CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Solving Quadratic Equations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Solving Quadratic Equations

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to solve quadratics in one variable. In particular, the lesson will help you identify and help students who have the following difficulties:

- Making sense of a real life situation and deciding on the math to apply to the problem.
- Solving quadratic equations by taking square roots, completing the square, using the quadratic formula, and factoring.
- Interpreting results in the context of a real life situation.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

A-REI: Solve equations and inequalities in one variable.
A-CED: Create equations that describe number or relations.
G-SRT: Define trigonometric ratios and solve problems involving right triangles.
G-MG: Apply geometric concepts in modeling situations.

This lesson also relates to **all** the *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, and 4:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

- Before the lesson students attempt the problem individually. You then review their work and write questions for students to answer to help them improve their solutions.
- At the start of the lesson, students work alone, answering your questions. Students are then grouped and work collaboratively on the same task. In the same small groups, students are given some sample solutions to comment on and evaluate. In a whole-class discussion, students explain and compare the different solution strategies they have seen and used.
- In a follow-up lesson students review what they have learned.

MATERIALS REQUIRED

- Each student will need a copy of *Cutting Corners* and the *How Did You Work?* questionnaire.
- Each small group of students will need an enlarged copy of the *Cutting Corners* task and a copy of each of the *Sample Responses to Discuss*.
- Calculators should be available for students who require them.
- There is also a projector resource to help with whole-class discussions.

TIME NEEDED

15 minutes before the lesson, a 90-minute lesson (or two shorter lessons), and 15 minutes in a follow-up lesson. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Cutting Corners* (15 minutes)

Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, to find out the kinds of difficulties students have with it and to see the mathematics they choose to use. You should then be able to target your help more effectively in the next lesson.

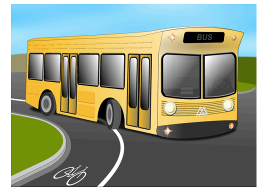
Give each student a copy of the *Cutting Corners* task.

Introduce the task and help the class to understand the problem and its context. You may like to show a short movie to help your introduction, such as: http://tinyurl.com/BusTurning.

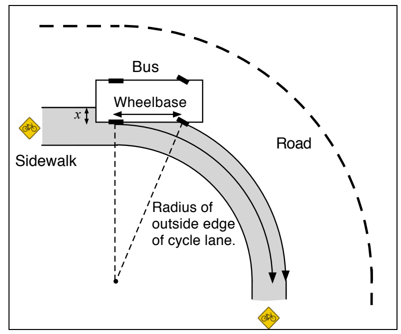
Cutting Corners

When a bus turns a corner, it must swing out so that its rear wheels don't go into the cycle lane.

In this picture, as the bus goes round the corner, the front wheel is on the edge of the cycle lane, but the rear wheel cuts into the cycle lane.



The diagram below shows the geometry of this situation.



The distance between the front and rear wheel is called the wheelbase, w.
Let r represent the radius of the outside edge of the cycle lane.
The distance marked x shows the amount by which the rear wheels cuts into the cycle lane.

> *Think about the wheels on the front and back of a bus.*
> *What can you tell me about how they turn a corner?*
> *What remains the same?*
> *What changes? [The front ones turn to steer the bus, the rear wheels are fixed parallel to the body of the bus. The axle distance between each pair of wheels is fixed.]*
>
> *The fixed distance between the hubs of the front and back wheel is called the **wheelbase**.*
>
> *When the front wheels turn a corner, they describe a circle on the road.*
> *What happens to the back wheel at the same time? [The back wheel cuts the corner.]*

Students may find the dynamic situation hard to visualize. A physical demonstration using, for example, a bicycle, or a toy truck could be helpful. If the wheels do not slip sideways, they will always be tangential to the circle they describe. The back wheel changes direction as soon as the front wheel does, a wheelbase length behind the front wheel. The back wheel is pulled forwards and inwards, turning a smaller circle than the front wheel.

> *The diagram on your sheet uses math to represent the bus scenario.*
> *Read through the task. Read the diagram carefully. Try to answer the questions as carefully as you can. Show all your working so that I can understand your reasoning.*

It is important that students, as far as possible, are allowed to answer the questions without assistance. If students are struggling to get started then ask questions that help them understand what is required, but make sure you do not do the task for them.

Students who sit together often produce similar responses and then when they come to compare their work, they have little to discuss. For this reason we suggest that, when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on page T-4. We suggest that you make a list of your own questions, based on your students' work, using the ideas below. We recommend you:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students at the beginning of the lesson.

Common issues	Suggested questions and prompts
Has difficulty getting started For example: The student does not understand the diagram.	• Describe fully the position in the road of the front and back wheels at the start of the turn. Describe fully the position in the road of the front and back wheels at the end of the turn. • What do you know? What do you need to find out? Try labelling the diagram. • What shape can you see in the diagram?
Relevant mathematics is not identified For example: The student does not refer to the Pythagorean Theorem (Q1).	• What mathematics have you studied that connects with the situation? • What shape can you see in the diagram? • What kind of equation are you given?
Incorrect labelling of the diagram (Q1) For example: In labelling the radius of the circle produced by the back wheel, the student adds x to r, rather than subtracting x from r, producing the equation $r^2 = (r + x)^2 + w^2$.	• What does r represent? What does x represent? • What is the connection between r and x?
Algebraic errors For example: The student mistakenly changes the sign of a term when clearing brackets: $(r-x)^2 + w^2 = r^2 \Rightarrow r^2 - 2rx - x^2 + w^2 = r^2$	• Check your algebra manipulation.
Explanation is unclear or incomplete For example: The student does not label the diagram, or the student does not explain the math used. Or: The student does not explain why they select just one root of the quadratic (Q2).	• Suppose someone in another class is reading your work to figure out the solution. Would they be able to understand your work? • Now explain why you have just used this root in your answer.
Finding not related to the context For example: The student does not explain which solution is relevant given the context.	• Look again at the diagram. Do your answers make sense? How do the numbers you have found help you to answer the question?
Inefficient method used For example: The student tries to use 'guess and check' to solve a quadratic (Q2).	• Can you think of a more efficient method? • What kind of equation are you working with? Can you think of other ways of solving that equation?
Only positive square roots are accounted for	• Check your solution to the quadratic. Could x be any other value?
No answer for Q2b	• Draw a diagram of the situation. • What lengths do you already know? • The values of w and r have not changed, but what will the value of x be now?

