CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Representing Linear and Exponential Growth

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Representing Linear and Exponential Growth

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to interpret exponential and linear functions and in particular, to identify and help students who have the following difficulties:

- Translating between descriptive, algebraic, tabular, and graphical representation of the functions.
- Recognizing how and why a quantity changes per unit interval.

To achieve these goals students work on simple and compound interest problems.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

A-SSE: Interpret the structure of expressions.
Write expressions in equivalent forms to solve problems.
F-LE: Construct and compare linear, quadratic, and exponential models and solve problems.
Interpret expressions for functions in terms of the situation they model.
F-BF: Build a function that models a relationship between two quantities.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 4, 7 and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding and difficulties working with linear and exponential functions. You review their responses and create questions for students to consider, to help them improve their work.
- After a whole-class interactive introduction, students work in small groups on a series of collaborative card matching tasks.
- In a whole-class discussion, students review the main mathematical concepts of the lesson and the strategies used.
- Students then return to the original task, consider their own responses and the questions posed and use what they have learned to complete a similar task.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks *Making Money?* and *Making Money?(revisited),* a calculator, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need cut-up copies of *Card Set: Investment Plans and Formulas, Card Set: Graphs*, *Card Set: Tables*, and *Card Set: Statements*, a large sheet of poster paper, a marker, and a glue stick.
- There is a projector resource to help introduce activities and support whole-class discussion.

TIME NEEDED

20 minutes before the lesson, a 100-minute lesson (or two 55-minute lessons), and 20 minutes in a follow-up lesson. Timings are approximate. Exact timings will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Making Money?* (20 minutes)

Ask the students to do this task, in class or for homework, a day or more before the lesson. This will give you an opportunity to assess their work and to find out the kinds of difficulties they have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the *Making Money?* task. Introduce the task briefly, and help the class to understand the context:

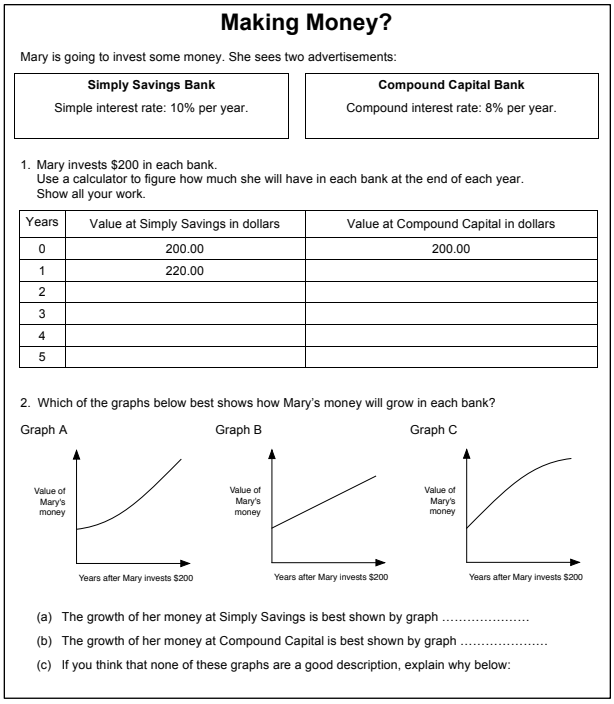
Why do we put money in a bank? [To keep it safe and gain interest.]

What does interest mean? [The money the bank adds to the investment.]

What is an interest rate? [The percentage by which the money grows each year. This is often called the APY 'Annual Percentage Yield'.]

Can you see why the $200 in Simply Savings grows to $220 after one year?

Spend 15 minutes on your own, reading through the questions and trying to answer them as carefully as you can.



Making Money?

Mary is going to invest some money. She sees two advertisements:

Simply Savings Bank	**Compound Capital Bank**
Simple interest rate: 10% per year.	Compound interest rate: 8% per year.

1. Mary invests $200 in each bank.
   Use a calculator to figure how much she will have in each bank at the end of each year.
   Show all your work.

Years	Value at Simply Savings in dollars	Value at Compound Capital in dollars
0	200.00	200.00
1	220.00	
2		
3		
4		
5		

2. Which of the graphs below best shows how Mary's money will grow in each bank?

Graph A

Graph B

Graph C

(a) The growth of her money at Simply Savings is best shown by graph .....

(b) The growth of her money at Compound Capital is best shown by graph .....

(c) If you think that none of these graphs are a good description, explain why below:

3. Write down a formula for calculating the amount of money in each of these banks after n years.

.....

.....

.....

.....

.....

4. Mary wants to invest some money for 5 years or more.
   Which bank should she choose?
   Give full reasons for your answer.

.....

It is important that, as far as possible, students are allowed to answer the questions without assistance. Students should not worry too much if they cannot understand or do everything because, in the next lesson, they will engage in a similar task that should help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions like these confidently. This is their goal.

Assessing students' responses

Collect student's responses to the task and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of any difficulties students may experience during the lesson itself, so that you can prepare carefully.

We suggest that you do not score students' work. Research shows that this will be counterproductive as it will encourage students to compare their scores and will distract their attention from the mathematics. Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on the next page.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or

- give each student a printed version of your list of questions and highlight appropriate questions for individual students.

If you do not have time to do this, you could select a few questions that will be of help to the majority and write these on the board when you return the work to the students in the follow-up lesson.

