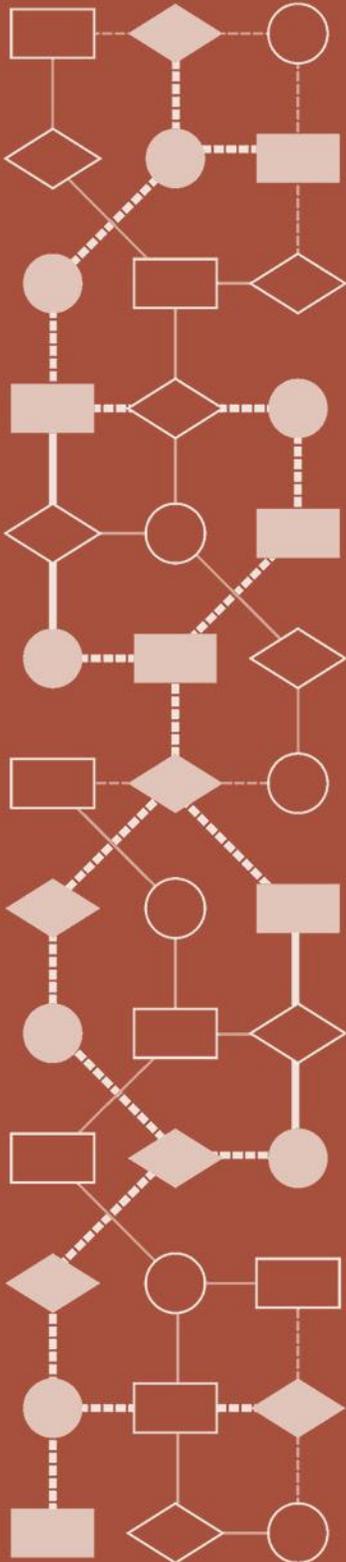


Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Interpreting Data: *Muddying the Waters*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

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Interpreting Data: *Muddying the Waters*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Interpret data and evaluate statistical summaries.
- Critique someone else’s interpretations of data and evaluations of statistical summaries.

The lesson also introduces students to the dangers of misapplying simple statistics in real-world contexts and illustrates some of the common abuses of statistics and charts found in the media.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2, 3, and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

- S-ID: Summarize, represent, and interpret data on two categorical and quantitative variables. Interpret linear models.
- S-IC: Understand and evaluate random processes underlying statistical experiments. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

INTRODUCTION

This lesson unit is structured in the following way:

- In the first lesson, students work individually on a task designed to reveal their current understanding. You review their solutions, creating questions to help them to improve their work.
- During the second lesson, students role-play a scene that provides the background to the lesson task. Next, students work in small-groups on a collaborative discussion task. They extract information from evidence and critically assess how it was gathered and presented.
- After a whole-class discussion students work alone on a new task, similar to the original task.

MATERIALS REQUIRED

- Each student will need a copy of *Muddying the Waters* and *Unhappy Campers*.
- Each small group of students will need a copy of the role-play script *A Case of Muddying the Waters* and the *Case Notes* worksheet.
- There is a projector resource to support the role-play and whole-class discussion. Use of a computer and projector is recommended. You could print the images onto overhead projector slides instead. If you do not have access to a projector, print a copy of Slides *Exhibits 1, 2, 3, 4*, *Scene 1: The Assistant DA’s office*, and *Scene 2: At the Factory* for each small group of students.
- A teaspoon may be useful to help illustrate the concept of ‘mg per cubic meter’.

TIME NEEDED

30 minutes before the lesson, a 1-hour lesson, and 15 minutes in a follow-up lesson. All timings are approximate. Exact timings will depend on the needs of the students.

BEFORE THE LESSON

Interactive whole-class introduction (10 minutes)

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the *Muddying the Waters* task. Ask students to read through the task carefully. Some teachers ask students to take turns to read parts of the task aloud.

Introduce the task, asking questions to help students to understand the problem and its context:

This task is concerned with the river pollution and its effect on the environment.

What do I mean by river pollution?

What does it mean when someone says that the level of pollution in a river is illegal?

Does anybody know of a river that is polluted? What was the source of the pollution?

How can you tell that a river is polluted?

In particular, explain how chemical pollution in a river is measured. You could use a teaspoon to help illustrate this.

Chemical pollution is measured in milligrams per cubic meter of river water.

Does anyone know how much a milligram is?

Does anyone know how much a cubic meter is?

A teaspoon of sugar is about 4,000 mg. This classroom is about 300 cubic meters. [Use figures for your own room.]

Assessment task: Muddying the Waters (15 minutes)

Now ask students to work through the task:

Spend fifteen minutes working individually on this task.

Don't worry if you can't complete everything. There will be a lesson [tomorrow] that should help you understand the math better.

Muddying The Waters

The manager of the Riverside Center is concerned about visitor numbers. He is certain the Center's popularity has been badly affected by an increase in river pollution. He feels the local Environmental Agency should do something about it.

To support his argument he measured the chemical concentration in the river each month. He also counted the number of people visiting the Center over several months. He used the results to draw this chart.

Concentration of chemical in river (mg per cubic meter)	Number of visitors to River Center each month
1	130
2	129
4	128
5	127
6	126
7	127
10	125
12	124
14	123
15	123

Scatter chart: Chemical concentration and number of visitors

At the same time the manager asked 18 visitors this question:
 'The odor you can smell originates from the pollution in the river. Is it spoiling your enjoyment of the Center?'

He displayed the results as a pie chart.

Response	Percentage
Yes	72%
Not sure	22%
No	6%

Pie chart showing the percentage of visitors whose enjoyment was spoiled.