SUGGESTED LESSON OUTLINE

Individual work (10 minutes)

Give students back their work. If you did not add questions to individual pieces of work, write your list of questions on the board. Students are to select questions appropriate to their own work and spend a few minutes answering them.

Recall what we were looking at in a previous lesson. What was the task about?

Today we are going to work together trying to improve on these initial attempts.

I have read your solutions and have some questions about your work.
I would like you to work on your own to answer my questions for about ten minutes.

Collaborative activity (20 minutes)

Organize the class into small groups of three students. Give each group an enlarged copy of the *Cutting Corners* task.

Deciding on a Strategy

Ask students to share their ideas about the task and plan a joint solution. A summary of how students are to work together is shown on Slide P-2:

Planning a Joint Solution

1. Take turns to explain how you did the task and how you now think it could be improved.
2. Listen carefully to explanations.
   – Ask questions if you don't understand.
   – Discuss with your partners:
     • What you like/dislike about your partner's math.
     • How clear their work is.
     • How their work could be improved.
3. Once everyone in the group has explained their solution, plan a joint method that is better than each of the separate solutions.

Everyone in the group must agree on the method

Implementing the Strategy

Students are now to write their joint solution on the handout.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different approaches to the task

Notice how students talk about the context and whether and how they link this to the diagram.

Notice the math students choose to use on the problem. Do they notice that the lengths given form a right triangle? Do they then realize that the Pythagorean Theorem is relevant? What method are students using to solve the quadratic equations? Note if and how students use algebra. What errors do students make when manipulating equations? Are there any errors in their numerical calculations? Do they find both solutions when solving a quadratic?

Notice whether students reinterpret their numerical solutions in terms of the context. Do they notice when one solution does not make sense, given the question?

Attend also to students' mathematical decisions. Do they talk about their own progress? Do they notice if they have chosen a strategy that does not seem to be productive? If so, what do they do?

You can use the information you find out about students to focus the whole-class discussion at the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students clarify their own thinking and promote further progress. Encourage students to check their work and detect errors. Encourage them to explain their answers to each other and question each other's explanations. You may find it helpful to use some of the questions in the *Common issues* table.

If the whole-class is struggling on the same issue, you could write a couple of relevant questions on the board, or hold a brief whole-class discussion. You could also give any students really struggling to get started one of the *Sample Responses to Discuss*.

Whole-class discussion (10 minutes)

After students have had enough time to attempt the problem you may want to hold a brief whole-class discussion. Have students solved the problem using a variety of methods? Or have you noticed some interesting ways of working or some incorrect methods, if so, you may want to focus the discussion on these. If you have noticed different groups using similar strategies but making different assumptions you may want to compare solutions.

Collaborative analysis of *Sample Responses to Discuss* (30 minutes)

This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

Give each group a copy of the *Sample Responses to Discuss*. If there is not time for all groups to look at all solutions, be selective about what you give out. For example, groups that have successfully completed the task using one method will benefit from looking at different approaches. Groups that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

Here are some solutions to the Cutting Corners task produced by students in another class. I would like you to choose one of the solutions and read it together, carefully.

Try to understand the method the student has chosen and any errors he or she has made. Answer the questions on the sheet, to correct the work and comment on the accuracy of the work.

Then choose the next response to work on. Work your way through the responses one by one.

Slide P-3 of the projector resource provides details of how students should work together:

Evaluating Sample Student Responses

1. Take turns to work through a student's solution.
   - Try to understand the method the student has chosen and any errors he or she has made.
2. Explain your answer to the rest of the group.
3. Listen carefully to explanations.
   - Ask questions if you don't understand.
4. Once everyone is satisfied with the explanations, write the answers to the questions below the student's solution.
   - Make sure the student who writes the answers is not the student who explained them.

The questions on each sample response should encourage students to do more than check to see if the answer is correct.

During the small group work, support the students as before. Encourage students to focus on the math of the student work, rather than surface features such as neatness and spelling. Check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample responses and those methods the students used in the group work.

Guy's Solution

Guy correctly substitutes into the equation $x^2 - 2xr + w^2 = 0$ the values for r and w. He initially tries to factorize the quadratic. After three attempts Guy realizes this method is not suitable so he changes to 'completing the square' to figure out a value for x.

The last line of Guy's solution is incorrect. It should be:

$x - 17 = 13.75$ or $x - 17 = -13.75$

Therefore $x = 30.75$ feet or $x = 3.25$ feet.

Guy has not considered the practical implications of $x = 30.75$ feet. The radius of the outside edge of the cycle lane is only 17 feet. Therefore the bus cuts the cycle lane by 3.25 feet or 3 feet 3 inches.

$r = 17 \quad w = 10$

$x^2 - 34x + 100 = 0$

$(x-10)(x-10)$

$(x-20)(x-5)$

$(x-25)(x-4)$

$x^2 - 34x + 100 = 0$

$(x-17)^2 - 289 + 100 = 0$

$(x-17)^2 = 189$

$x - 17 = 13.75 \qquad \underline{x = 30.75}$

Where has Guy made a mistake? What does he need to do to correct this mistake?