Common issues:	**Suggested questions and prompts:**
Assumes simple interest for both investments For example: The student makes the Compound Capital investment grow by $16 per year (Q1). Or: The student selects Graph B for both banks (Q2).	• Can you explain the difference between compound interest and simple interest? • For the investment in Compound Capital, what is the amount in the bank at the end of the first year? What will be the interest after one year? Can you explain why the interest changes for the second year?
Figures out the interest on $100 (Q1) For example: The student writes the Year 1 value of the savings at Compound Capital as $208.	• What is 8% of $200?
Writes a general formula (Q3) For example: $A = P + \frac{RP}{100}$ or $A = P\left(1 + \frac{R}{100}\right)^n$	• Can you write a formula that includes the interest rates for each of the banks?
Writes the formula incorrectly (Q3) For example: $A = 200 + 10n$; $A = 200 \times 10n$ (Simply Savings.) $A = 200 + 1.08^n$; $A = 200 \times 8n$ (Compound Capital.)	• How can you check your answer? • Try substituting values of n into your formula to check your answers.
Assumes that Simply Savings will always be a better investment (Q4)	• Which is the better investment after 5 years? • Which is the better investment after 6/7/8 years? How do you know? • What will happen to the difference in amounts over a longer period of time?
Uses an inefficient method (Q4) For example: Instead of using a formula, the student compounds the interest each year.	• Can you think of a quicker method for calculating the amount of money in the bank after say, 10 years?
Completes the task The student needs an extension task.	• How long would it take each savings plan to double your investment? • Does this doubling time depend on the size of the initial investment? Why? / Why not? • What would Mary get from each plan if she took out her money after 6 months?

SUGGESTED LESSON OUTLINE

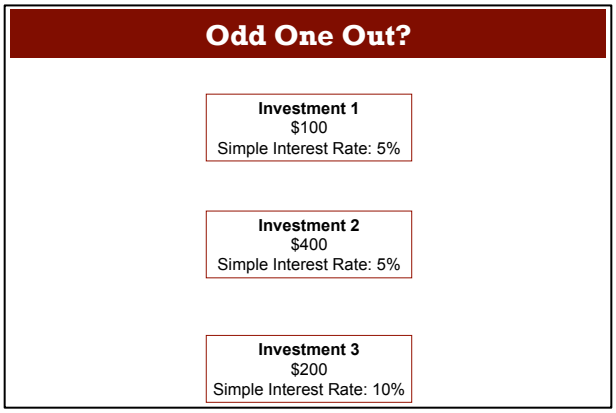
Whole-class interactive introduction (15 minutes)

Give each student a calculator, a mini-whiteboard, a pen, and an eraser.

Today we will investigate two ways to invest money: simple interest and compound interest.

What is the difference between simple and compound interest? [Simple interest calculates a percentage of the original investment amount and adds it on each year. Compound interest calculates a percentage of the amount in the account.]

To introduce simple interest, show Slide P-1 of the projector resource:



Look at these three investments. Which is the odd one out?
Write down your reasoning on your mini-whiteboards.

Ask one or two students to explain their answers. Most students will answer that Investment 3 is the odd one out because it has a different interest rate. Prompt them to consider other possibilities:

I think Investment 1 is the odd one out. Why do I think this? [Investment 1 will increase by $5 a year, whereas Investments 2 and 3 will both increase by $20 a year.]

Ask students to explain their answers. If students are struggling to answer the question, ask:

Investment 1 and 2 have the same interest rate, does this mean the investments will increase by the same amount each year? How much will each investment increase by each year?

Now ask the students to represent the description of an investment algebraically.

How can you represent Investment 2 as a formula? Start the formula with A = ..., where A is the amount in the bank. Use n to represent the number of years the money is invested.

Allow students a few minutes to think about the question individually and then ask them to discuss the problem with a partner before sharing ideas with the whole-class. (We sometimes refer to this as the 'think-pair-share' strategy.)

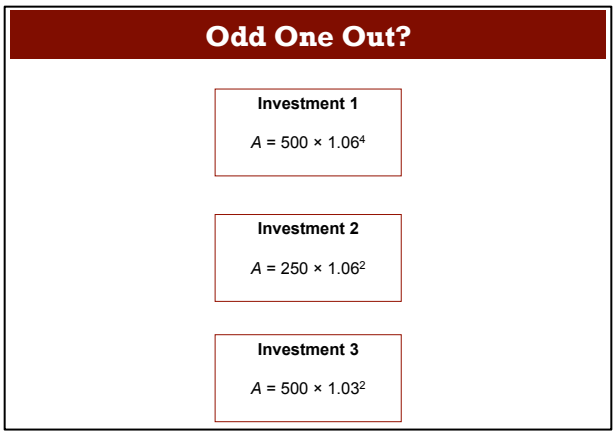
Ask students to show you their formula using their mini-whiteboards. Ask students with different answers to justify them. Encourage the rest of the class to challenge these explanations.

Look for students that use the interest rate in their formula, instead of interest: e.g. $A = 400 + 5n$ or $A = 400 + 0.05n$

Can you use your formula to figure out how much is in the bank after 5 years?

Can you check this answer by using the description of the investment plan?

To introduce compound interest, show Slide P-2 of the projector resource:



In each expression, *A* shows the value of an amount of money that has been invested for a given period of time.

How do you know that these represent compound interest, not simple interest?

What does each expression mean?

Which is the odd one out? Write down your reason on your mini-whiteboards.

Encourage students to discuss this and then ask a few with different answers to justify them. Students may reason that:

- Investment 1 is the odd one out because the money is invested over a longer period than the other two investments.
- Investment 2 is the odd one out because the initial investment is different from the other two investments.
- Investment 3 is the odd one out because the interest rate is different from the other two investments.

Try to make sure that students can see the significance of each number in the expression by asking specific questions:

What is the initial investment?

How long is the money invested for?

What is the interest rate?

Encourage students to justify their answers. Look out for students that assume the interest rate is 1.06% or 1.03%.

Can you use your calculator to work out the amount of money in each investment, after the specified period? [Investment 1: A = $631.24; Investment 2: A = $280.90; Investment 3: A = $530.45.]