The Center Manager writes to the Environmental Officer to try to get something done about the river pollution:

Dear Environmental Officer,

Please find enclosed two charts.

The scatter chart clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.

The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.

The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business. Please let me know what action you intend to take.

Yours faithfully,
 Manager, Riverside Center

Tasks

- Describe in detail what you think the two charts show.

- Do you think the Riverside Center Manager's argument is fair? Explain your reasoning.

Your goal is to be able to answer questions like these confidently by the end of the next lesson.

At this stage, do not help students with the task or comment on their work. Stop them after 15 minutes, whether or not they have finished. Collect in students' papers for analysis.

Preparation for the next lesson (5 minutes)

Explain the theme for the next lesson. You need three copies of the role-play script *A Case of Muddying the Waters*.

Next lesson we are going to continue the theme of river pollution with some role-play.

The role-play sets the scene for the task. An Assistant DA talks to a local Environmental officer and a Factory owner about the pollution of a river. Who wants to play these three roles?

Give each volunteer a copy of the role-play script *A Case of Muddying the Waters*.

Ask the volunteers to read over their parts before the next lesson. If you wish, enrich the role-play by asking the volunteers to bring in a few theatrical props.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of this is to forewarn you of issues that will arise during the lesson itself, so that you may prepare carefully.

We suggest that you do not score students' work. Research shows that this is counterproductive, as it encourages students to compare scores and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues:

Suggested questions and prompts:

<p>Does not understand that there are alternative interpretations of data and statistics, some of which may be biased.</p> <p>For example: The student copies or paraphrases the Riverside Manager’s interpretations.</p>	<ul style="list-style-type: none"> • What does the chart show? What does the Riverside Manager say the chart shows? Is there a difference? • In what ways is what the Riverside Manager said misleading?
<p>Does not recognize that things can happen together without one causing the other</p> <p>For example: The student does not contradict the causal claim made about the scatter plot.</p>	<ul style="list-style-type: none"> • If two things happen at the same time, does that mean one made the other happen? • Write down another reason that there might be a correlation. • What other interpretations of the correlation can you find?
<p>Does not understand that survey questions may push respondents towards a particular response</p> <p>For example: The student does not recognize that the phrasing of the statement biases respondents towards thinking of the river as polluted and smelly.</p>	<ul style="list-style-type: none"> • Does the way this question is asked make a ‘yes’ response more likely than ‘no’ or ‘maybe’? Why do you think that? Does it matter? • Try writing this question in a way that doesn’t push the respondent towards a particular answer.
<p>Does not recognize that statistics may be compiled in ways that push readers towards a biased interpretation</p> <p>For example: The student does not recognize that on the scatter plot, starting the ‘number of visitors’ scale at 122 (rather than 0) distorts perceptions of the proportional change in the number of visitors.</p> <p>Or: The student does not recognize that it is inappropriate to draw conclusions about the whole population from such a small sample size.</p>	<ul style="list-style-type: none"> • Notice that the scale on this graph starts at 122. How different would the graph look if you drew the axis showing the whole range? How might that affect your interpretation? • How many visitors were there overall? When was the survey conducted? Can you think of a more convincing way to set the survey? • How do the ways data was collected affect your interpretation of the results?

SUGGESTED LESSON OUTLINE

Review individual solutions to *Muddying the Waters* (10 minutes)

Remind students of their work on the assessment task.

*Recall the work you did in the last lesson on river pollution.
In this lesson you will build on that work.*

Return the papers to the students. If you chose to write questions on the board rather than on individual papers, display them now.

*I read your papers and I've some questions about them.
I'd like you to work on improving your answers for a few minutes, using my questions.*

Ask students to work on their own for a few minutes, answering your questions.

Interactive role-play introduction (10 minutes)

To introduce the lesson task, use a projector and Slides P-3 to P-8 from the projector resource: *Exhibits 1, 2, 3, 4, Scene 1: The Assistant DA's office, and Scene 2: At the Factory*. If you do not have a projector, hand out printed copies of these Slides. You also need four copies of the role-play script, *A Case of Muddying The Waters* (one copy for each actor and one copy for yourself.)

In the next section of this lesson you will be working on river pollution again. The role-play sets the scene.

Ask the actors to read out the script. Advise them to talk slowly and to pause at the end of each sentence, as the script contains a lot of information. Encourage the students to listen carefully to the facts being presented about the river pollution.

Collaborative small-group work (25 minutes)

Once the students have acted out the scenes turn to the class and say:

The case goes to court. The Assistant DA prosecutes the Factory Owner for polluting the river.

What does 'prosecution' mean?

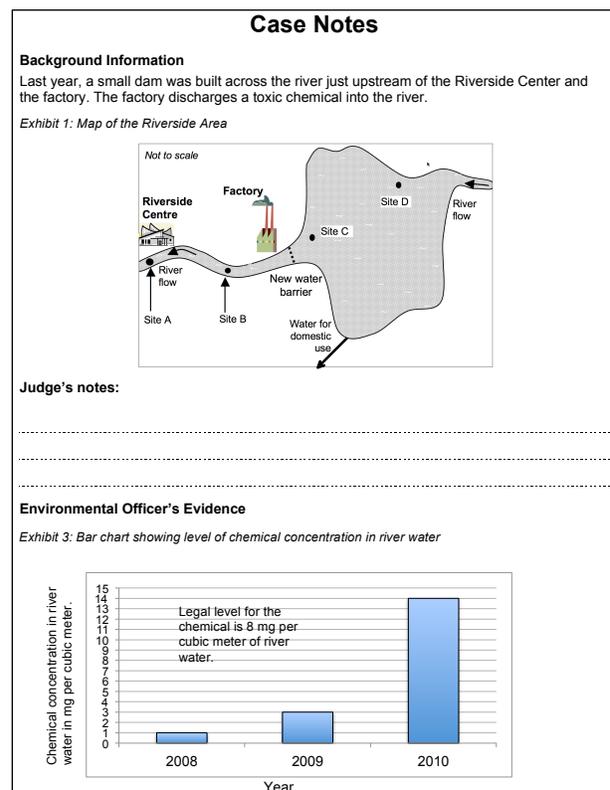
Your task now is to be the judges. You have to reach a fair judgment about who wins the case.