Ryan's Solution

Ryan correctly substitutes into the equation $x^2 - 2xr + w^2 = 0$ the values for r and w. He uses the 'quadratic formula' to figure out values for x. However he makes a mistake when substituting into this formula. The correct solution for x is:

$$x = \frac{34 \pm \sqrt{34^2 - 4 \times 1 \times 100}}{2 \times 1}$$

$$= \frac{34 \pm \sqrt{756}}{2}$$

$$x = 30.75 \text{ or } x = 3.25$$

Ryan has not considered the practical implications of a negative value for x.

$x^2 - 2xr + w^2 = 0$

$r = 17 \quad w = 10$

$x^2 - 34x + 100 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-34 \pm \sqrt{34^2 - 4.1.100}}{2}$

$= \dfrac{-34 \pm \sqrt{756}}{2}$

$x = -30.75 \quad \text{or} \quad x = -3.25$

Does Ryan's result make practical sense? Why/Why not?

Donna's Solution

Donna uses guess and check. Her method is not very efficient and her answer is not very accurate. Donna could organize her work into a table. Donna has not considered the practical implications of x =31 feet. The radius of the outside edge of the cycle lane is only 17 feet.

There is another value for x that fits into the equation.

Try $x = 10$

$10^2 - 34 \times 10 + 100 = -140$

$x = 20$

$20^2 - 34 \times 20 + 100 = -180$

$x = 30$

$30^2 - 34 \times 30 + 100 = -2($

$x = 40$

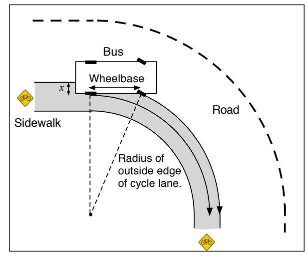
$40^2 - 34 \times 40 + 100 = 340$

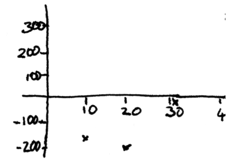
x must be between 30 and 40. Try $x = 35$

$35^2 - 34 \times 35 + 100 = 135$

$x = 32$ $\quad 32^2 - 34 \times 32 + 100 = 36$

$x = 31$ $\quad 31^2 - 34 \times 31 + 100 = 7$ $\quad x = 31$





Does Donna's result make practical sense? Why/Why not?

Sketch the graph from $x = 0$ to $x = 50$.

What value of x should Donna try?

Liam's Solution

Liam has used an efficient method, however he has not labeled the diagram.

Liam should explain that a represents the radius of the circle turned by the rear wheel.

Liam makes a mistake in his manipulation of the equation. The solution should be:

$a^2 = 289 - 100 = 189$

$a = 13.75$ or $a = -13.75$.

$17^2 = a^2 + 10^2$

$a^2 = 289 + 100$

$= 389$

$a = 19{,}72$

A negative value for a is not possible in this context, therefore $x = 17 - 13.75 = 3.25$.

The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches).

Extension Task

If any students complete the task quickly, ask them to write about practical changes that the bus driver could adopt to help prevent the bus cutting into the cycle lane.

Whole-class discussion: comparing different approaches (20 minutes)

Organize a whole-class discussion to consider the different approaches used in the *Sample Responses to Discuss*. Ask the students to compare the different solution methods.

Which of the student's methods did you think was the most effective? Why?

Which approach did you find difficult to understand? Why?

How could the work be improved?

Focus the discussion on parts of the small group tasks students found difficult. To critique the different strategies, you can use the questions on the *Sample Responses to Discuss* sheets and Slides P-4 to P-8 of the projector resource.

Follow-up lesson: review solutions to *Cutting Corners* (15 minutes)

Give each student a copy of the *How Did You Work?* questionnaire. This is intended to help students monitor and review their progress.

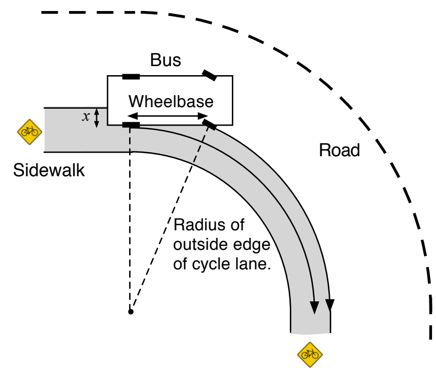
> *Read through your first solution. Think about all the work you've done on this problem: the first time you tried it, when you used my questions working alone, when you worked with your partners.*
>
> *Fill in the questions as you reflect on your experience.*

Some teachers give this task as a homework task.

If you have time you may also want ask students to use what they have learned to attempt the task again.

SOLUTIONS

1. The radius of the circle formed by the back wheel is $r-x$. This radius is perpendicular to the tangent to the circle, the line segment formed by the wheelbase. Thus r, $r-x$ and w form a right-angled triangle.



Using the Pythagorean Theorem:

$$r^2 = (r-x)^2 + w^2$$

$$r^2 = r^2 - 2xr + x^2 + w^2$$

$$x^2 - 2xr + w^2 = 0$$

2a. If we take $w = 10$ and $r = 17$ and substitute into the equation:

$$x^2 - 34x + 100 = 0$$

There is a range of methods students might use to figure out the value for x.

Completing the square:

$$(x-17)^2 - 172 + 100 = 0$$

$$(x-17)^2 - 189 = 0$$

$$(x-17)^2 = 189$$

$x - 17 = 13.75$ or $x - 17 = -13.75$ (Taking the square root of both sides.)

$x = 30.75$ **or** $x = 3.25$.

The outside edge of the cycle lane is a circle of radius only 17 feet, therefore $x = 30.75$ feet is not a possible solution in this context. (See T-12 for a discussion of this issue.)