Collaborative activity 1: *Card Set: Investment Plans and Formulas* (15 minutes)

Organize students into groups of two or three.

Give each group cut-up *Card Set: Investment Plans and Formulas*.

You have two sets of cards, one with descriptions of investment plans and one with the formulas. Some of the investment plans use simple interest and some use compound interest.

Using what you have learned from our discussion, take turns to match a formula with a corresponding investment plan.

There are two spare investment plan cards. Write on the blank cards the formula for these two plans.

Some students may not be able to match all the cards. Later in the lesson they will be given more cards that should help them to complete all of the matches.

Card Set: Investment Plans and Formulas

P1 Investment: $400 Simple Interest Rate: 16%	P2 Investment: $400 Compound Interest Rate: 2%
P3 Investment: $400 Simple Interest Rate: 8%	P4 Investment: $200 Compound Interest Rate: 2%
P5 Investment: $400 Compound Interest Rate: 8%	P6 Investment: $400 Simple Interest Rate: 2%

F1 $A = 400 \times 1.08^n$	F2 $A = 400 + 32n$
F3 $A = 400 \times 1.02^n$	F4 $A = 400 + 8n$
F5	F6

Whilst students work on the collaborative activity you have two tasks: to notice students' approaches and difficulties and to support student reasoning:

Note different student approaches to the task

Listen and watch students carefully. Are they matching the cards correctly? Do students within a group use the same strategies for matching the cards? Are they starting with an investment plan and looking for a formula that matches or are they interpreting the formula and then linking this with the description of the plan? How do students go about completing the blank cards? Do they refer to their already matched cards and if so, which ones? Notice whether students are addressing the difficulties they experienced in the assessment task. You may want to use the questions in the *Common issues* table to help address misconceptions.

Support student reasoning

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking. It is important that students are encouraged to engage with their partner's explanations and take responsibility for each other's understanding.

Pippa, you matched these two cards. Gita, can you explain why Pippa matched these cards?

Encourage students to think about how the formula relates to the investment plan:

[Select a formula card.] For each year, will the interest change or remain the same? How can you check your answer is correct?

Can you explain what the number 400 relates to in this formula?

How can you work out the amount in the bank for this investment plan/formula after, say, 3 years?

[Select a formula card.] If you substitute a value for n *into this formula what do you get?*

[Select an investment plan card of a simple interest investment.]
How can you calculate the interest made each year for this plan? How is this represented in a formula?

[Select a formula card.] What can you tell me about the interest rate or the interest for this investment?

Some students may assume that for simple interest, the interest added each year is the coefficient of n.

What does 32 represent in this formula? [Card F2: the amount of interest added each year.]

How can you check this? Is this the same as the interest rate?

What is the formula for an investment of $100 and a simple interest rate of 8%?

How can you check that your answer is correct?

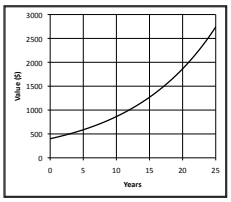
How does the formula change if the investment is $200/$400?

Collaborative activity 2: *Card Set: Graphs* and *Card Set: Tables* (20 minutes)

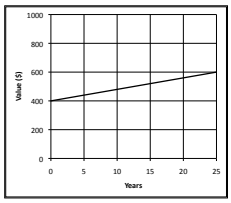
As the groups finish matching the cards, give them cut-up *Card Set: Graphs* and *Card Set: Tables*. These cards should help students check their existing matches.

Card Set: Graphs

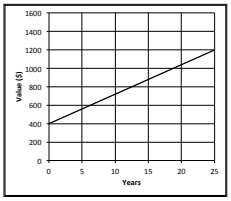
G1



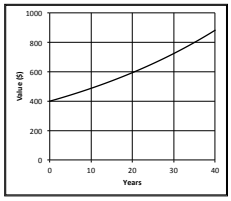
G2



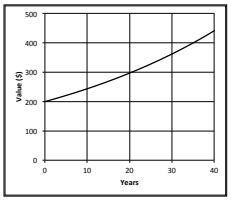
G3



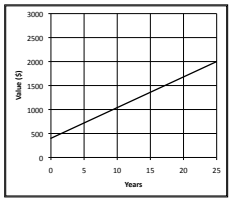
G4



G5



G6



Card Set: Tables

T1

Years	Value ($)
0	400.00
1	432.00
2	466.56
3	
4	544.20
5	587.73

T2

Years	Value ($)
0	200.00
1	204.00
2	
3	212.24
4	216.49
5	220.82

T3

Years	Value ($)
0	400.00
1	408.00
2	
3	424.00
4	432.00
5	440.00

T4

Years	Value ($)
0	400.00
1	408.00
2	
3	424.48
4	432.97
5	441.63

T5

Years	Value ($)
0	400.00
1	432.00
2	464.00
3	
4	528.00
5	560.00

T6

Years	Value ($)
0	400.00
1	464.00
2	
3	592.00
4	656.00
5	720.00

Now match the Graphs and Tables cards with the cards already matched.

You must also calculate the missing value in each table.

Observe the different strategies that students use as they do this and encourage them to try different methods and to draw links between the different representations. Try to avoid making all the connections for the students.

Some may think about the shapes of the graphs or the differences between rows in the tables:

Can you separate these graphs and tables into those that represent simple interest and those that represent compound interest? [Some are linear and some are exponential.]

Which investments go up by equal amounts each year and which go up by increasing amounts each year? How does the plan/formula/table/graph show this? What does this tell you about the investment plan?

In trials, some students called the non-linear curves 'quadratic'. You may need to help your students distinguish between quadratic and exponential functions.

Some students may substitute into an equation (e.g. $n = 0$ or $n = 25$) and check if the answer matches a point on one of the graphs.

John, you used substitution to calculate the value of the investment after 25 years. Can you now think of a different method? [Students could compare slopes of two graphs.]