Organize students into groups of two or three. Provide each small group with a copy of the *Case Notes* worksheet. The *Case Notes* contain the map, information from the script, and arguments made in court by the Environmental Officer and the Factory Owner.

Help students to understand the task and its context:

I'm giving you a copy of the arguments presented in court.

Read through the information carefully.



Write notes on what you think the data and statistics show.

In particular, ask them to focus on critical analysis of the information presented.

Explain why you agree or disagree with the arguments people made, using math.

The important thing is to look critically at all of the information. Do not just accept what people say as fact.

At the end of the lesson, you will use your work to decide together whether the factory owner is guilty, or not guilty, of polluting the river.

These instructions are reproduced on Slide P-9, *Judge's Instructions*.

During small-group work you have two tasks: to notice strengths and weaknesses you see in students' work and to support their thinking.

Notice strengths and weaknesses in students' work

Find out about students' current levels of understanding and the difficulties they encounter in the task. Students may be used to interpreting statistical diagrams, but may find it more difficult to critique someone else's biased reading of information. Students may fail to notice a bias in a question, or may struggle to understand the issue of small sample size. You can use the information about common difficulties to focus the whole-class discussion towards the end of the lesson.

Support student thinking

Try not to solve students' difficulties for them. Instead, ask them questions to help them move their thinking on.

You could strengthen your argument if you did some math on the data you've been given.

Is there another way to present this data?

Could you redraw that chart so it displays the important features of the data better?

Questions similar to those in the *Common issues* table on page T-4 were found to be useful in lesson trials.

For students who are struggling, it may help to ask some specific questions about aspects of the mathematics:

Describe this chart.

Is there another way to present this data?

The Environmental Officer/Factory Owner drew this conclusion. Can you draw any different conclusions from this evidence?

Encourage students to explain their reasoning to others in the group before writing it down. Other group members may question and refine the explanations.

Whole-class discussion: reaching a judgment (10 minutes)

Organize a whole-class discussion, focusing on the mathematical practice of critiquing the reasoning of others.

Choose a group to present their argument about one piece of evidence. Instruct the other groups to listen and write down questions about the group's argument.

Hani, does this evidence help the factory owner show he is not guilty? Tell us why you think that.

If you disagree with the group's interpretation of the evidence, write why and challenge them at the end of their presentation.

Once the group has presented their case, other groups get a chance to challenge the details of their argument. If the challenge is not based on mathematics, you can rule it out of court.

That is not a mathematical argument. As there is no good evidence, it can't be accepted in court.

Once you have modeled this process for students, give them responsibility for deciding whether there is an evidential base for each claim and challenge.

Is that a mathematical argument? Is there good evidence for what [Shelley] has just said?

Does the evidence support her conclusion?

Once students have worked through the four pieces of evidence, ask them to come to a collective decision about the verdict.

Do you find the factory owner guilty or not guilty?

If there is disagreement, take arguments from both sides. You may find you cannot reach a collective decision. In that case, suggest students send the defendant for a retrial.

Whole-class discussion: summing up (5 minutes)

Point out that an important message of this lesson is that it is easy to 'get it wrong' when interpreting statistics, especially in complicated real-world situations. In reality, most of the 'evidence' in this lesson is too vague to draw any firm scientific conclusions. A lot of questions are left unanswered.

How, exactly, were the wildlife surveys conducted? Why did the second survey look at so many more fish than the first? How do you 'count the number of invertebrates' at a site?

The dam has reduced the flow of the river by 80%. Even without the pollution, is it possible that this could affect the wildlife or the popularity of the Riverside Center?

Can you see any other problems with the data collection? / with the statistics that have been calculated?

If you think it is appropriate to the class, you could mention that if they study statistics further they will learn how to calculate **significance**: the likelihood that a difference in two results is not just 'the luck of the draw'. Most serious scientific studies will do this, but you do not often find it in news reports!

Follow-up lesson: Unhappy Campers (15 minutes)

Give each student a copy of the *Unhappy Campers* task. Explain that this task uses very similar math to the lessons on river pollution, but in a new context.

Help students to read through the task sheet and use questions to help them understand the context:

*What is a wind turbine?
What are they used for?*

What do decibels measure?

How loud is 50 decibels? 30 decibels? [0 decibels is the threshold for human hearing. A whisper in a quiet library is about 25 decibels. Normal conversation at about five feet is around 60 decibels.]

Ask students to work on their own on the assessment task, bearing in mind what they have learned during the previous lesson.

*I want you to work on this task, using those same ideas about a fair, mathematical critique.
Remember not to believe all the arguments someone gives you using statistics.*

After the assessment, you may find it useful to ask students to compare their responses to the first and second assessment tasks, so they can see the progress they have made.

Some teachers give this for homework.