The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches.)

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1,\ b = -34,\ \text{and } c = 100$$

$$x = \frac{34 \pm \sqrt{34^2 - 4 \bullet 1 \bullet 100}}{2 \bullet 1}$$

$$= \frac{34 \pm \sqrt{756}}{2}$$

$$x = 30.75 \text{ or } x = 3.25$$

The radius of the outside edge of the cycle lane is only 17 feet, therefore $x = 30.75$ feet is impossible. The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches.)

Replacing $r - x$ with the variable a.

Note that a represents the radius of the circle described by the rear wheel.

$$r^2 = a^2 + w^2$$

$$17^2 = a^2 + 10^2$$

$$a^2 = 289 - 100 = 189$$

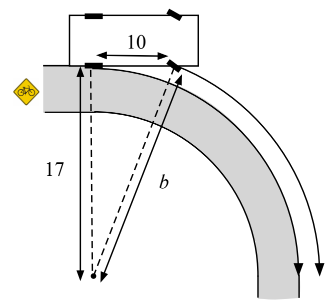
$a = 13.75$ **or** $a = -13.75$.

$x = r - a \quad \Rightarrow \quad x = 17 - 13.75$ **or** $x = 17 + 13.75$

$x = 3.25$ **or** $x = 30.75$

The radius of the outside edge of the cycle lane is only 17 feet, therefore x = 30.75 feet is impossible. The bus cuts into the cycle lane by 3.25 feet (3 feet 3 inches.)

2b. The student now needs to reinterpret the situation, to find the value of r when $x = 0$.



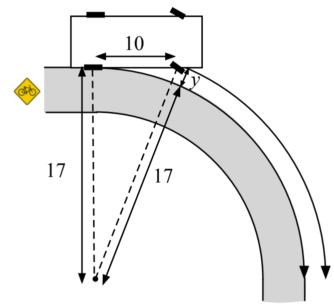
Using the Pythagorean Theorem:

$b^2 = 17^2 + 10 = 389$

$b = 19.72$ or $b = -19.72$.

A negative value for b is impossible given the context, therefore the front wheel is:

19.72 – 17 = 2.72 feet (approx. 2 feet 9 inches) from the outside edge of the cycle lane.



Students may also solve the quadratic equation:

$(17 + y)^2 = 17^2 + 10^2$

$289 + 34y + y^2 = 289 + 100$

$y^2 + 34y - 100 = 0$.

They could do this by either completing the square or using the quadratic formula.

Alternatively, students could start with the equation from Q1 and, instead of substituting in $r = 17$, substitute $r = 17 + x$.

Simplifying this leads to the same quadratic equation as above.

Completing the square:

$(y + 17)^2 - 289 - 100 = 0$

$(y + 17)^2 = 389$

$y + 17 = 19.72$ **or** $y + 17 = -19.72$

$y = 19.72 - 17 = 2.72$ **or** $y = -19.72 - 17 = -36.72$

A negative value for y is impossible, therefore the front wheel is 2.72 feet (approx. 2 feet 9 inches) from the outside edge of the cycle lane.

Using the quadratic formula:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1,\ b = 34,\ \text{and } c = -100$$

$$y = \frac{-34 \pm \sqrt{34^2 - 4 \bullet 1 \bullet -100}}{2 \bullet 1}$$

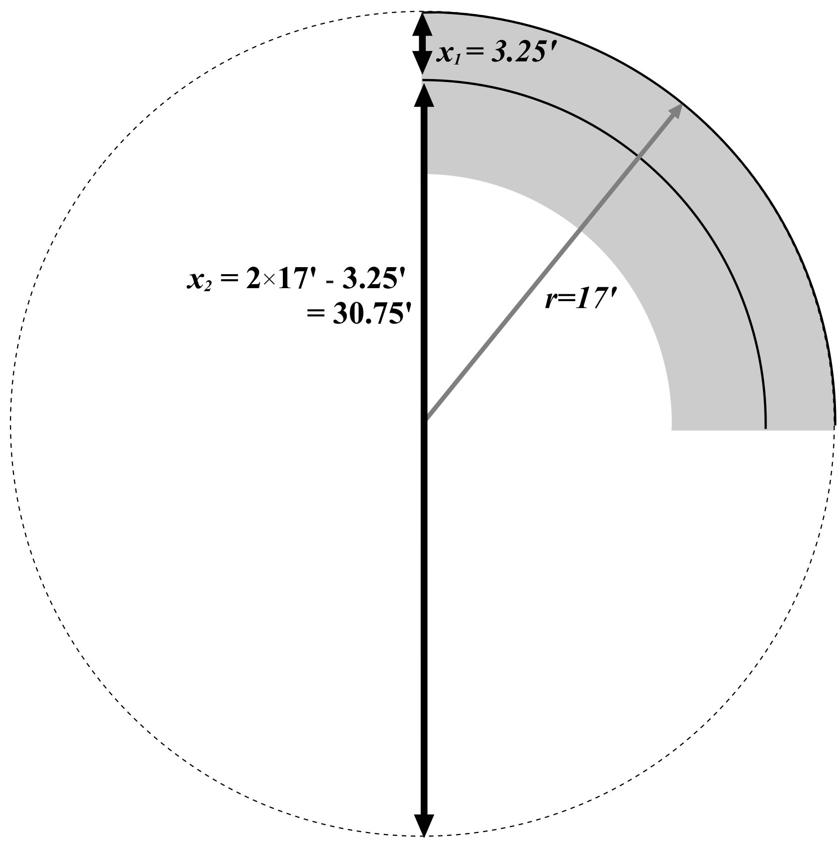
$$= \frac{-34 \pm \sqrt{1556}}{2}$$

$$y = 2.72 \text{ or } y = -36.72$$

A negative value for y is impossible, therefore the front wheel is 2.72 feet (approx. 2 feet 9 inches) from the outside edge of the cycle lane.