Some students may assume that the slope represents the interest rate.

[Select a linear graph card.] What does the slope of this graph represent? [The interest added each year. The slope can be calculated by multiplying the simple interest rate by the amount invested.]

Extending the lesson over two days

If you are taking two days to complete the unit you might want to end the first lesson here. Before the end of the lesson, ask students to make a note of the cards they have successfully matched so far. Then, at the start of the second day, allow the students time to recreate their matches and familiarize themselves with their work, before they share their work with another group.

Sharing work (15 minutes)

When students have completed the task, ask them to check their work against that of a neighboring group.

Check to see which matches are different from your own.
If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

You may then need to consider whether to make any changes to your own work.

It is important that everyone in both groups understands the math. You are responsible for each other's learning.

Collaborative activity 3: *Card Set: Statements* (20 minutes)

Give each group cut-up *Card Set: Statements*, a large sheet of paper for making a poster, and a glue stick.

In your groups you are now going to match one of these statements to the cards already on your desk.

Statement Card S1 matches two sets of cards.

You may need to explain to students that the phrase 'return for your money' means the interest gained for each $100 originally invested.

Students may find it helpful to sketch the graphs for two different plans onto the same set of axes when comparing investments. They can use their mini-whiteboards to do this.

Card Set: Statements

S1 These two investments will take the same time to double your money.	S2 This investment will double your money in 12 years 6 months.
S3 This investment gives the worst return for your money over two years or more.	S4 This investment is the best one over 10 years.
S5 This investment is the best one over 20 years.	

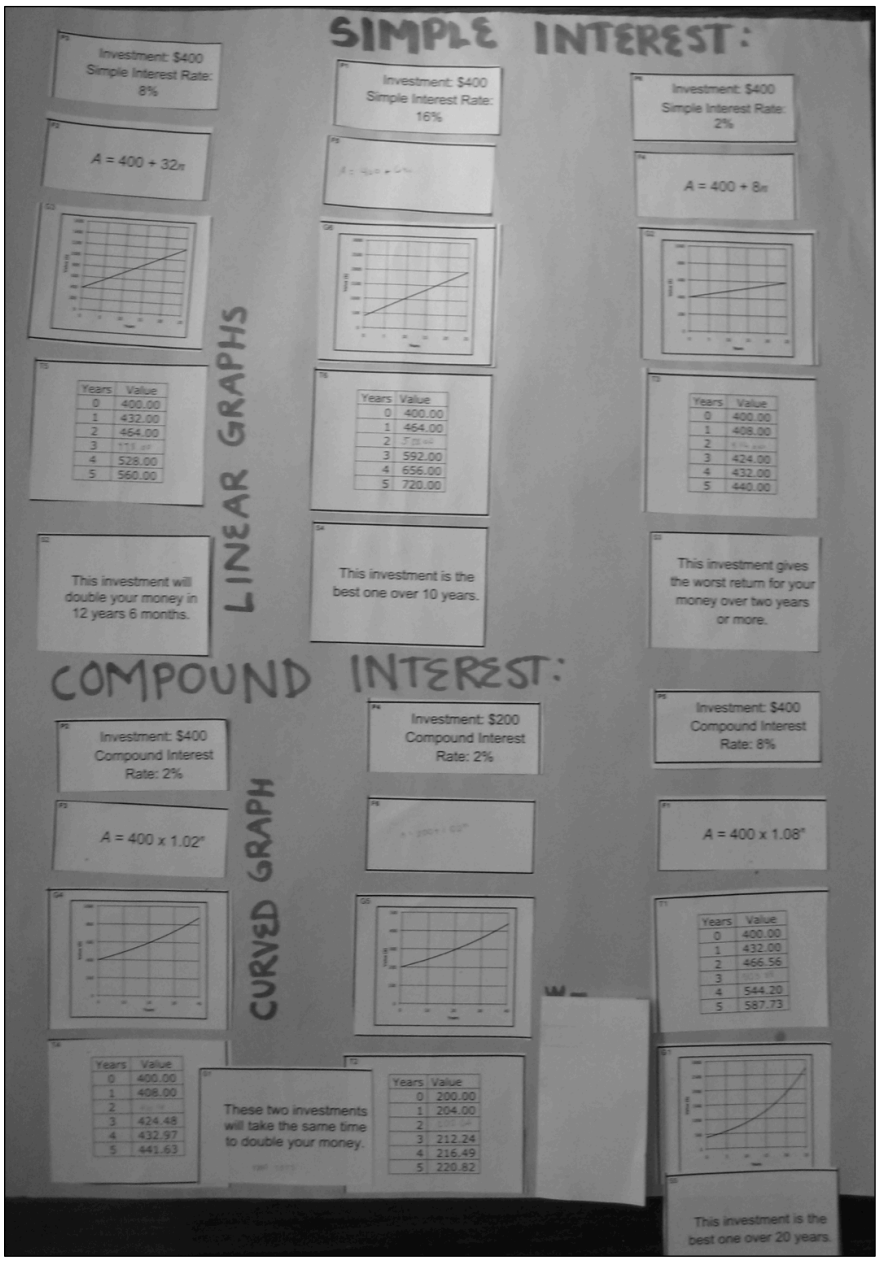
Encourage students to check their matches. For example, if they use graphs to match the statements, ask:

> *Chris, you used this graph card to match this statement. Can you now use a different card to check your pairing is correct?*

When students have completed the task, ask them to glue the cards on to their poster paper, writing next to the cards an explanation for the matched cards.