SOLUTIONS

Assessment task: *Muddying the Waters*

Question 1:

Interpreting the scatter chart

- The water was tested on a monthly basis for 10 months and at the same time the number of visitors to the Center was recorded.
- There is a negative correlation.
- The number of visitors per month varies from 122 to 130. The range is 8 and the mean is 126.
- Chemical concentration varies from 1 to 14 mg per m³. The range is 13 mg per m³ and the mean is 7.4 mg per m³.

Interpreting the pie chart

- Eighteen people are involved in the survey. 13 replied yes, 1 replied no and 4 were unsure.

Question 2:

The Riverside Manager's argument is misleading in various ways.

- The scatter plot has a misleading scale. It gives the impression that correlated with a rise in pollution there has been a massive drop in visitor numbers. In fact, there is a fall of only 8. Overall the decrease is 6%.
- There is a negative correlation on the scatter chart. This may not be causal as there are many other reasons why the visitor numbers fell, such as change in season. If the dam was reducing the amount of water in the river, this might have made it less attractive to visitors. The survey was over 10 months, not a year.
- The pie chart is based on a survey that uses a biased question: people may not have noticed a smell until they were asked about it.
- The sample size for the pie chart is small. The results of the survey are unlikely to be a true representation of all the visitors to the Center. Providing the number of people as well as the percentages in each response category would be helpful.

Lesson task: *Case Notes*

The concentration of the chemical in the river has risen above the legal limit.

The bar chart is appropriate and it clearly shows that three distinct tests have been carried out. The concentration of the chemical in the river **is** now above the legal limit.

The levels were within the limit in the previous two years. The factory is discharging the same amount of chemical, but the flow rate of the water has reduced, meaning that the **concentration** is now above the legal limit.

Students may have calculated the concentration of the chemical in the water:

Last year:

$$\frac{60}{20} = 3\text{mg}/\text{m}^3.$$

This year:

$$\frac{60}{4} = 15\text{mg} / \text{m}^3.$$

Arguing that the mean concentration is within the legal limit is a misuse of statistics, because the low measurements in the first two years disguise the much higher figure in the last year:

$$\frac{1+3+14}{4} = 6\text{mg} / \text{m}^3.$$

A more sympathetic judge might argue that there are not enough test sites to prove that the pollution was not caused by another source: it might have been useful to test the water upstream of the factory, to find out whether that water was polluted to start with.

There has been an increase in the number of diseased fish due to the rise in chemical pollution.

The survey is misleading because the sample sizes are different. Arguing that there are now ten times more diseased fish is incorrect because it ignores the sample sizes. If students have calculated proportions or percentages, they will get a better sense of the data than if they rely on numbers:

Two years ago:

$$\frac{6}{300} \times 100 = 2\%$$

This year:

$$\frac{64}{1600} \times 100 = 4\%$$

Using this to argue that the number of diseased fish has doubled is still a misleading use of statistics: finding slightly more or fewer diseased fish in either survey (due to weather, the way the fish were caught, or just ‘the luck of the draw’) would make a big difference to the percentages.

The judge could argue that the survey is poor evidence because the sample sizes are too small to detect a difference in such a small percentage of diseased fish. Or the judge might argue that there are not enough survey sites to show whether being downstream of the factory makes a difference. The reduced flow rate of the river might have affected the health of the fish regardless of the pollution. Why did the second survey look at so many more fish than the first: were the fish harder to find the first time?

The number of invertebrates has not changed.

There has been hardly any change in the mean number of invertebrates. Two years ago the mean across four sites was 21, and now it is 19.

However, two years ago the range was 4. Now the range is 20. This is a big increase.

The sites most likely to have been affected by the pollution are A and B, downstream from the factory. Two years ago the mean number of invertebrates at these sites was 21, now it is 12. This is quite a large decrease. In contrast, the mean at sites C and D has increased.

Arguing that the mean number of invertebrates has hardly changed is a misuse of statistics: taking the mean of all four sites (including two which would not have been affected by pollution from the factory) hides the possibly significant reduction at the polluted sites.

The number of birds has increased.

Using a line graph to represent this data is inappropriate because it gives the impression that the birds were continually monitored. A bar chart with two bars would be more appropriate.

The scale on the line graph is misleading because it gives the impression that there has been a dramatic increase in the number of birds. The data show that there has only been an increase of 6 birds (about 7%). This is insignificant, especially without more details of how the birds were counted or at what time of year.

Arguing that the chart shows that the number of birds has increased dramatically in the last two years is a misleading use of statistics.

Assessment task: *Unhappy Campers*

Question 1:

Interpreting the scatter chart:

- There are fourteen data points on the scatter chart; the survey took place over a two-week period. There is a negative correlation.
- The number of visitors ranges from 70 to 78, a range of 8 with a mean average of 75. The noise level ranges from 10 to 60 decibels, with a range of 50 decibels and a mean average of 35 decibels¹.

Interpreting the pie chart:

- The number of campers surveyed was 50.
- 80% of the sample responded yes, 16% unsure, and 4% no.
- The numbers of respondents are 40, 8, and 2 respectively.

Question 2:

The camp manager's argument is biased in several ways.

Her choice of math introduces bias:

- The scatter plot has a misleading scale. The scale on the 'number of campers' axis starts at 40 rather than 0. It gives the impression that correlated with the rise in noise level there has been a large drop in visitor numbers. The number of campers only varies by 8 across the fourteen-day period, decreasing by about 10% between the quietest and noisiest day.
- The survey statement and question is biased. Stating that the noise is 'loud' and assuming that the respondent can hear the noise pushes the respondent towards a positive response; the use of 'spoiling enjoyment' in the question also introduces potential bias.
- The pie chart is based on a relatively small sample (50 campers). The survey took place on one day. On only 2 days on the scatter chart were there 50 campers. Both days were particularly noisy. Surveying only on a noisy day produces potential bias in the survey responses. It would have been helpful to show the number of respondents, not just percentages, on the pie chart, to aid interpretation of the results.

