CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Inscribing and Circumscribing Right Triangles

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Inscribing and Circumscribing Right Triangles

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to use geometric properties to solve problems. In particular, it will help you identify and help students who have difficulty:

- Decomposing complex shapes into simpler ones in order to solve a problem.
- Bringing together several geometric concepts to solve a problem.
- Finding the relationship between radii of inscribed and circumscribed circles of right triangles.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

G-SRT: Define trigonometric ratios and solve problems involving right triangles.
G-C: Understand and apply theorems about circles.
A-CED: Create equations that describe numbers or relationships.

This lesson also relates to **all** the *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, 5, 6, and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson is designed to enable students to develop strategies for describing relationships between right triangles and the radii of their inscribed and circumscribed circles.

- Before the lesson students attempt the problem individually. You review their work and formulate questions for students to answer in order to improve their solutions.
- At the start of the lesson, students work alone answering your questions. They then work in groups in a collaborative discussion of the same task before evaluating and commenting on some sample solutions.
- In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, in a follow-up lesson students review what they have learned.

MATERIALS REQUIRED

- Each student will need a mini-whiteboard, pen, and eraser and a copy of *Inscribing and Circumscribing Right Triangles, Circle Theorems*, and the *How Did You Work?* questionnaire.
- Each group of students will need an enlarged copy of *Inscribing and Circumscribing Right Triangles* and the *Sample Responses to Discuss*.
- Compasses, rules, pencils and protractors can be given to students if requested.

TIME NEEDED

15 minutes before the lesson, a 100-minute lesson (or two 50-minute lessons), and 10 minutes in a follow-up lesson. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Inscribing and Circumscribing Right Triangles* (15 minutes)

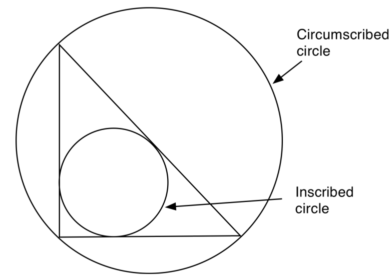
Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Inscribing and Circumscribing Right Triangles

The circle that passes through all three vertices of a triangle is called a circumscribed circle.

The largest circle that fits inside a triangle is called an inscribed circle.

This diagram is not drawn to scale



1. Figure out the radii of the circumscribed and inscribed circles for a right triangle with sides 5 units, 12 units, and 13 units.
   Show and justify every step of your reasoning.
   The sheet of *Circle Theorems* may help you.

2. Use mathematics to explain carefully how you can figure out the radii of the circumscribed and inscribed circles of a right triangle with sides of *any* length: a, b and c (where c is the hypotenuse.)

Give each student a copy of the *Inscribing and Circumscribing Right Triangles* task and the *Circle Theorems* sheet. Have compasses, pencils, rules, and protractors available for students who request them.

Briefly introduce the task and help the class to understand the problem. Make sure everyone understands the words 'inscribed' and 'circumscribed'.

Read through the task and try to answer it as carefully as you can.
Show all your working so that I can understand your reasoning.

It is important that, as far as possible, students are allowed to answer the questions without assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Students who sit together often produce similar answers and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

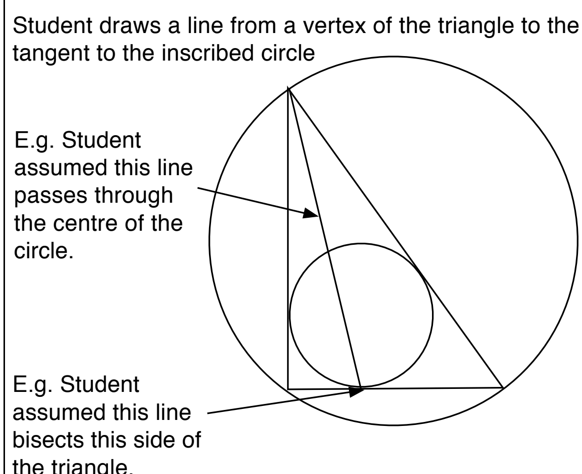
We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on pages T-4 and T-5. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues	Suggested questions and prompts
Has difficulty getting started (Q1)	• What do you know about the angles or lines in the diagram? How can you use what you know? What do you need to find out? • Which circle theorems are most relevant? • Can you add any helpful construction lines to your diagram? What do you know about these lines?
Explanation is unclear, irrelevant or incomplete (Q1 and Q2) For example: The student writes correct facts, such as the hypotenuse is the diameter of the circumscribed circle, but does not use theorems to support their answers. Or: The student does not explain why angles are 90° or why lengths are equal.	• Would someone unfamiliar with your type of solution easily understand your work? • Can you use one of the circle theorems to describe why the hypotenuse is the diameter of the circumscribed circle? • What circle theorems can you use to support your explanation? • What do the circle theorems tell you about tangents?
Makes incorrect assumptions based on perception (Q1 and Q2) For example: The student assumes that as the radius of the circumscribed circle is half the length of a tangent then the radius of the inscribed circle is half the length of one of its tangents. The student may write: “radius of inscribed circle is 5 ÷ 2 = 2.5 units.” (Q1)  	• Have you made any assumptions? • Can you use the circle theorems to support your work? • Why is the radius of the circumscribed circle half of 13? Does it make sense to apply this explanation to figure out the radius of the inscribed circle? [No.]

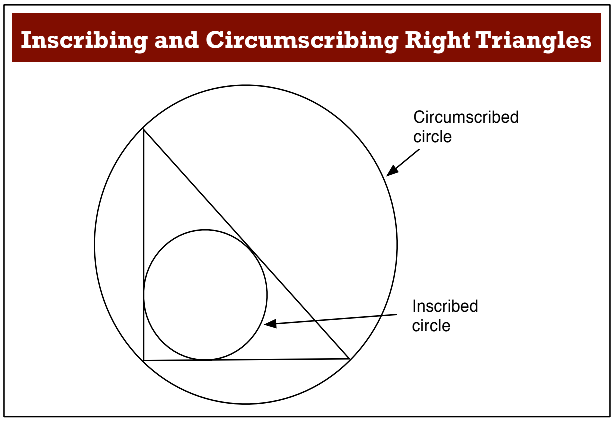
Common issues	Suggested questions and prompts
Uses geometric construction to find the relationships (Q2) For example: The student constructs two or three triangles and their inscribed and circumscribed circles and finds the answers by measuring.	• What are the advantages/disadvantages of constructing the shapes? • What do you notice about the hypotenuse of the triangle and the diameter of the circumscribed circle? Will this be the case for all right triangles with vertices on the circumference of a circle? Why? • What do you notice about the points where the triangle touches the inscribed circle? Can you use what you've noticed to figure out the radius of an inscribed circle of any right triangle?
Does not attempt to generalize (Q2) For example: The student has successfully figured out the radii for a specific circle, but makes no attempt to generalize the method.	• What do you notice about the lengths of the sides of the triangle and the radius of the circumscribed circle? Do you know for sure this is always the case? • Can you use your method from Question 1 for a right triangle that has sides of length a, b, and c?
Correctly answers all the questions	• Can you solve the problem using a different method? • How could you adapt your method if the triangle was not a right triangle, but scalene?