> *Once you have investigated the statement and are happy with your findings, glue the statement and investment plans on to your poster.*
>
> *Add reasons for all your matches.*

Completed posters may look something like this:

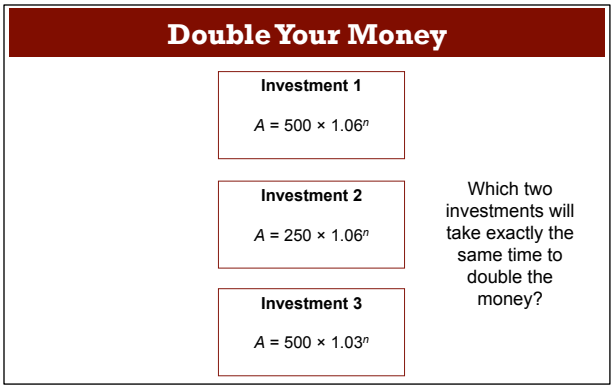


Whole-class discussion (15 minutes)

Organize a whole-class discussion about different strategies used to match the cards. You may first want to select a set of cards that most groups matched correctly. This approach can encourage good explanations. Then select one or two cards that most groups found difficult to match. Ask other students their views on which method is easiest to follow, as well as contributing ideas of alternative approaches.

If there is time, you may like to consider the following extension: *Double Your Money*.

Show Slide P-3 of the projector resource:



At the start of the lesson, we noted that Investment 1 and Investment 3 invest the same amount of money and Investment 1 and 2 have same interest rate. Now try to answer the question. [Investments 1 and 2 will double the money in the same time, as they have the same interest rate.]

Ask students to write their answers on their mini-whiteboards.

After a few minutes ask students with different answers to justify them. Encourage other students to challenge their explanations.

Does it matter how much money is invested? [No.]

How can you answer the question without doing any calculations?

In order to double your money, what should 1.06^n equal? [2.]

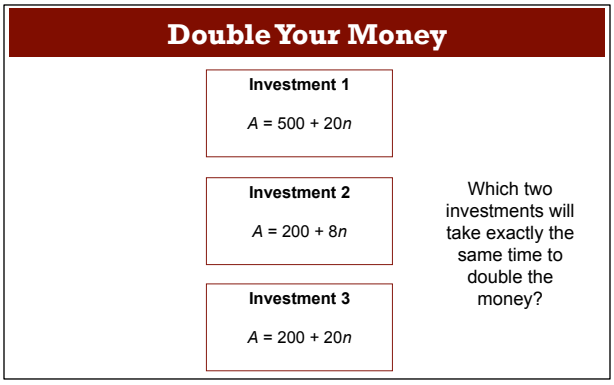
Show me two different compound investment plans that take the same time to double your money.

Show me two different compound interest investment plans that take different times to double your money. Which one would double your money first? How do you know?

Students who are confident using the formula could be encouraged to provide an algebraic solution. For example, to show that doubling any amount of money takes the same amount of time, writing the starting amount as x and the final amount as $2x$ gives the equation $(1.06^n)x = 2x$, the x's cancel out, leaving the same equation to be solved each time ($1.06^n = 2$).

Some students may ask how to find the value of n. If you have time and you think your students will understand, you may want to explain how logarithms can be used to make n the subject of the equation.

Now show Slide P-4 of the projector resource:



For Investment 1, what is the value of A when the initial investment is doubled? [$1000.]

Does the time it takes to double your money depend on how much money is invested? [The time it takes to double your money for simple interest investments = Amount Invested ÷ Interest. If the ***interest rates*** *are the same, then it does not matter how much money is invested.]*

Can you think of a quick way to answer the question?

Show me two different simple interest investment plans that take the same time to double your money.

Show me two different simple interest investment plans that take different times to double your money. Which one would double your money first? How do you know?

Follow-up lesson: *Making Money? (revisited)* (20 minutes)

Return their original assessment task to the students together with a copy of *Making Money? (revisited)*.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Look at your original response and read through my questions. Answer these questions and revise your response.

Now, using what you have learned, try to answer the questions on the new task, Making Money? (revisited).

Explain how you figure out your answers and record all your calculations.

Some teachers give this for homework.

Extension

One natural extension to this work would be to consider how much an investment will pay if it is withdrawn part way through a year. This leads to a consideration of the continuity of the growth function.

For example, if the annual compound interest rate is 8%, then:

$$\text{Approximate value after } n \text{ years } = A \times (1.08)^n = A \times (1.08^{\frac{1}{12}})^{12n}$$

Replacing $12n$ by m, this gives:

$$\text{Value after } m \text{ months} = A \times (1.0064)^m$$

SOLUTIONS

Assessment task: *Making Money?*

1.

Years	Value at Simply Savings ($)	Value at Compound Capital ($)
0	200.00	200.00
1	220.00	216.00
2	240.00	233.28
3	260.00	251.94
4	280.00	272.10
5	300.00	293.87

2. (a) The growth of her money at Simply Savings is best shown by graph B.

   (b) The growth of her money at Compound Capital is best shown by graph A.

   (c) Students may reason that if the interest is only added at the end of the year, then the graph would have discrete steps. These graphs assume that interest is added continuously.

3. $A = 200 + 20n$; $A = 200 \times 1.08^n$.

4. For the first five investment years Simply Savings Bank is the better plan, however at the end of Year 7 Compound Capital starts to perform better ($342.76 compared to $340.) Compound Capital is a better investment for savers wanting to invest for 7 years or more.

Assessment task: *Making Money? (revisited)*

1.

Years	Value at Compound Investments ($)	Value at Simple Investments ($)
0	300.00	300.00
1	315.00	318.00
2	330.75	336.00
3	347.29	354.00
4	364.65	372.00
5	382.88	390.00

2. (a) The growth of her money at Compound Investments is best shown by graph C.

   (b) The growth of her money at Simple Investments is best shown by graph B.