Her interpretations of her data and statistics are incorrect.

Peggy claims that the noise from the turbines has **caused** a drop in camper numbers. The correlation between the noise level in decibels and the number of campers does not show there is a causal relationship between the two variables. There may be other explanations of why the number of campers and the noise level correlate. For example, the turbine noise increases with the wind level, so you would expect fewer people to want to camp at noisy times, because it is windier then.

Although most of the people surveyed did state that the wind turbines spoil their enjoyment, the questionnaire was biased, the sample was small, and the survey took place on a noisy day. As the results of the survey are dubious, there is no evidence to support Peggy's interpretation that most people coming to the camp would agree with the results of the survey. She cannot generalize from a small, biased sample and she cannot rely on responses to a biased question.

¹From the US Environmental Protection Agency website:

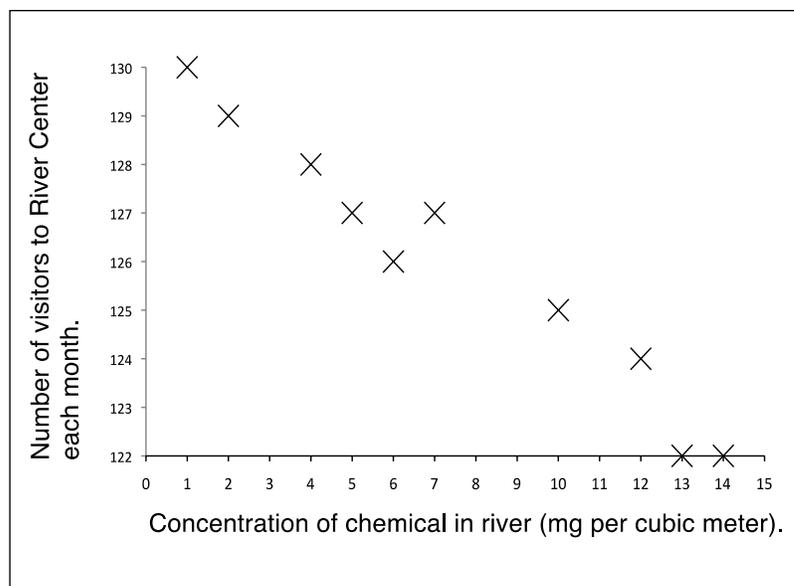
The [document](#) identifies a 24-hour exposure level of 70 decibels as the level of environmental noise which will prevent any measurable hearing loss over a lifetime. Likewise, levels of 55 decibels outdoors and 45 decibels indoors are identified as preventing activity interference and annoyance. These levels of noise are considered those which will permit spoken conversation and other activities such as sleeping, working and recreation, which are part of the daily human condition.

<http://www2.epa.gov/aboutepa/epa-identifies-noise-levels-affecting-health-and-welfare>

Muddying The Waters

The manager of the Riverside Center is concerned about visitor numbers. He is certain the Center's popularity has been badly affected by an increase in river pollution. He feels the local Environmental Agency should do something about it.

To support his argument he measured the chemical concentration in the river each month. He also counted the number of people visiting the Center over several months. He used the results to draw this chart:



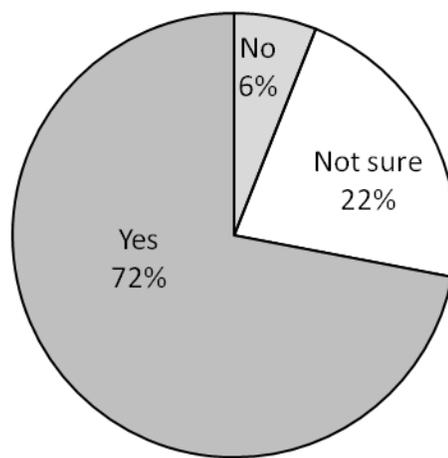
Scatter chart: Chemical concentration and number of visitors

At the same time the manager asked 18 visitors this question:

'The odor you can smell originates from the pollution in the river. Is it spoiling your enjoyment of the Center?'

He displayed the results as a pie chart.

Pie chart showing the percentage of visitors whose enjoyment was spoiled.



The Center Manager writes to the Environmental Officer to try to get something done about the river pollution:

Dear Environmental Officer,

Please find enclosed two charts.

The scatter chart clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.

The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.

The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business. Please let me know what action you intend to take.