SUGGESTED LESSON OUTLINE

Individual work (10 minutes)

Give each student a mini-whiteboard, pen, and eraser.

Begin the lesson by briefly reintroducing the problem. You may want to show the class Slide P-1 of the projector resource:



Do you remember the problem I asked you to do last time? What was the task about?

Return to students their work on the *Inscribing and Circumscribing Right Triangles* task.

If you did not add questions to individual pieces of work, write your list of questions on the board. Students should select questions appropriate to their own work and spend a few minutes answering them. Some students may struggle to identify which questions they should be considering. If this is the case, it may be helpful to give students a printed version of the list of questions so that you can highlight the ones that you want them to focus on.

Today you are going to work together to try to improve your initial attempts at this task. First, though, I would like you to work individually again.

I looked at your work and I have some questions on it. On your own, carefully read through the questions I have written.

I would like you to use the questions to help you to think about ways of improving your own work. Use your mini-whiteboards to make a note of your answers to these questions.

It is helpful to ask students to write their ideas on their mini-whiteboards, as you will be able to monitor their work more easily. It will also help students to share their ideas easily later in the lesson.

Collaborative activity (30 minutes)

Organize the class into small groups of two or three students.

Give each group an enlarged copy of the *Inscribing and Circumscribing Right Triangles* task.

Deciding on a Strategy

Ask students to share their individual work and ideas about the task and plan a joint solution.

I want you to share your work with your group.
Take turns to explain how you did the task and how you now think it could be improved.

Listen carefully to explanations. Ask questions if you don't understand or agree with the method. (You may want to use some of the questions I have written on the board.)

Once students have evaluated the relative merits of each approach, ask them to write their strategy on the second side of the handout.

> *Once everyone has shared, plan a joint approach that is better than your individual solutions. Write an outline of your plan on the back of the task sheet.*

Slide P-2 of the projector resource summarizes these instructions:

Planning a Joint Solution

1. Take turns to explain how you did the task and how you now think it could be improved.

2. Listen carefully to explanations.
   - Ask questions if you don't understand.
   - Discuss with your partner(s):
     - What you like/dislike about your partner's math.
     - Any assumptions your partner has made.
     - How their work could be improved.

3. Once everyone in the group has explained their solution, plan a joint approach that is better than each of the individual solutions.
   - On the second side of your sheet of paper write a couple of sentences, outlining your plan.

Implementing the Strategy

Students are now to write their joint solution on the front of the handout:

> *Now that you have decided on a joint strategy, use your agreed approach to produce a joint solution to the task and answer the questions on the sheet.*

While students work in small groups you have two tasks, to note different student approaches to the task and to support student problem solving.

Note different student approaches to the task

Listen and watch students carefully. In particular, note any common mistakes. Are students making any incorrect assumptions? What circle theorems are students using? Are students constructing the diagram? Note if and how students use algebra.

Attend also to students' mathematical decisions. Do they track their own progress? Do they notice if they have chosen a strategy that does not seem to be productive? If so, what do they do?

You can use this information to focus the whole-class discussion at the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students clarify their thinking, promote further progress and encourage students to check their work and detect errors. You may want to use some of the questions in the *Common issues* table to support your own questioning or, if the whole-class is struggling on the same issue, write relevant questions on the board and hold a brief whole-class discussion. You could also give any struggling students one of the sample responses.

Reviewing the strategy

As students finish working on the problem, give them the *How Did You Work?* review questionnaire.

Ask students to answer **Questions 1** and **2** of this questionnaire.

Whole-class discussion (10 minutes)

You may now want to hold a brief whole-class discussion. Have students solved the problem using a variety of methods? Have you noticed some interesting ways of working or some incorrect methods? If so, you may want to focus the discussion on these. Equally, if you have noticed different groups using similar strategies but making different assumptions, you may want to compare solutions.

Extending the lesson over two days

If you are taking two days to complete the lesson unit then you may want to end the first lesson here. At the start of the second day, allow students the opportunity to familiarize themselves with their individual and collaborative work on the task before moving on to the analysis of sample responses.

Collaborative analysis of *Sample Responses to Discuss* (30 minutes)

This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy. Research has shown that exposing students to multiple perspectives of a solution can deepen their understanding of the mathematics.

After students have had sufficient time to solve the problem, give each group the *Sample Responses to Discuss*. If your students are not familiar with geometric constructions, you may want to withhold Aiden's work. We found in trials of this lesson that on reviewing Aiden's response some students tried to construct the diagram, this left little time to evaluate the three remaining student responses.

Also, there may not be time for all groups to look at all four solutions. If this is the case, be selective about what you give out. For example, groups that have successfully completed the task using one method will benefit from looking at different approaches. Other groups that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

Have compasses, pencils, rules, and protractors available for students who request them.

To encourage students to do more than check to see if the answer is correct, ask them to answer the questions below each sample piece of work. Students should focus on the math of the student work, not whether the student has neat writing etc.

Imagine you are the teacher and have to assess this work.

Answer the questions below each response.

Slide P-3 of the projector resource describes how students should work together:

Evaluating Sample Responses

1. Imagine you are the teacher and have to assess the student's work.
2. Take turns to work through a student's solution.
   – Write your answers on your mini-whiteboards.
3. Explain your answers to the rest of the group.
4. Listen carefully to explanations.
   – Ask questions if you don't understand.
5. Once everyone is satisfied with the explanations, write the answers below the student's solution.
   – Make sure the student who writes the answers is not the student who explained them.

During the small group work, support the students as before. Check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the small group work.