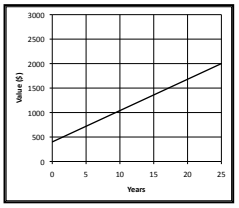
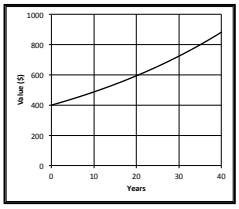
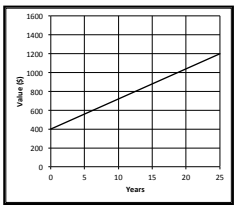
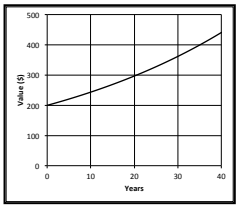
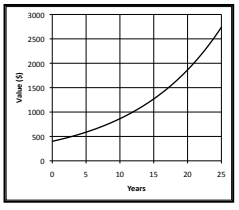
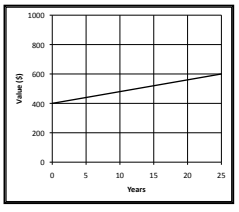
   (c) Students may reason that if the interest is only added at the end of the year, then the graph would have discrete steps. These graphs assume that interest is added continuously.

3. $A = 300 \times 1.05^n$; $A = 300 + 18n$.

4. For the first five investment years Simple Investments is the better plan, however at the end of Year 9 Compound investments starts to perform better ($465.40 compared to $462.00.) Compound Investments is a better investment scheme for savers wanting to invest for 9 years or more.

Collaborative activity

The parts in **bold** are to be provided by the student.

<table>
<tr>
<td>P1
Investment: $400
Simple Interest
Rate: 16%</td>
<td>F6
A = 400 + 64n</td>
<td>G6
 
</td>
<td>T6
<table>
<tr><th>Years</th><th>Value</th></tr>
<tr><td>0</td><td>400.00</td></tr>
<tr><td>1</td><td>464.00</td></tr>
<tr><td>2</td><td>528.00</td></tr>
<tr><td>3</td><td>592.00</td></tr>
<tr><td>4</td><td>656.00</td></tr>
<tr><td>5</td><td>720.00</td></tr>
</table></td>
<td>S4
This investment is the best one over 10 years.</td>
</tr>
<tr>
<td>P2
Investment: $400
Compound
Interest Rate: 2%</td>
<td>F3
A = 400 x 1.02ⁿ</td>
<td>G4
 
</td>
<td>T4
<table>
<tr><th>Years</th><th>Value</th></tr>
<tr><td>0</td><td>400.00</td></tr>
<tr><td>1</td><td>408.00</td></tr>
<tr><td>2</td><td>416.16</td></tr>
<tr><td>3</td><td>424.48</td></tr>
<tr><td>4</td><td>432.97</td></tr>
<tr><td>5</td><td>441.63</td></tr>
</table></td>
<td>S1 (P2 and P4)
These two investments will take the same time to double your money.</td>
</tr>
<tr>
<td>P3
Investment: $400
Simple Interest
Rate: 8%</td>
<td>F2
A = 400 + 32n</td>
<td>G3
 
</td>
<td>T5
<table>
<tr><th>Years</th><th>Value</th></tr>
<tr><td>0</td><td>400.00</td></tr>
<tr><td>1</td><td>432.00</td></tr>
<tr><td>2</td><td>464.00</td></tr>
<tr><td>3</td><td>496.00</td></tr>
<tr><td>4</td><td>528.00</td></tr>
<tr><td>5</td><td>560.00</td></tr>
</table></td>
<td>S2
This investment will double your money in 12 years 6 months.</td>
</tr>
<tr>
<td>P4
Investment: $200
Compound
Interest Rate: 2%</td>
<td>F5
A = 200 x 1.02ⁿ</td>
<td>G5
 
</td>
<td>T2
<table>
<tr><th>Years</th><th>Value</th></tr>
<tr><td>0</td><td>200.00</td></tr>
<tr><td>1</td><td>204.00</td></tr>
<tr><td>2</td><td>208.08</td></tr>
<tr><td>3</td><td>212.24</td></tr>
<tr><td>4</td><td>216.49</td></tr>
<tr><td>5</td><td>220.82</td></tr>
</table></td>
<td>S1 (P2 and P4)
These two investments will take the same time to double your money.</td>
</tr>
<tr>
<td>P5
Investment: $400
Compound
Interest Rate: 8%</td>
<td>F1
A = 400 x 1.08ⁿ</td>
<td>G1
 
</td>
<td>T1
<table>
<tr><th>Years</th><th>Value</th></tr>
<tr><td>0</td><td>400.00</td></tr>
<tr><td>1</td><td>432.00</td></tr>
<tr><td>2</td><td>466.56</td></tr>
<tr><td>3</td><td>503.88</td></tr>
<tr><td>4</td><td>544.20</td></tr>
<tr><td>5</td><td>587.73</td></tr>
</table></td>
<td>S5
This investment is the best one over 20 years.</td>
</tr>
<tr>
<td>P6
Investment: $400
Simple Interest
Rate: 2%</td>
<td>F4
A = 400 + 8n</td>
<td>G2
 
</td>
<td>T3
<table>
<tr><th>Years</th><th>Value</th></tr>
<tr><td>0</td><td>400.00</td></tr>
<tr><td>1</td><td>408.00</td></tr>
<tr><td>2</td><td>416.00</td></tr>
<tr><td>3</td><td>424.00</td></tr>
<tr><td>4</td><td>432.00</td></tr>
<tr><td>5</td><td>440.00</td></tr>
</table></td>
<td>S3
This investment gives the worst return for your money over two years or more.</td>
</tr>
</table>

Making Money?