Yours faithfully,

Manager, Riverside Center

Tasks

1. Describe in detail what you think the two charts show.

2. Do you think the Riverside Center Manager's argument is fair?
Explain your reasoning.

A Case Of Muddying The Waters: Role-Play Scripts

Scene 1: Environmental Officer talking to the Assistant DA in the DA's office

- Teacher A year ago a small dam was built across the river to provide extra water for local domestic use. It was positioned just upstream of the Riverside Center and a factory [*show class **Exhibit 1** and point out the landmarks*]. This factory continually discharges a toxic chemical into the river. Prompted by the letter [*show class **Exhibit 2***] from the Manager of the Riverside Center, the local Environmental Officer [*indicate the student playing this role*] checks the concentration of the chemical in the river. Unhappy with the result he arranges to meet the Assistant DA at his office [*indicate the student playing this role*]. Here is how the meeting went [*show class the photograph of the DA's office*]:
- Environmental Officer Each year I test the river water for this toxic chemical and up until this year the concentration levels have been fine.
- Assistant DA But there's a problem now?
- Environmental Officer That's right. As you can see from this chart, [*show class **Exhibit 3***] the concentration of the chemical in the river is now disturbingly high.
- Assistant DA What can account for this change?
- Environmental Officer Well, downstream of the barrier the flow of the river has decreased from 20 to 4 cubic meters per second.
- Assistant DA Go on.
- Environmental Officer This decrease does affect the concentration of the chemical in the river. To calculate this concentration you divide the chemical discharge from the factory by the rate of flow of the river.
- Assistant DA Oh yeah, right [*not understanding a word*]. Have you noticed any other changes?
- Environmental Officer Yes, the number of diseased fish in the river. Two years ago we only found 6 diseased fish in the waters near the factory, but just last week we found 64. I'm sure you'll agree that's a massive increase.
- Assistant DA Mmm ... I think I better take a ride out to the factory and see what the owner has to say about all this.

Scene 2: Assistant DA interviewing the Factory Owner in the Factory Owner's office

Show the photograph of the factory.

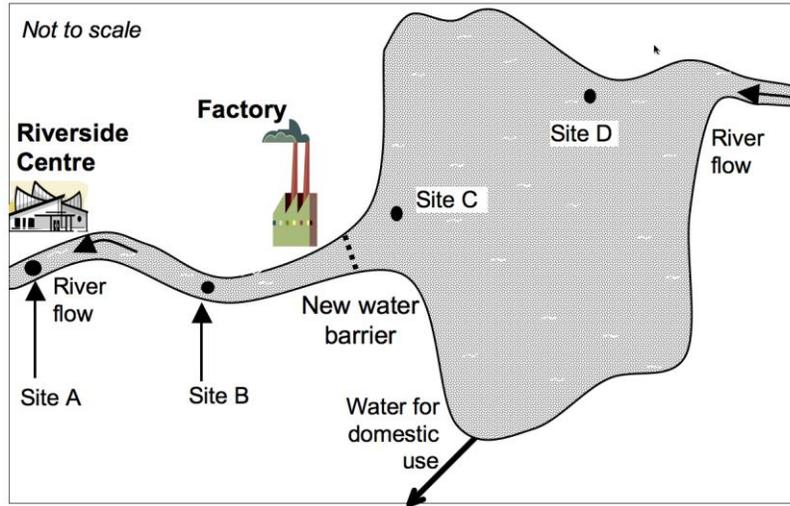
- Teacher So the assistant DA then drove out to the factory to interview its owner
[*indicate the student playing this role to the class*].
- Assistant DA Do you know you are discharging a toxic chemical into the river?
- Factory owner Sure we know that, but we're only discharging it at the rate of 60 milligrams per second. When I first opened the factory some years ago, my manager checked that this was within the legal limits and this rate has not changed since then.
- Assistant DA I see. But the fact is, the concentration of the chemical in the river is now above the legal limit and you are the only factory in the area producing the stuff.
- Factory owner Well, it's a mystery to me. Something else must have caused the increase.
- Assistant DA I hear what you're saying, but I still think you're responsible.
- Factory owner Do you reckon – because I don't. What is important, though, is the health of the river. As you probably know, a good sign of a healthy river is the variety of the invertebrates in it. I've been keeping an eye on them in our river.
- Assistant DA What, you mean you've actually been counting the number of snails and water insects in the river?
- Factory owner Well yes, my people have counted all animals without a backbone. They've been counted at four different sites and the average number has barely changed in the last two years [*show class Exhibit 1 and point to the 4 sites*]. That's a good sign, don't you think?
- Assistant DA That is interesting. Have you checked anything else?
- Factory owner I've also kept a close watch on the number of birds around the factory and as you can see from the chart, there has been a dramatic increase [*show class Exhibit 4*]. Another sure sign of a healthy river.
- Assistant DA Well, I will get my experts to check all this. But from what I've heard, there is a case to answer. So I'll see you in court!

Case Notes

Background Information

Last year, a small dam was built across the river just upstream of the Riverside Center and the factory. The factory discharges a toxic chemical into the river.

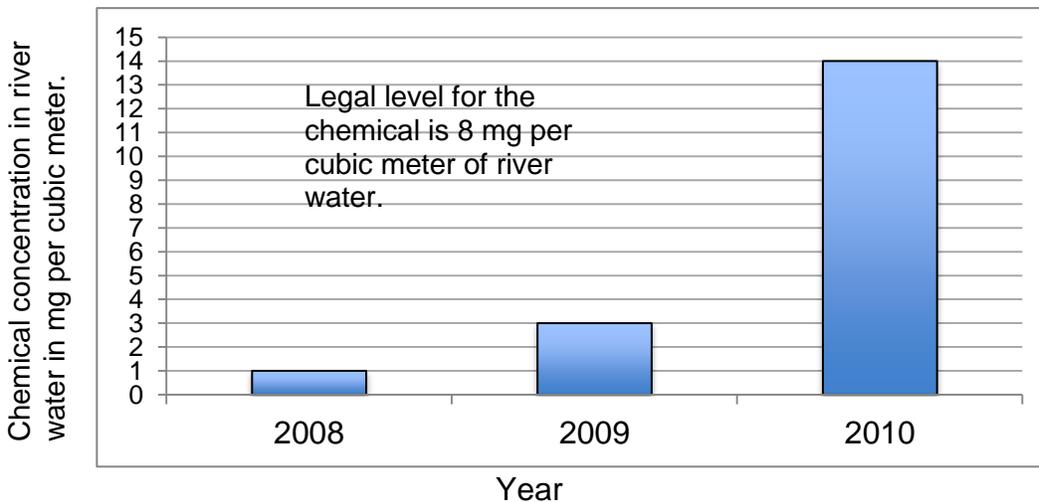
Exhibit 1: Map of the Riverside Area



Judge's notes:

Environmental Officer's Evidence

Exhibit 3: Bar chart showing level of chemical concentration in river water



“Each year, I test the river water for this toxic chemical and up until this year the concentration levels have been fine.

