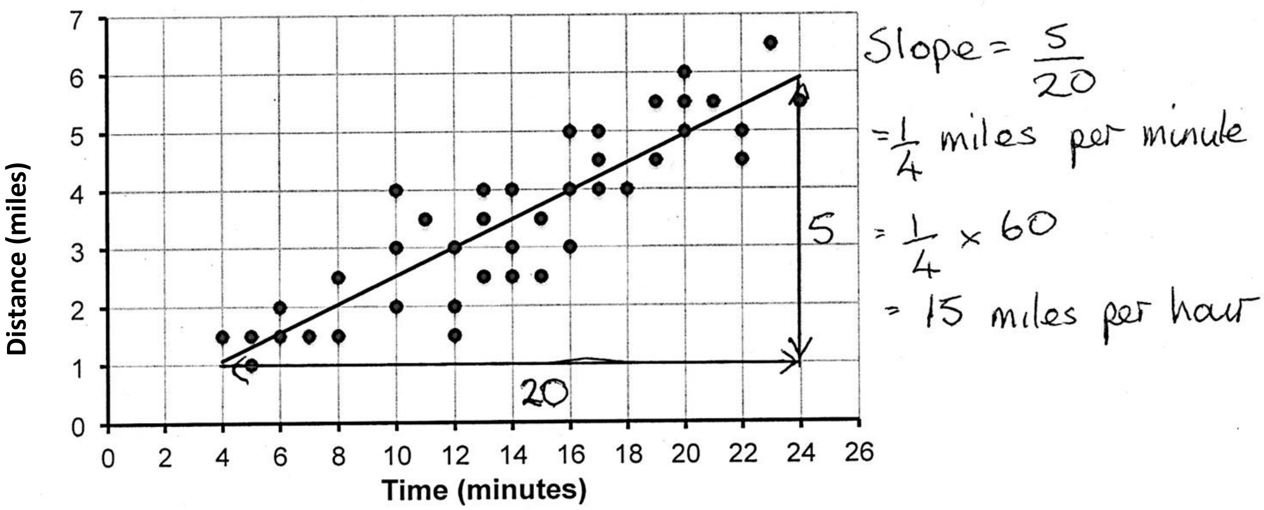
Each minute : 30 ÷ 60 = $0.5
In 564 minutes 564 × 0.5 = $282 (140.5 miles)

You don't need to check Nick's arithmetic. It is correct!

Clearly explain the reasoning behind Nick's method.

Use Nick's method to solve Q2 of the *Taxi Fares* problem.

# Sample Responses to Discuss: *Lydia*




Clearly explain how Lydia used the graph to figure out the speed.

Use Lydia's method to solve Q2 of the *Taxi Fares* problem.

# How Did You Work?

1. Compare the sample responses and your group response. What are the advantages and disadvantages of each approach?

| | Advantages | Disadvantages |
|---|---|---|
| Nick | | |
| Lydia | | |
| Our group work | | |

2. Now that you have seen Nick's and Lydia's work, what would you do if you started the task again?

3. What do you think are the difficulties someone new to the task will face?

# Sharing Individual Solutions

1. Take turns to share with your partner(s) your own individual solution to the task and the notes you made on your mini-whiteboard about how you might improve your work.

2. Listen carefully to each other, asking questions if you don't understand.

3. Notice any similarities or differences between the methods described.

# Joint Solution: Making Posters

1. In your group agree on the best method for completing Q2 of the problem.

2. Produce a poster that shows a joint solution to Q2 of the *Taxi Fares* task, that is better than your individual work.

3. State on your poster any assumptions you have made.

4. Give clear reasons for your choice of method.

# Sharing Posters

1. One person from each group get up and visit a different group.

2. If you are staying with your poster, explain your work to the visitor, giving reasons for your choice of method.

3. If you are the visitor, look carefully at the work, asking clarifying questions to help you to understand the method used.

4. Discuss whether or not the method described on the poster is similar to the visitor's method.

5. The visitor is to write on a post-it note suggestions on how the work could be improved.

# Sample Responses to Discuss

1. Read each piece of sample student work carefully.

2. Try to understand what they have done. You may want to add annotations to the work to make it easier to follow.

3. Take turns explaining your thinking to your partner.

4. Listen carefully and ask clarifying questions.

5. When your group has reached its conclusions, write your answers to the questions underneath the work.

6. Finally, compare the sample responses.
   - What are the strengths and weaknesses of each?
   - Which do you prefer?
   - Justify your answers.