Mary is going to invest some money. She sees two advertisements:

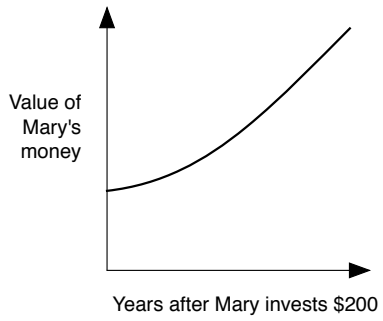
Simply Savings Bank	**Compound Capital Bank**
Simple interest rate: 10% per year.	Compound interest rate: 8% per year.

1. Mary invests $200 in each bank.
   Use a calculator to figure how much she will have in each bank at the end of each year.
   Show all your work.

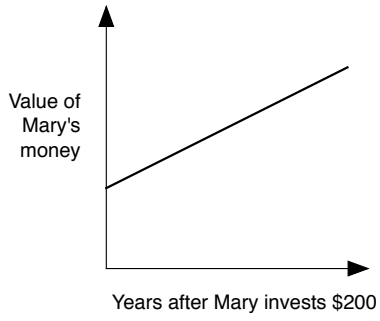
Years	Value at Simply Savings in dollars	Value at Compound Capital in dollars
0	200.00	200.00
1	220.00	
2		
3		
4		
5		

2. Which of the graphs below best shows how Mary's money will grow in each bank?

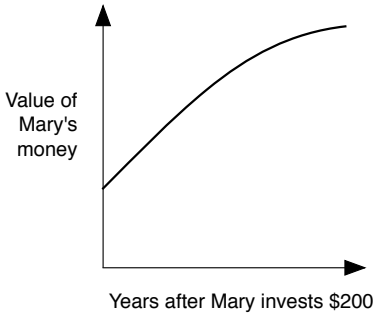
Graph A



Graph B



Graph C



(a) The growth of her money at Simply Savings is best shown by graph ……

(b) The growth of her money at Compound Capital is best shown by graph ……

(c) If you think that none of these graphs are a good description, explain why below:

3. Write down a formula for calculating the amount of money in each of these banks after n years.

4. Mary wants to invest some money for 5 years or more.
   Which bank should she choose?
   Give full reasons for your answer.

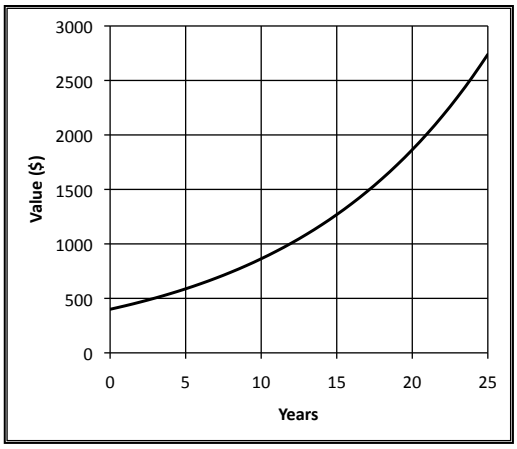
Card Set: Investment Plans and Formulas

P1 Investment: $400 Simple Interest Rate: 16%	**P2** Investment: $400 Compound Interest Rate: 2%
P3 Investment: $400 Simple Interest Rate: 8%	**P4** Investment: $200 Compound Interest Rate: 2%
P5 Investment: $400 Compound Interest Rate: 8%	**P6** Investment: $400 Simple Interest Rate: 2%

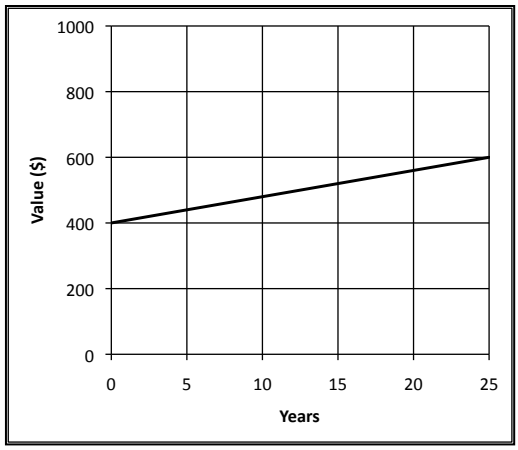
F1 $A = 400 \times 1.08^n$	**F2** $A = 400 + 32n$
F3 $A = 400 \times 1.02^n$	**F4** $A = 400 + 8n$
F5	**F6**