Extension task

For Natalie's and Logan's work students are asked to figure out a general formula for the radius of an inscribed circle of any right triangle. These formulas look different. You may want to ask students to investigate whether these formulas represent the same radius.

What can you figure out if you assume the formulas are equal?

The equivalence of Logan's and Natalie's formulas is shown as follows:

If:

$$\frac{ab}{a+b+c} = \frac{a+b-c}{2}$$

Then:

$$2ab = (a+b+c)(a+b-c)$$

$$2ab = a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$$

$$2ab = a^2 + b^2 - c^2 + 2ab$$

$$a^2 + b^2 = c^2$$

This shows that if the two equations are equivalent, then the triangle is right-angled, which is indeed the case.

Only Logan's method and formula will work in a situation where there is not a right triangle.

Whole-class discussion: comparing different approaches (20 minutes)

Now hold a whole-class discussion to consider the different approaches used in the sample student work. Focus the discussion on parts of the tasks students found difficult. Ask the students to compare the different solution methods.

Which approach did you like best? Why?

Which approach did you find most difficult to understand?

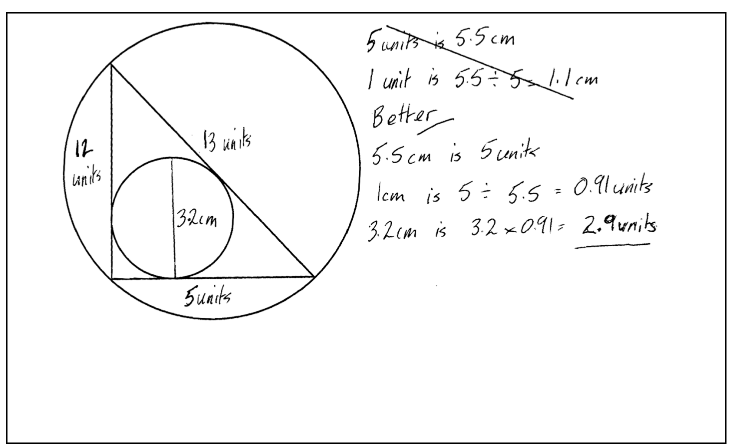
To critique the different strategies use the questions on the *Sample Responses to Discuss* sheets and Slides P-4 to P-7 of the projector resource.

Some issues that might be discussed are given below:

Luke's method Q2

Luke has treated the diagram as a scale drawing. He has figured out the diameter not the radius. The diagram is not to scale. To figure out the radius of the inscribed circle Luke needs to draw a scale diagram.

Using a scale drawing is a useful introduction to the problem, but it will not provide a general solution and so cannot be taken further.



Natalie's method Q2

Natalie correctly states that DOFC is a square with sides of length r, but does not support this with an explanation. (Segments CD and CF are tangents and are therefore congruent. $\angle ACB = 90°$.)

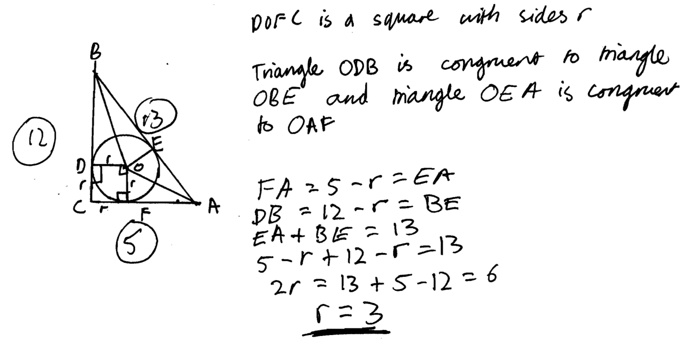
Natalie correctly states the congruency of the triangles, but again there is no supporting explanation.

For example, Δ BOD is congruent to Δ BOE because OB is common, OD = OE = r, DB = BE. (If two segments from the same exterior point are tangent to a circle, then the segments are congruent.)

Natalie's method is correct, however she makes a mistake manipulating the equation.

The correct solution is:

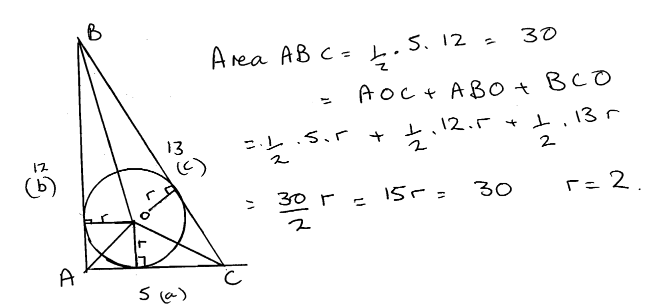
$2r = -13 + 5 + 12$
$2r = 4$
$r = 2$.



Logan's method Q2

Logan has split the right triangle ABC into three smaller ones: AOB, BOC and AOC. Logan correctly explains his method: add together the areas of the 3 smaller triangles. This total is equal to the area of triangle ABC.

Logan correctly labels right angles in the diagram but does not explain why they are 90º (tangents to the circle.)



Aiden's method Q2

Aiden has correctly identified the center of the circle.

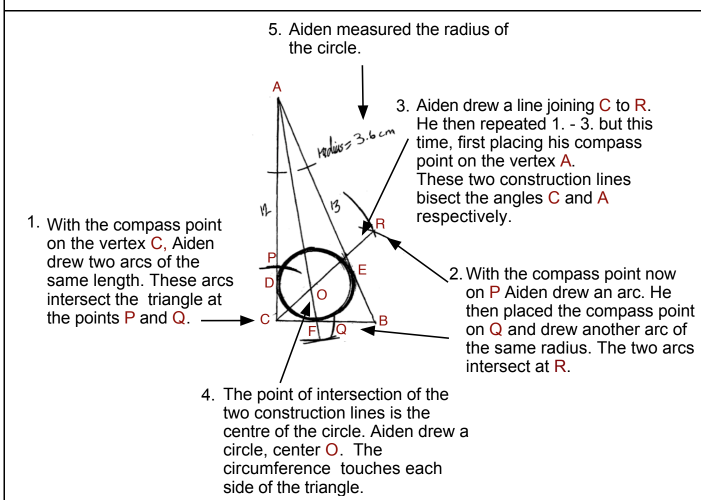
Aiden uses circle theorems to construct the inscribed circle because:

If two segments from the same exterior point are tangent to a circle, then the segments are congruent.