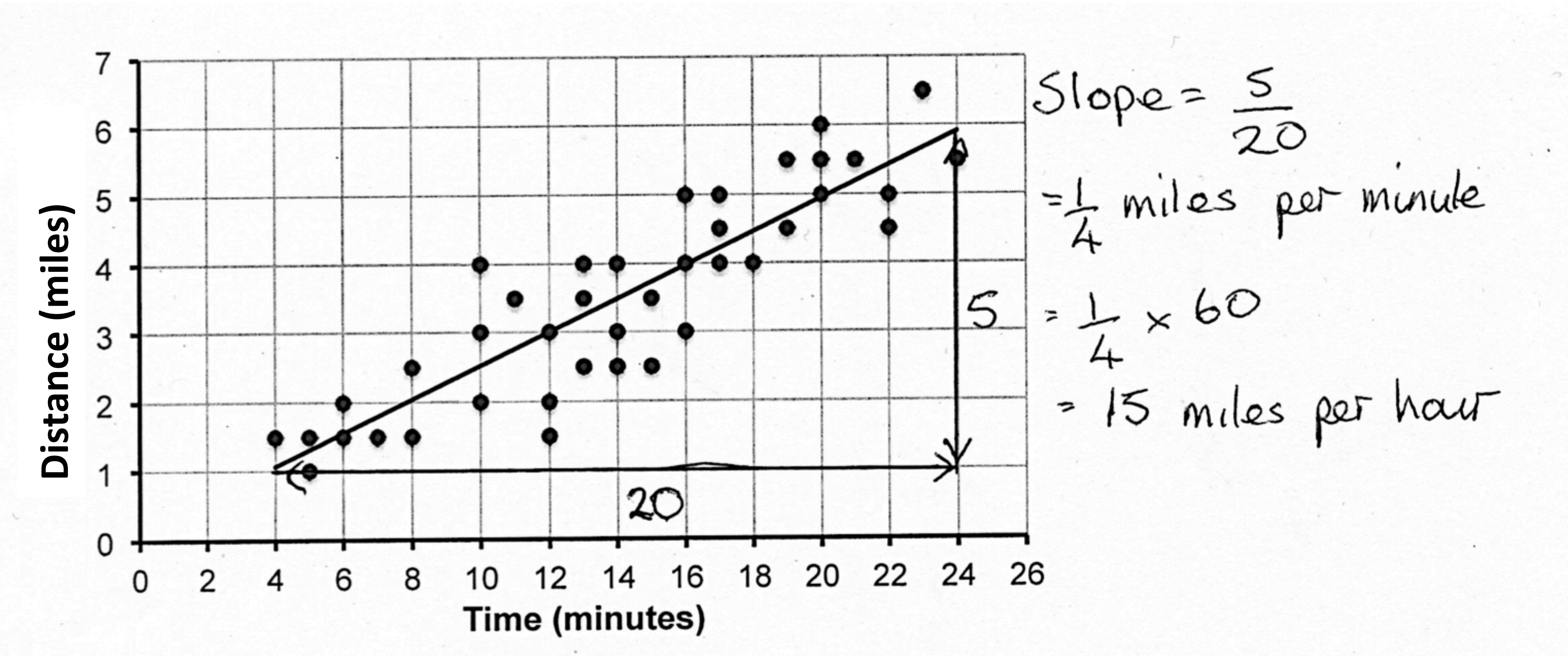
# Sample Responses to Discuss: *Nick*

Distance = 1.5 + 1.5 + 1 + 1.5 + 2 + 1.5 + 1.5 + 2.5 + 2 + 3 + 4
+ 3.5 + 1.5 + 2 + 3 + 2.5 + 4 + 3.5 + 2.5 + 3 + 4 + 2.5 + 3.5
+ 3 + 4 + 5 + 4 + 4.5 + 5 + 4 + 4.5 + 5.5 + 5 + 5.5 + 6 + 5.5 +
+ 4.5 + 5 + 6.5 + 5.5 = 140.5 miles

Time = 4 + 6 + 5 + 5 + 6 + 7 + 8 + 8 + 10 + 10 + 10 + 11 + 12 + 12 + 12 + 13
+ 13 + 13 + 14 + 14 + 14 + 15 + 15 + 16 + 16 + 16 + 17 + 17 + 17 + 18 + 19
+ 19 + 20 + 20 + 20 + 21 + 22 + 22 + 23 + 24 = 564 minutes

Each minute : 30 ÷ 60 = $0.5
In 564 minutes 564 × 0.5 = $282 (140.5 miles)

# Sample Responses to Discuss: *Lydia*



Mathematics Assessment Project

# Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

**Malcolm Swan,**
**Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster,** and **Marie Joubert**
with
**Hugh Burkhardt, Rita Crust, Andy Noyes,** and **Daniel Pead**

We are grateful to the many teachers and students, in the UK and the US,
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