Note: What does the 'impossible' solution mean?

If you extend the cycle path into a complete circle, you'll see that there are two ways you could measure the distance from the rear wheel of the bus to the circle representing outside of the cycle path. The solution you threw away was actually the distance from the rear wheel to the far side of the circle:



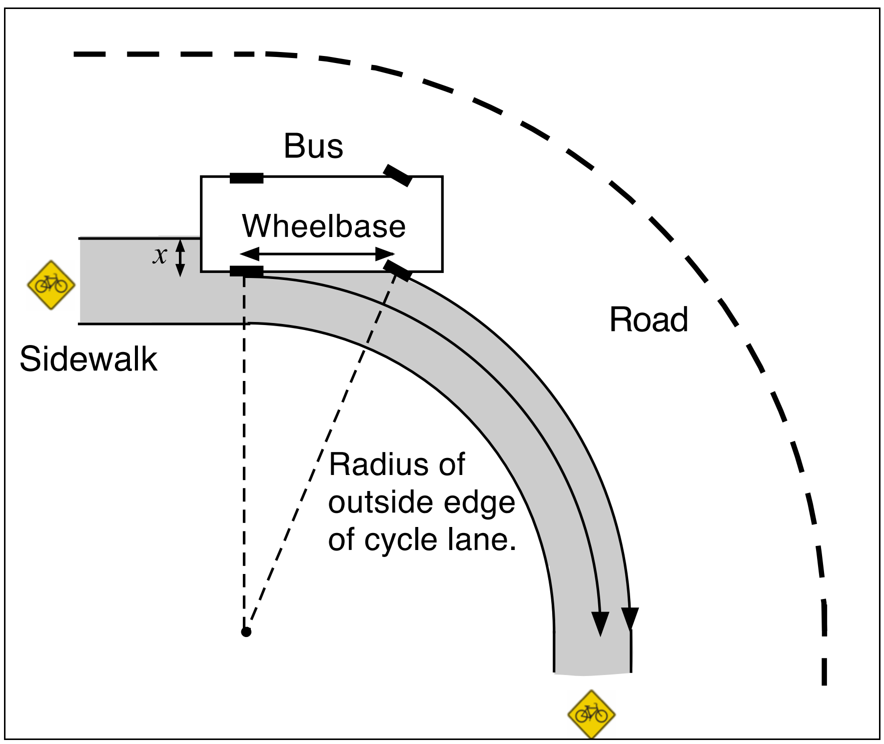
It is clear from the description of the problem that this is not the distance we're interested in.

Cutting Corners

When a bus turns a corner, it must swing out so that its rear wheels don't go into the cycle lane.

In this picture, as the bus goes round the corner, the front wheel is on the edge of the cycle lane, but the rear wheel cuts into the cycle lane.

The diagram below shows the geometry of this situation.



The distance between the front and rear wheel is called the wheelbase, w.

Let r represent the radius of the outside edge of the cycle lane.

The distance marked x shows the amount by which the rear wheels cuts into the cycle lane.

Questions on Cutting Corners

1. Use the diagram to show that $x^2 - 2xr + w^2 = 0$.

2. Let $w = 10$ feet and $r = 17$ feet.

(a) Figure out how much the rear wheel cuts into the cycle lane.

(b) Figure out how far the front wheel must be from the outside edge of the cycle lane for the rear wheel **not** to cut into the cycle lane.

Sample Responses to Discuss: Guy

$r = 17 \quad w = 10$

$\cancel{x^2 - 34x + 100 = 0}$

$\cancel{(x-10)(x-10)}$

$\cancel{(x-20)(x-5)}$

$\cancel{(x-25)(x-4)}$

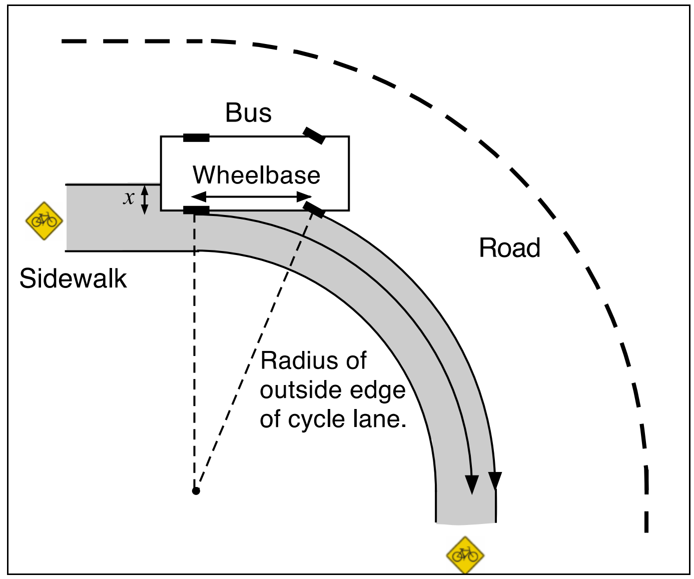
$x^2 - 34x + 100 = 0$

$(x-17)^2 - 289 + 100 = 0$

$(x-17)^2 = 189$

$x - 17 = 13.75$

$\underline{x = 30.75}$



Check Guy's work carefully and correct any errors you find.

What methods has Guy used?

Why has Guy changed his method?

Do you think Guy has chosen a good method? Explain your answer.

How can Guy improve his explanations?