Card Set: Graphs

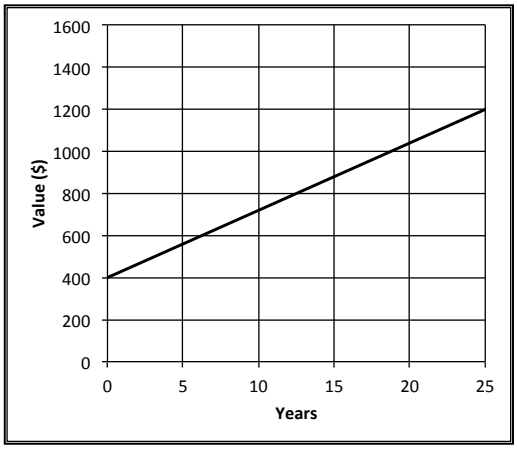
G1



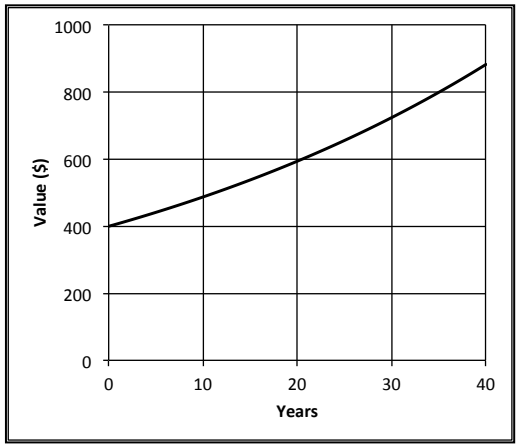
G2



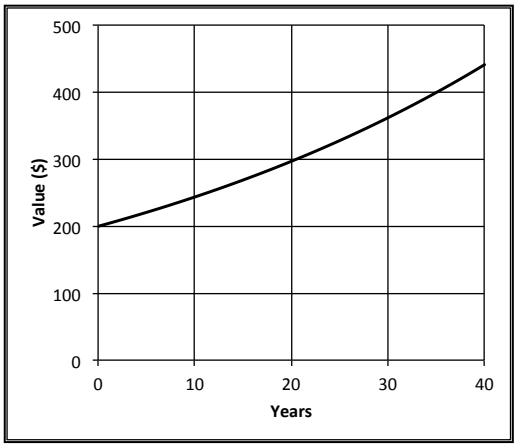
G3



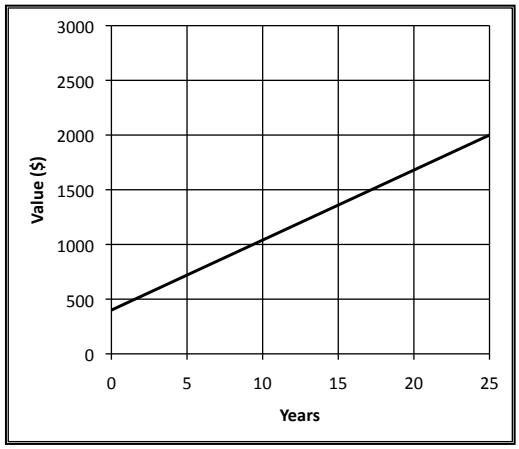
G4



G5



G6



Card Set: Tables

T1

Years	Value ($)
0	400.00
1	432.00
2	466.56
3	
4	544.20
5	587.73

T2

Years	Value ($)
0	200.00
1	204.00
2	
3	212.24
4	216.49
5	220.82

T3

Years	Value ($)
0	400.00
1	408.00
2	
3	424.00
4	432.00
5	440.00

T4

Years	Value ($)
0	400.00
1	408.00
2	
3	424.48
4	432.97
5	441.63

T5

Years	Value ($)
0	400.00
1	432.00
2	464.00
3	
4	528.00
5	560.00

T6

Years	Value ($)
0	400.00
1	464.00
2	
3	592.00
4	656.00
5	720.00

Card Set: Statements

S1 These two investments will take the same time to double your money.	S2 This investment will double your money in 12 years 6 months.
S3 This investment gives the worst return for your money over two years or more.	S4 This investment is the best one over 10 years.
S5 This investment is the best one over 20 years.	

Making Money? (revisited)

Jack is going to invest some money. He sees two advertisements:

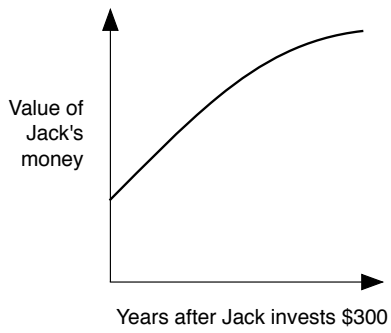
Compound Investments	**Simple Investments**
Compound interest rate: 5% per year.	Simple interest rate: 6% per year.

1. Jack invests $300 in each scheme.
   Use a calculator to figure how much he will have in each scheme at the end of each year.
   Show all your work.

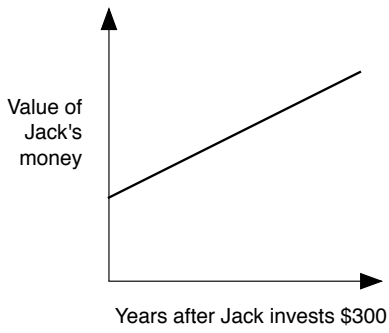
Years	Value at Compound Investments ($)	Value at Simple Investments ($)
0	300.00	300.00
1		
2		
3		
4		
5		

2. Which of the graphs below best shows how Jack's money will grow in each scheme?

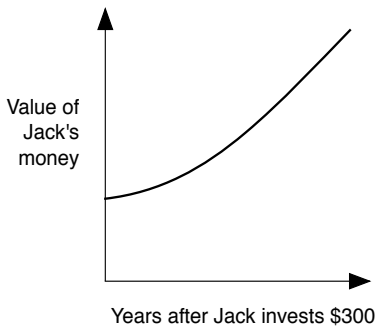
Graph A



Graph B



Graph C



(a) The growth of his money at Compound Investments is best shown by graph ……

(b) The growth of his money at Simple Investments is best shown by graph ……

(c) If you think that none of these graphs are a good description, explain why below:

3. Write down a formula for calculating the amount of money in each of these schemes after n years.

4. Jack wants to invest some money for 5 years or more.
   Which scheme should he choose?
   Give full reasons for your answer.

Odd One Out?

Investment 1
$100
Simple Interest Rate: 5%

Investment 2
$400
Simple Interest Rate: 5%

Investment 3
$200
Simple Interest Rate: 10%

Odd One Out?

Investment 1

$A = 500 \times 1.06^4$

Investment 2

$A = 250 \times 1.06^2$

Investment 3

$A = 500 \times 1.03^2$

Double Your Money

Investment 1

$A = 500 \times 1.06^n$

Investment 2

$A = 250 \times 1.06^n$

Investment 3

$A = 500 \times 1.03^n$

Which two investments will take exactly the same time to double the money?

Double Your Money

Investment 1

$A = 500 + 20n$

Investment 2

$A = 200 + 8n$

Investment 3

$A = 200 + 20n$

Which two investments will take exactly the same time to double the money?

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and **Daniel Pead**

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Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org