Therefore:

AD = AE, EB = FB, and CF = DC.

Δ DOA is congruent to Δ EOA (DO = OE, AD = AE and AO is common.)



Therefore the line that bisects the angle at this point is equal distance from the two tangents and passes through the center of the inscribed circle (tangents to circles.)

Aiden has measured the diameter not the radius.

Constructing the circles and the triangles can be a useful introduction to the problem, but it will not provide a general solution and so cannot be taken further.

If you have time, you may want to ask students to investigate applying the general formulas derived from Logan's and Natalie's work to any triangle. [Only Logan's method can be used.]

Follow-up lesson: individual review (10 minutes)

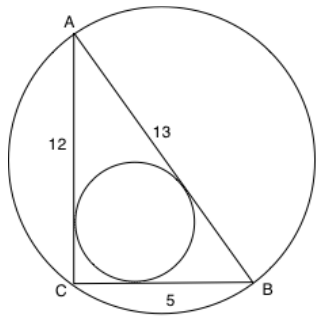
Ask students to complete the remaining questions (Questions 3 and 4) of the *How Did You Work?* review questionnaire. The questionnaire should help students monitor and review their progress both during and at the end of an activity.

If you have time you may also want to ask your students to read through their original solution and using what they have learned, attempt the task again.

Some teachers give this task for homework.

SOLUTIONS

1.



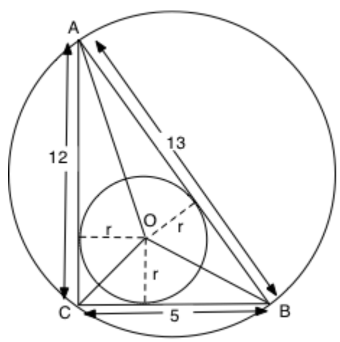
Radius of circumscribed circle

$\angle BCA = 90°$.

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc (or central angle.)

Therefore the central angle is 180º. It then follows that AB is a diameter.

Radius of circumscribed circle = $13 \div 2 = 6.5$



Radius of inscribed circle - method 1

The lines AB, BC and CA are all tangents to the circle and so are perpendicular to the radii at the point they touch the circle.

Area Δ ABC = area Δ BCO + area Δ ACO + area Δ ABO

$$\frac{1}{2} \times 12 \times 5 = \frac{1}{2} \times 5 \times r + \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 13 \times r$$

$$30 = \frac{1}{2}(5r + 12r + 13r)$$

$$30 = 15r$$

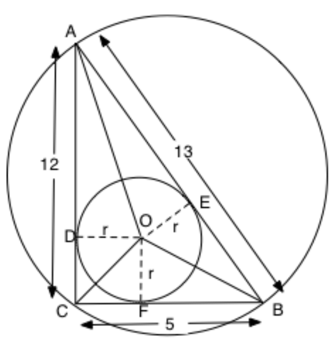
$$r = 2$$

Radius of inscribed circle - method 2

If two segments from the same exterior point are tangent to a circle, then the segments are congruent. Therefore:

AD = AE; EB = FB; CF = DC.

Δ DOC is congruent to Δ COF (DO = OF = r, DC = FC, and CO is common.)



Therefore $\angle OCF = 90º \div 2 = 45º$.
$\tan \angle OCF = \tan 45º = 1 = r \div CF$
$CF = r = DC$
It follows that:

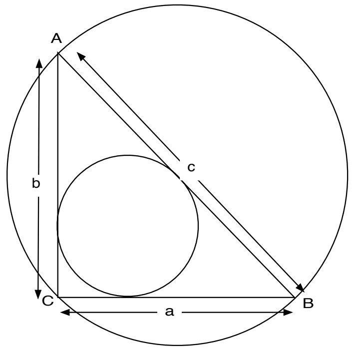
$5 = CF + FB = r + FB$

$12 = AD + DC = AD + r$

$13 = AE + EB$

$5 + 12 - 13 = r + FB + AD + r - AE - EB$
$4 = 2r$ (FB = EB; AD = AE)
$r = 2$

2. Describe carefully a method for working out the radius of the circumscribed and inscribed circle of a right triangle with sides of length a, b and c.



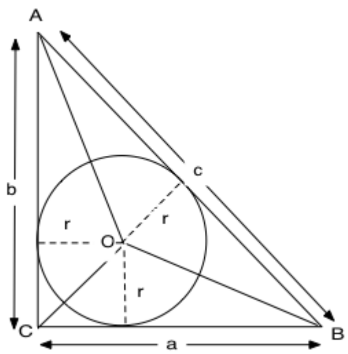
Radius of circumscribed circle

$\angle BCA = 90°$.

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc (or central angle.)

Therefore the central angle is 180º. It then follows that AB is a diameter.

Radius of circumscribed circle = $\frac{c}{2}$



Radius of an inscribed circle

Logan's method:

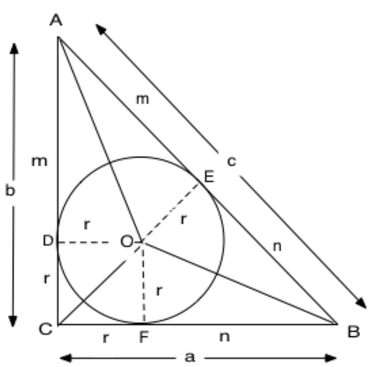
The lines AC, BC and CA are all tangents to the circle and so are perpendicular to the radii at the point they touch the circle.

Area Δ ABC = area Δ BCO + area Δ ACO + area Δ ABO

$$\frac{1}{2}ab = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$$

$$\frac{1}{2}ab = \frac{1}{2}r(a + b + c)$$

$$r = \frac{ab}{a + b + c}$$



Natalie's method:

If two segments from the same exterior point are tangent to a circle, then the segments are congruent. Therefore:

AD = AE;   EB = FB;   CF = DC

Δ DOC is congruent to Δ COF (DO = OF, DC = FC, and CO is common.)

$\angle BCA = 90^\circ$
Therefore $\angle OCF = 90^\circ \div 2 = 45^\circ$.