Sample Responses to Discuss: Ryan

$$x^2 - 2xr + w^2 = 0$$

$$r = 17 \quad w = 10$$

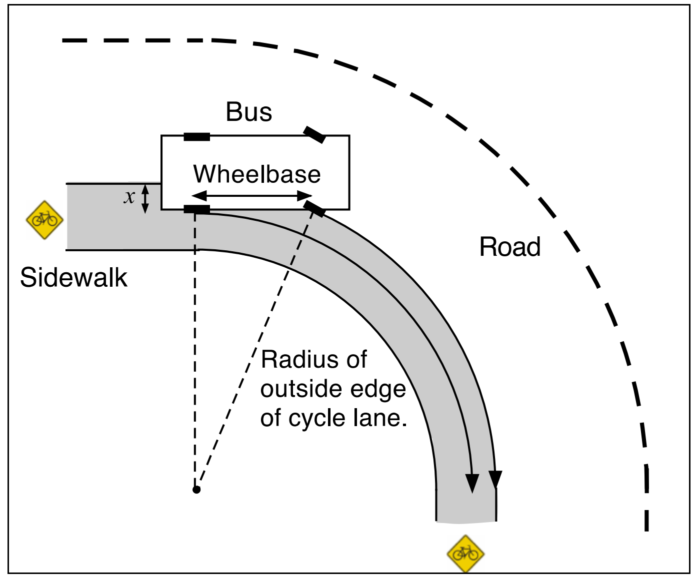
$$x^2 - 34x + 100 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-34 \pm \sqrt{34^2 - 4.1.100}}{2}$$

$$= \frac{-34 \pm \sqrt{756}}{2}$$

$$x = -30.75 \quad \text{or} \quad x = -3.25$$



Check Ryan's work carefully and correct any errors you find.

What method has Ryan used?

Do you think Ryan has chosen a good method? Explain your answer.

How can Ryan improve his explanation?

Sample Responses to Discuss: Donna

Try $x = 10$

$10^2 - 34 \times 10 + 100 = -140$

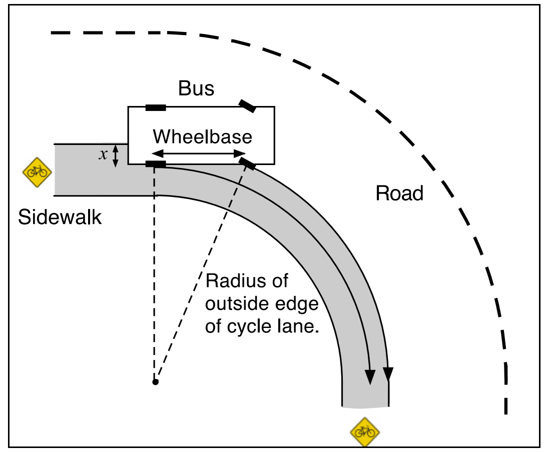
$x = 20$

$20^2 - 34 \times 20 + 100 = -180$

$x = 30$

$30^2 - 34 \times 30 + 100 = -20$

$x = 40$



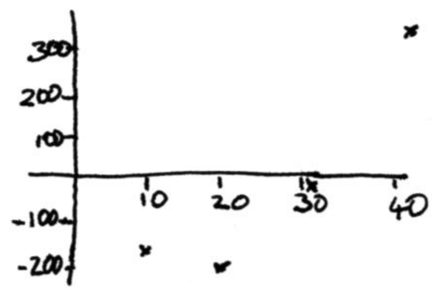
$40^2 - 34 \times 40 + 100 = 340$

x must be between 30 and 40. Try $x = 35$

$35^2 - 34 \times 35 + 100 = 135$

$x = 32$ $\quad 32^2 - 34 \times 32 + 100 = 36$

$x = 31$ $\quad 31^2 - 34 \times 31 + 100 = 7$ $\quad \underline{\underline{x = 31}}$



Check Donna's work carefully and correct any errors you find.

What method has Donna used?

Why do you think Donna has sketched a graph?

How could Donna improve her work?

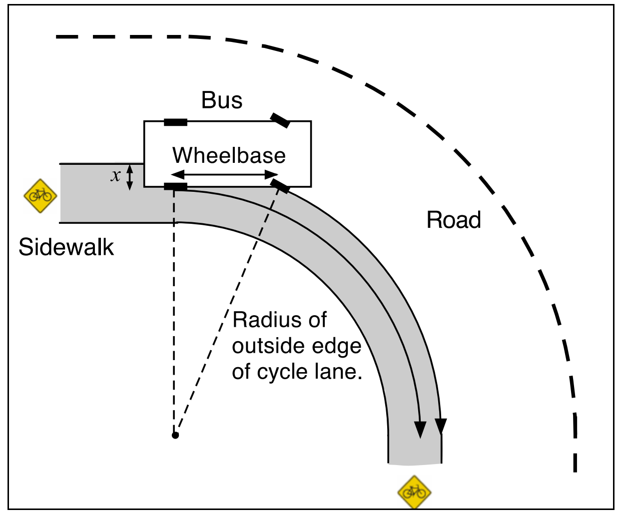
Sample Responses to Discuss: Liam

$17^2 = a^2 + 10^2$

$a^2 = 289 + 100$

$= 389$

$a = \underline{19.72}$



Check Liam's work carefully and correct any errors you find.

What method has Liam used?

Do you think Liam has chosen a good method? Explain your answer

What isn't clear about the work?

Why is Liam's work incomplete?

How Did You Work?

This questionnaire concerns your solutions to question 2 (b). Tick the boxes and complete the sentences that apply to your work.

1. Our group solution was better than my own work ☐

   Our group solution was better because ....

2. We checked our solution: ☐

   We checked our solution by ....

3. a. Our solution is similar to one of the sample responses ☐

   Our method is similar to .... *(add name of the student)*

   OR Our solution is different from **all** of the sample responses ☐

   I prefer our solution ☐ **OR** I prefer the solution of .... *(add name of the student)*

   b. I prefer **our solution** / **the student's sample response** (*circle)*

   This is because ....