The chart shows that the concentration of the chemical in the river is now disturbingly high.

Downstream of the barrier, the flow of the river has decreased from 20 to 4 cubic meters per second.

This affects the concentration of the chemical in the river.

To calculate this concentration you divide the chemical discharge from the factory by the rate of flow of the river.”

Judge’s notes:

Factory Owner’s Evidence

“We are discharging a toxic chemical, but only at the rate of 60 milligrams per second.

When I first opened my factory the manager checked that this is within the legal limits.

The rate has not changed since then.

If the increase is now outside the legal limits, someone else must have caused the change.”

Judge’s notes:

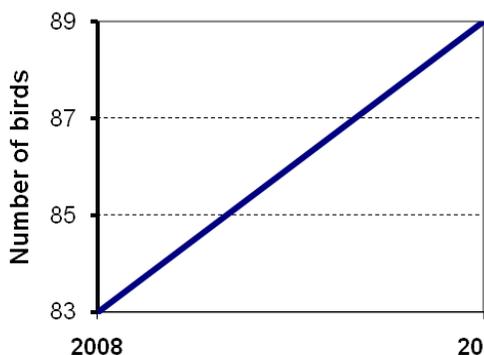
Factory Owner’s Evidence

“I’ve kept a close watch on the number of birds around the factory.

You can see from the chart that there has been a dramatic increase in the number of birds around here.

This is another sure sign of a healthy river.”

Exhibit 4: Survey of the number of birds seen next to the factory in one afternoon



Judge's notes:

Environmental Officer's Evidence

“There has been an increase in the number of diseased fish in the river.”

Exhibit 5: Survey of diseased fish near site A

Both surveys covered a period of five days.

- Two years ago 6 fish out of 300 were diseased.
- Last week 64 fish out of 1,600 were diseased.

Judge's notes:

The Factory Owner's Evidence

“It's important to think about the health of the river.

A sign of a good healthy river is the variety of invertebrates in it.

I've been counting the number of animals without a backbone – things like snails and water insects.”

Exhibit 6: Survey of numbers of invertebrates

	Two years ago	Now
Site A	20	15
Site B	22	9
Site C	19	23
Site D	23	29

“Invertebrates were counted at four different sites.

On the map (Exhibit 1), you can see where the sites are.

The average number of invertebrates has barely changed in the last two years - a good sign.”

Unhappy Campers

Peggy is the manager of a campground.

Some new wind turbines are built next to the camp.

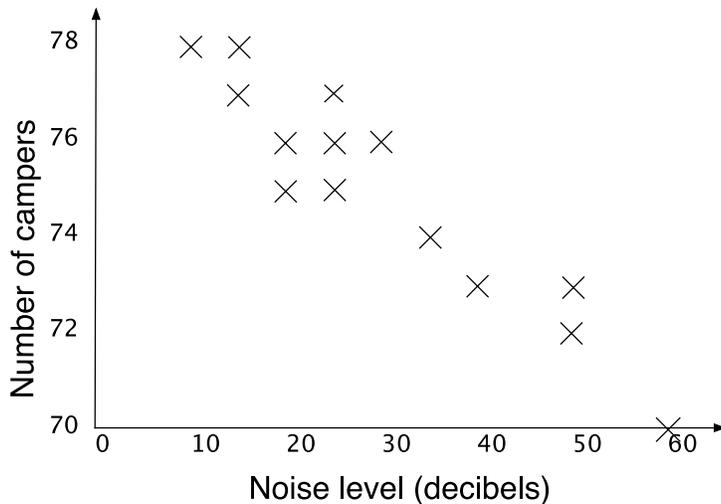
Peggy feels sure fewer people are visiting her site because of the turbines.

Peggy wants the Environmental Agency to get rid of the turbines.

To make her case she does some math.

For two weeks, Peggy records the number of visitors to the campsite each day. She also records the highest level of noise the turbines make each day.

She uses the information to draw a scatter chart.



Peggy also conducts a survey of the 50 campers who visit one day.

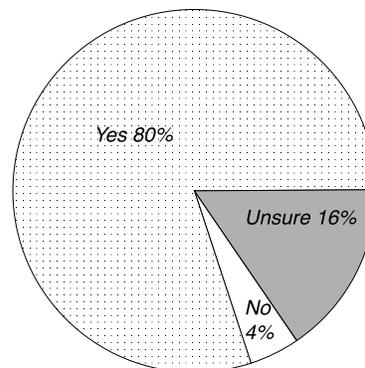
She asks the campers this question:

The loud noise you can hear comes from turbines on the wind farm.

Is the wind farm spoiling your enjoyment of the campsite? Yes No Unsure

Peggy uses the survey results to draw a pie chart.

Pie chart showing percentage of campers whose enjoyment was spoiled by turbine noise.



Peggy writes to the Environmental Officer.

Dear Environmental Officer,

Please find enclosed two charts.

The new wind farm is noisy and ugly and I have evidence to prove it.

The scatter chart shows that the wind farm drives campers away. The noise causes fewer people to come to camp.

The pie chart shows that most people don't like the wind farm. It's affecting their enjoyment of the camp.