$\tan \angle OCF = \tan 45^\circ = 1 = r \div CF$

CF = r = DC

It follows that:

$a = n + r$ $b = r + m$ $c = m + n$

$a + b - c = (n + r) + (r + m) - (m + n) = 2r$

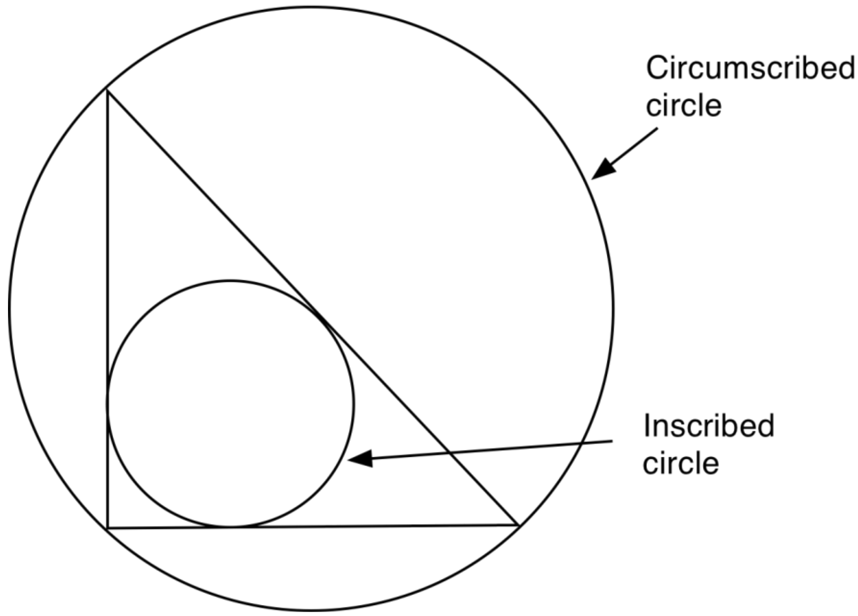
$$r = \frac{a + b - c}{2}$$

Inscribing and Circumscribing Right Triangles

The circle that passes through all three vertices of a triangle is called a circumscribed circle.

The largest circle that fits inside a triangle is called an inscribed circle.

This diagram is not drawn to scale



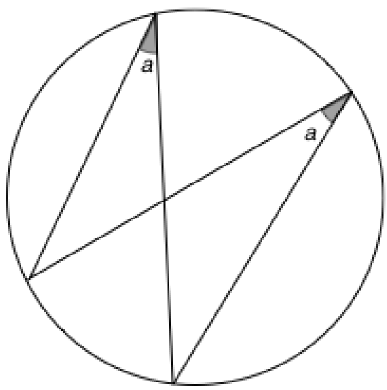
1. Figure out the radii of the circumscribed and inscribed circles for a right triangle with sides 5 units, 12 units, and 13 units.
   Show and justify every step of your reasoning.
   The sheet of *Circle Theorems* may help you.

2. Use mathematics to explain carefully how you can figure out the radii of the circumscribed and inscribed circles of a right triangle with sides of *any* length: a, b and c (where c is the hypotenuse.)

Circle Theorems

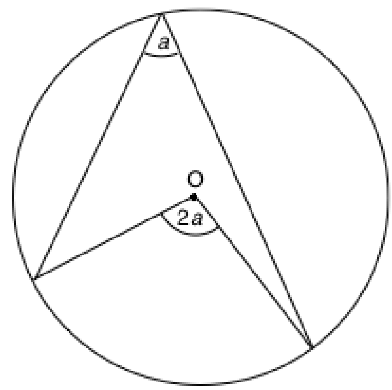
Theorem 1

If inscribed angles of a circle intercept the same arc, then they are congruent.



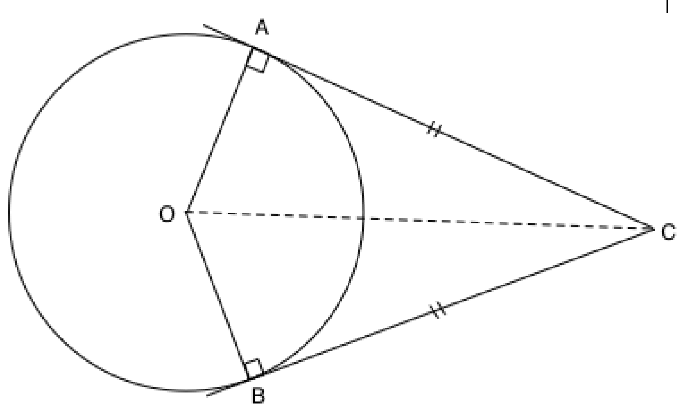
Theorem 2

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc (or central angle.)



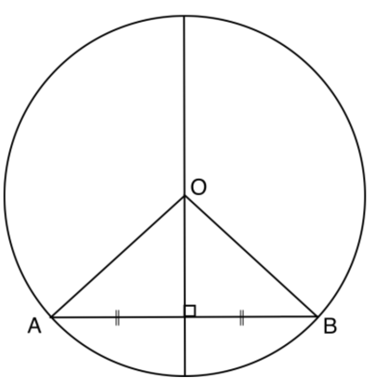
Theorem 3

If two segments from the same exterior point are tangent to a circle, then the segments are congruent.

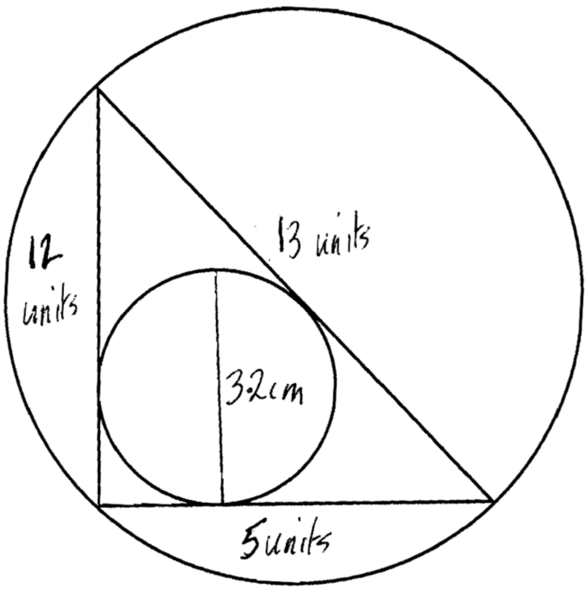


Theorem 4

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord.