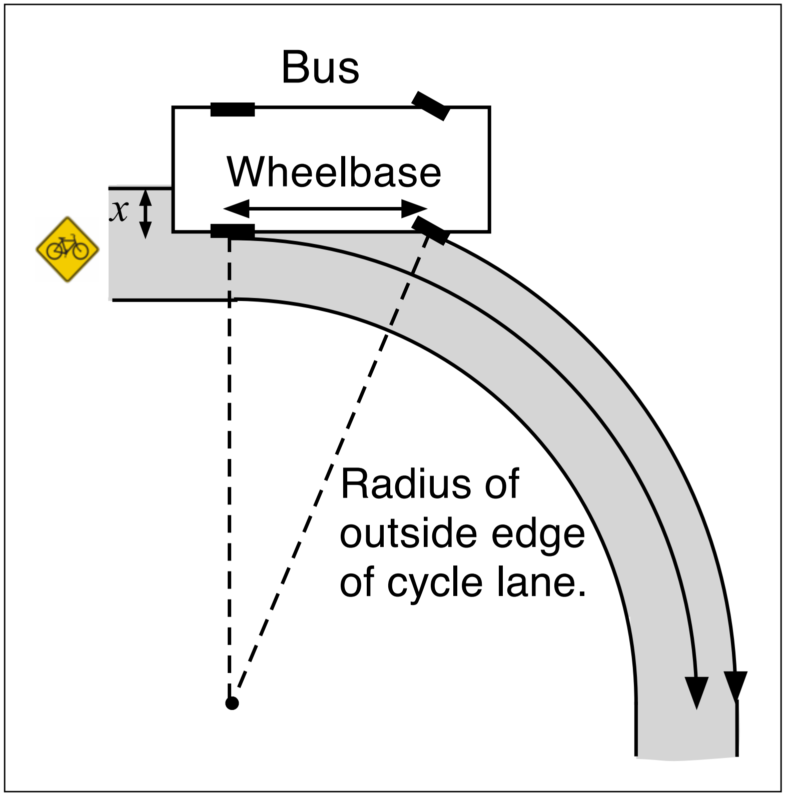
4. Which approach would you **now** use to answer Question 2b? .... *(add name of the student or your own name)*

   I prefer this approach because ....

5. What advice would you give someone new to the task?

   ....

Buses cut corners…beware!



Link to online movie: http://tinyurl.com/BusTurning

Planning a Joint Solution

1. Take turns to explain how you did the task and how you now think it could be improved.

2. Listen carefully to explanations.
   - Ask questions if you don't understand.

   - Discuss with your partners:
     - What you like/dislike about your partner's math.
     - How clear their work is.
     - How their work could be improved.

3. Once everyone in the group has explained their solution, plan a joint method that is better than each of the separate solutions.

Everyone in the group must agree on the method

Evaluating Sample Student Responses

1. Take turns to work through a student's solution.
   - Try to understand the method the student has chosen and any errors he or she has made.

2. Explain your answer to the rest of the group.

3. Listen carefully to explanations.
   - Ask questions if you don't understand.

4. Once everyone is satisfied with the explanations, write the answers to the questions below the student's solution.
   - Make sure the student who writes the answers is not the student who explained them.