Sample Responses to Discuss: Luke



~~5 units is 5.5 cm~~

~~1 unit is 5.5 ÷ 5 = 1.1 cm~~

Better

5.5 cm is 5 units

1cm is 5 ÷ 5.5 = 0.91 units

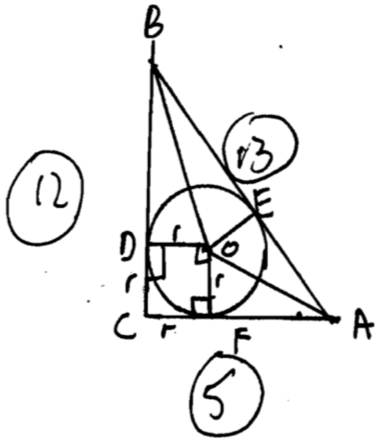
3.2cm is 3.2 × 0.91 = 2.9units

Why did Luke change his method?

How can Luke improve his work?

Can the improved method be used to answer **all** the task questions?

Sample Responses to Discuss: Natalie



DOFC is a square with sides r

Triangle ODB is congruent to triangle OBE and triangle OEA is congruent to OAF

$FA = 5 - r = EA$
$DB = 12 - r = BE$
$EA + BE = 13$
$5 - r + 12 - r = 13$
$2r = 13 + 5 - 12 = 6$
$\underline{r = 3}$

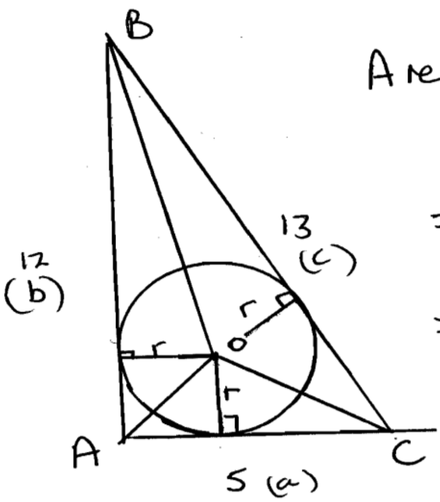
Check Natalie's work carefully and correct any errors you find.

What isn't clear about Natalie's work?

Which circle theorems has Natalie used?

Use the same method to figure out the radius of the inscribed circle when the sides of the triangle are a, b, and c.

Sample Responses to Discuss: Logan



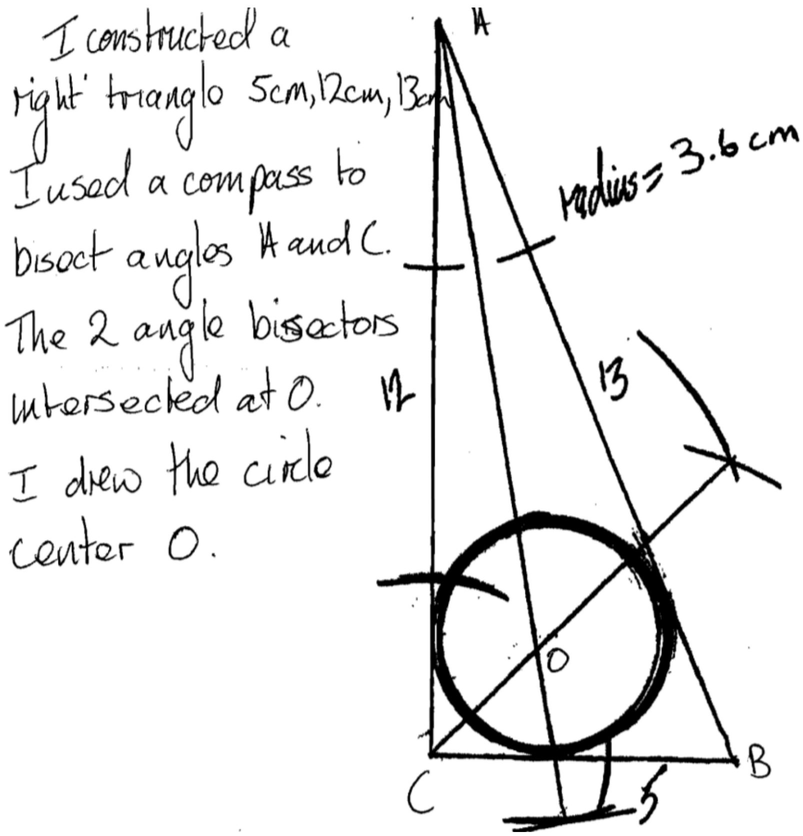
$$\text{Area } ABC = \tfrac{1}{2} \cdot 5 \cdot 12 = 30$$
$$= AOC + ABO + BCO$$
$$= \tfrac{1}{2} \cdot 5 \cdot r + \tfrac{1}{2} \cdot 12 \cdot r + \tfrac{1}{2} \cdot 13\,r$$
$$= \frac{30}{2} r = 15r = 30 \qquad r = 2.$$

Check Logan's work carefully and correct any errors you find.

What isn't clear about Logan's work?

Use the same method to figure out the radius of the inscribed circle when the sides of the triangle are a, b, and c.

Sample Responses to Discuss: Aiden



Check Aiden's work carefully and correct any errors you find.

Explain how Aiden uses circle theorems to construct the inscribed circle.

What are the problems with using construction when answering **all** the questions in the task?

How Did You Work?

Mark the boxes and complete the sentences that apply to your work.

1. Our group solution is different to my own solution ☐

   The differences are: ....

2. In our solution we showed where we used the following circle theorems: ....

3. Our group solution is similar to one of the sample responses ☐

   Our solution is similar to .... *(add name of sample response)*

   I prefer our solution / the sample response solution *(circle)* because: ....

OR Our group solution is different from all of the sample responses ☐

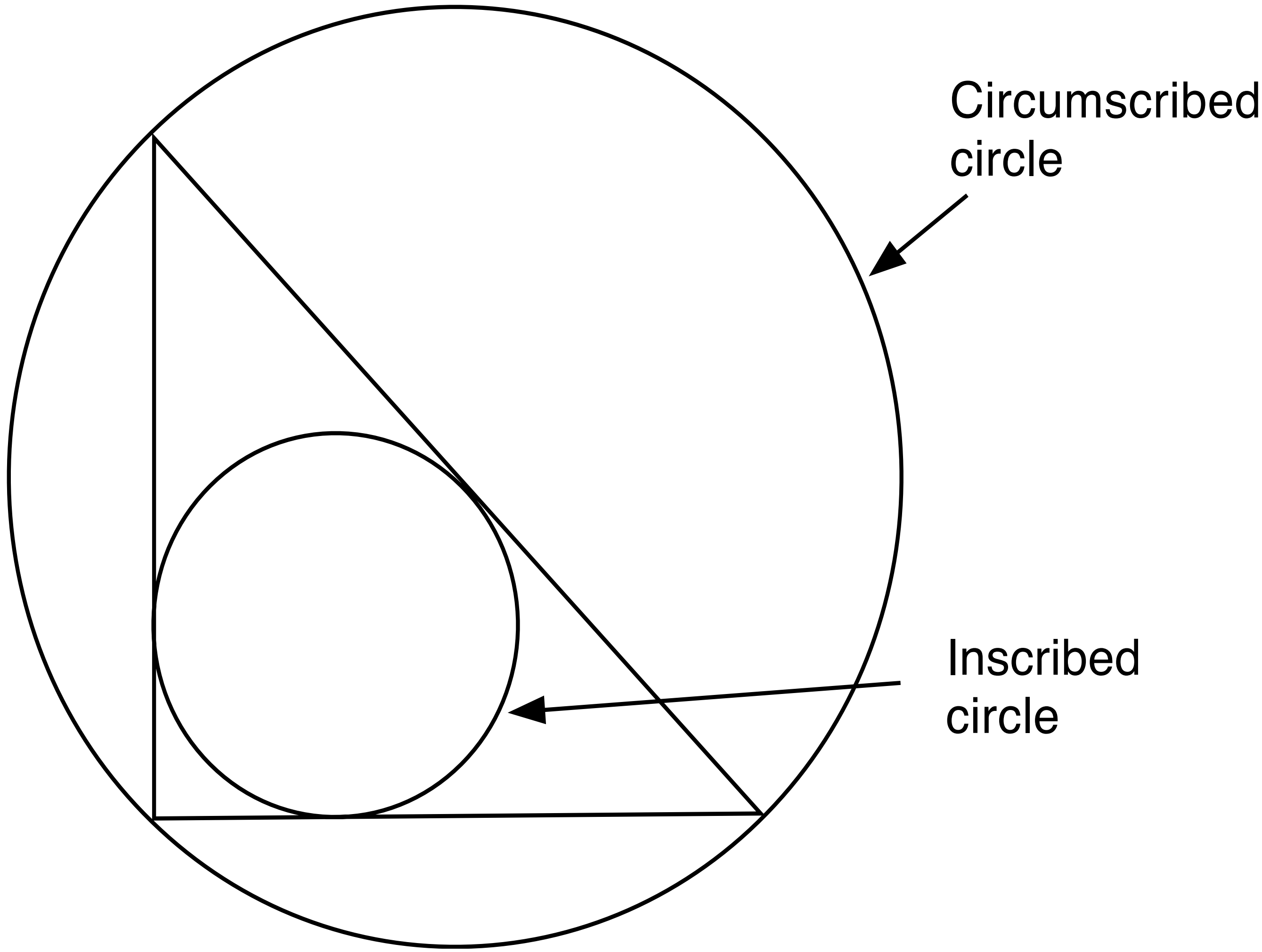
   because ....

4. Which of the sample response methods do you think is the clearest and easiest to understand?

   Luke's   Natalie's   Logan's   Aiden's

   Explain why. ....

Inscribing and Circumscribing Right Triangles



Planning a Joint Solution

1. Take turns to explain how you did the task and how you now think it could be improved.

2. Listen carefully to explanations.
   - Ask questions if you don't understand.
   - Discuss with your partner(s):
     - What you like/dislike about your partner's math.
     - Any assumptions your partner has made.
     - How their work could be improved.

3. Once everyone in the group has explained their solution, plan a joint approach that is better than each of the individual solutions.
   - On the second side of your sheet of paper write a couple of sentences, outlining your plan.

Evaluating Sample Responses

1. Imagine you are the teacher and have to assess the student's work.

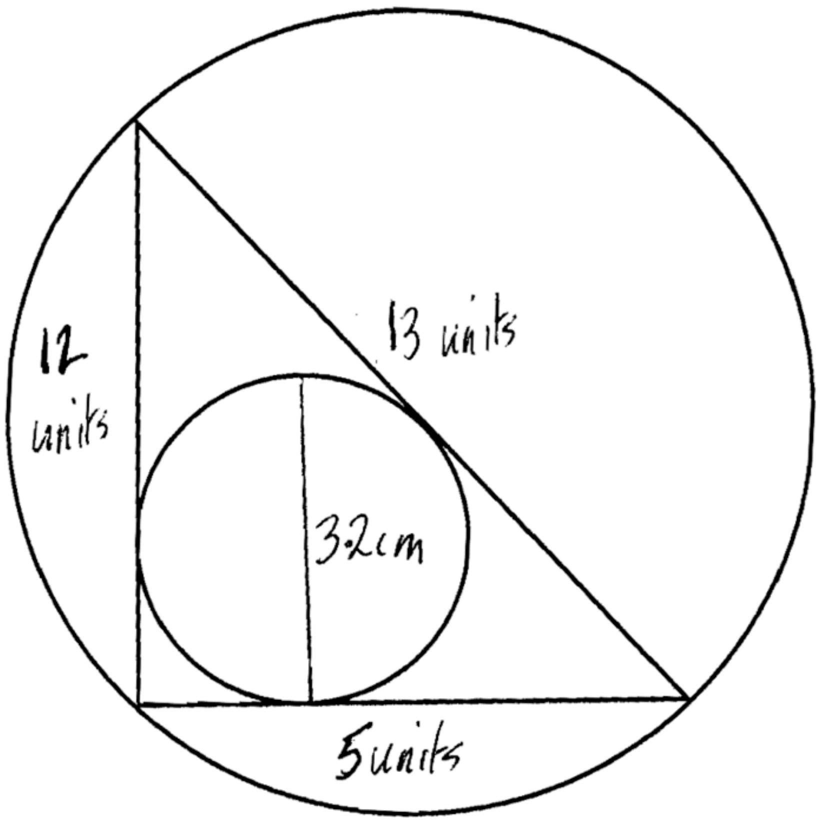
2. Take turns to work through a student's solution.
   - Write your answers on your mini-whiteboards.

3. Explain your answers to the rest of the group.

4. Listen carefully to explanations.
   - Ask questions if you don't understand.

5. Once everyone is satisfied with the explanations, write the answers below the student's solution.
   - Make sure the student who writes the answers is not the student who explained them.