Sample Responses to Discuss: Guy

$r = 17 \quad w = 10$

$\cancel{x^2 - 34x + 100 = 0}$

$\cancel{(x-10)(x-10)}$

$\cancel{(x-20)(x-5)}$

$\cancel{(x-25)(x-4)}$

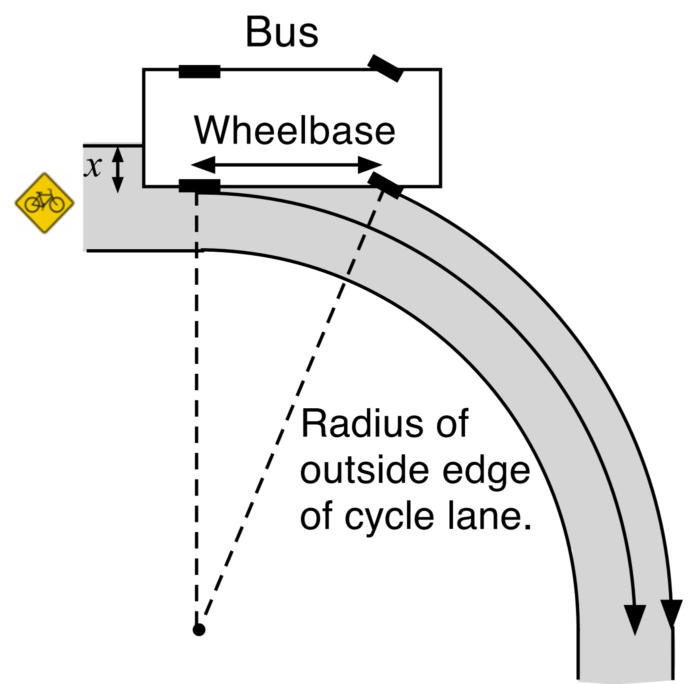
$x^2 - 34x + 100 = 0$

$(x-17)^2 - 289 + 100 = 0$

$(x-17)^2 = 189$

$x - 17 = 13.75$

$\underline{x = 30.75}$



Sample Responses to Discuss: Ryan

$$x^2 - 2xr + w^2 = 0$$

$$r = 17 \quad w = 10$$

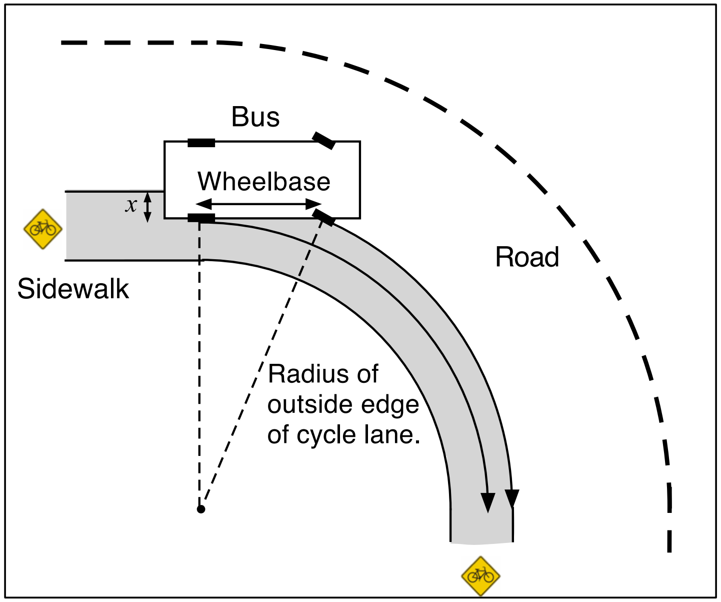
$$x^2 - 34x + 100 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

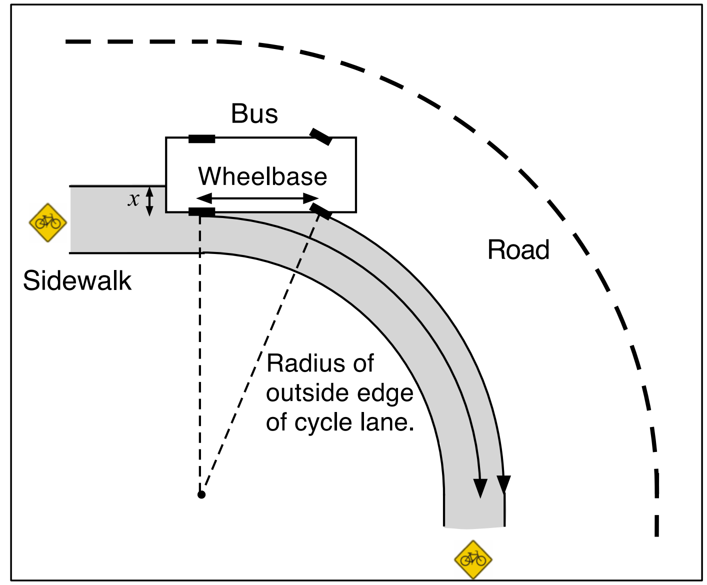
$$= \frac{-34 \pm \sqrt{34^2 - 4.1.100}}{2}$$

$$= \frac{-34 \pm \sqrt{756}}{2}$$

$$x = -30.75 \quad \text{or} \quad x = -3.25$$



Sample Responses to Discuss: Donna



Try $x = 10$

$10^2 - 34 \times 10 + 100 = -140$

$x = 20$

$20^2 - 34 \times 20 + 100 = -180$

$x = 30$

$30^2 - 34 \times 30 + 100 = -2$

$x = 40$

$40^2 - 34 \times 40 + 100 = 340$

x must be between 30 and 40. Try $x = 35$

$35^2 - 34 \times 35 + 100 = 135$

$x = 32$ $\quad 32^2 - 34 \times 32 + 100 = 36$

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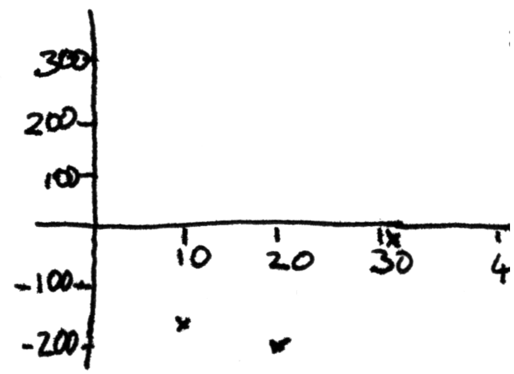


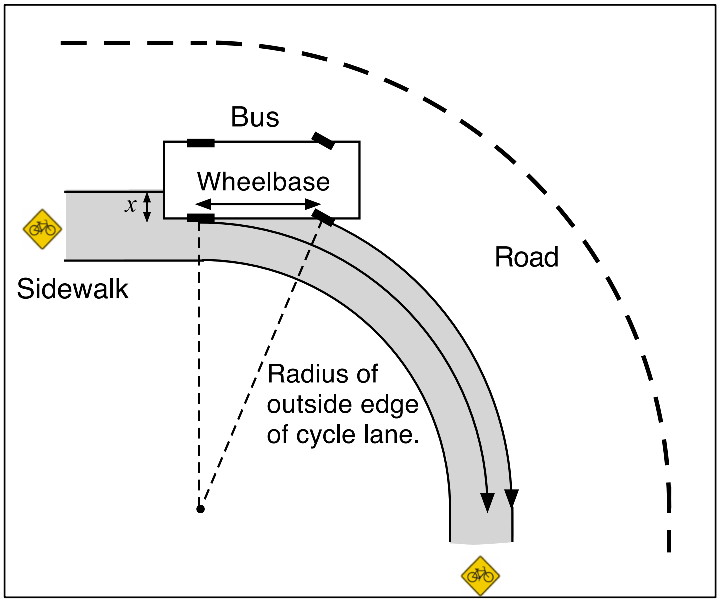
Sample Responses to Discuss: Liam

$$17^2 = a^2 + 10^2$$

$$a^2 = 289 + 100$$

$$= 389$$

$$a = 19.72$$



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
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Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org