Sample Responses to Discuss: Luke



~~5 units is 5.5cm~~

~~1 unit is $5.5 \div 5 = 1.1$ cm~~

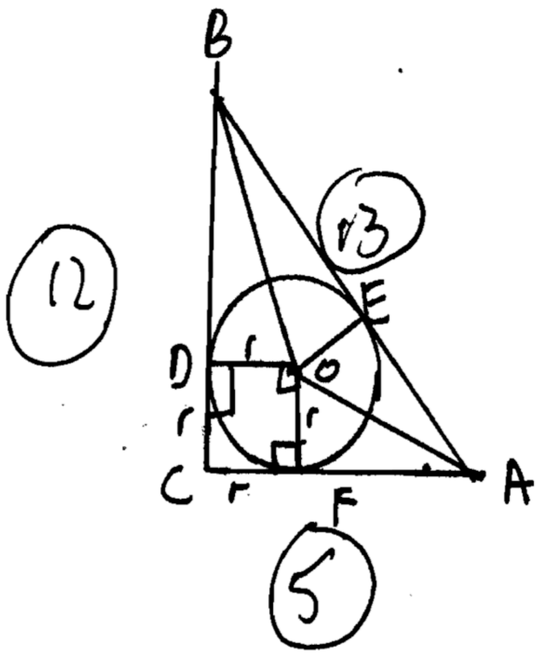
Better

5.5cm is 5 units

1cm is $5 \div 5.5 = 0.91$ units

3.2cm is $3.2 \times 0.91 =$ <u>2.9units</u>

Sample Responses to Discuss: Natalie

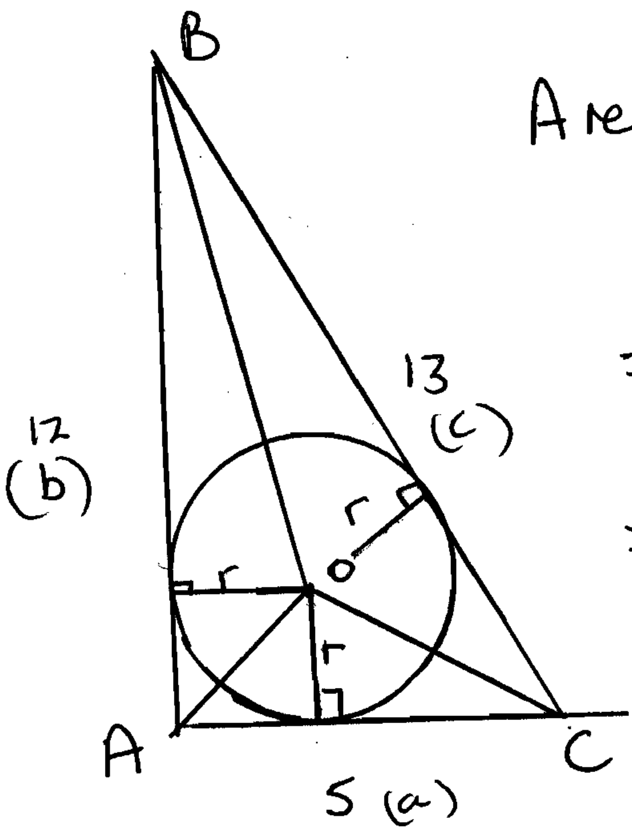


DOFC is a square with sides r

Triangle ODB is congruent to triangle OBE and triangle OEA is congruent to OAF

$FA = 5 - r = EA$
$DB = 12 - r = BE$
$EA + BE = 13$
$5 - r + 12 - r = 13$
$2r = 13 + 5 - 12 = 6$
$\underline{r = 3}$

Sample Responses to Discuss: Logan



$$\text{Area } ABC = \frac{1}{2} \cdot 5 \cdot 12 = 30$$

$$= AOC + ABO + BCO$$

$$= \frac{1}{2} \cdot 5 \cdot r + \frac{1}{2} \cdot 12 \cdot r + \frac{1}{2} \cdot 13\, r$$

$$= \frac{30}{2} r = 15r = 30 \qquad r = 2.$$

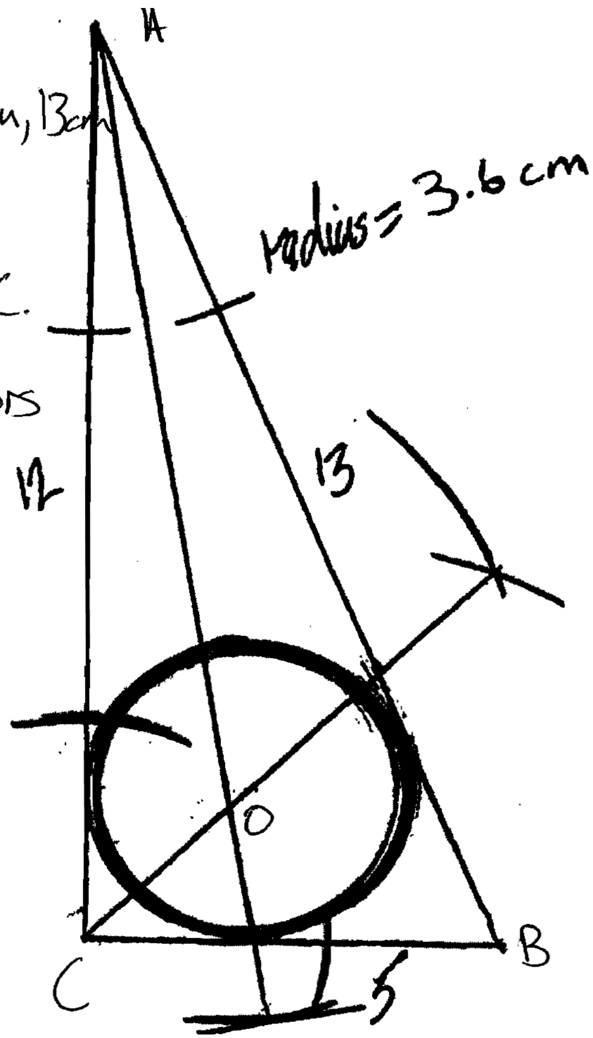
Sample Responses to Discuss: Aiden

I constructed a right triangle 5cm, 12cm, 13cm

I used a compass to bisect angles A and C.

The 2 angle bisectors intersected at O.

I drew the circle center O.



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and **Daniel Pead**

We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org