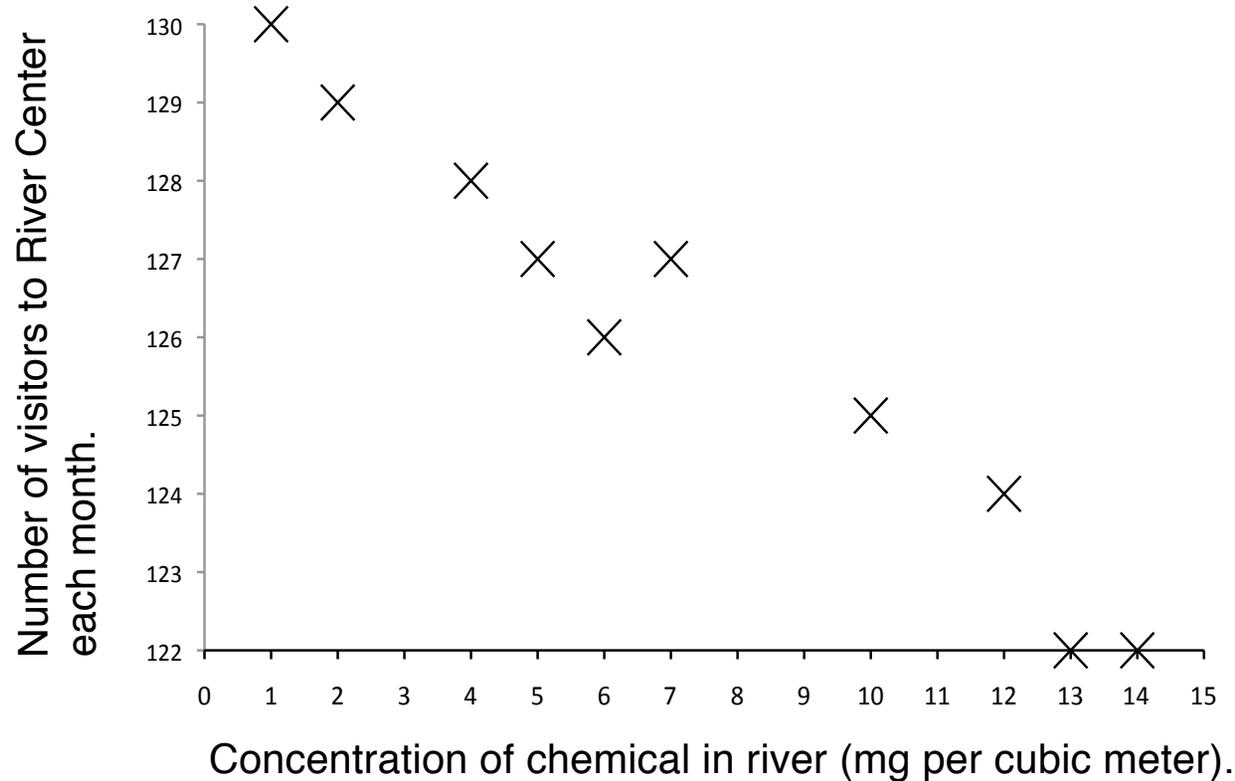
You should close the wind farm because it is spoiling the local environment.

Yours faithfully,

1. Describe what you think is shown by the scatter chart and pie chart.

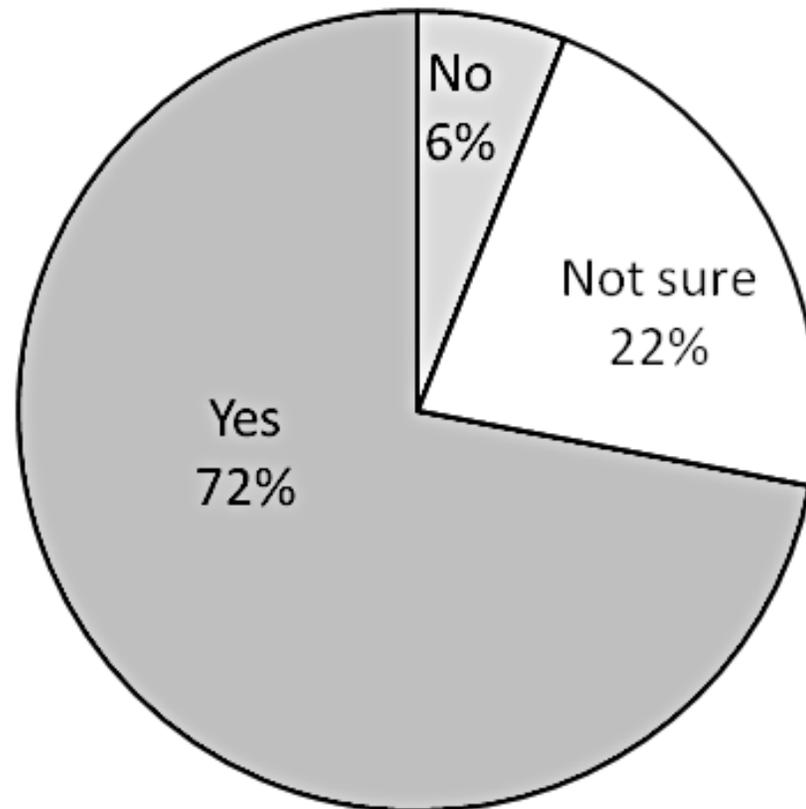
2. Do you think the Camp Manager makes a fair argument? Explain your answer.

Muddying the Waters: Scatter Chart



Scatter chart: Chemical concentration and number of visitors.

Muddying the Waters: Pie Chart



Pie chart showing the percentage of visitors whose enjoyment was spoiled.

Exhibit 1: Map of Riverside Area

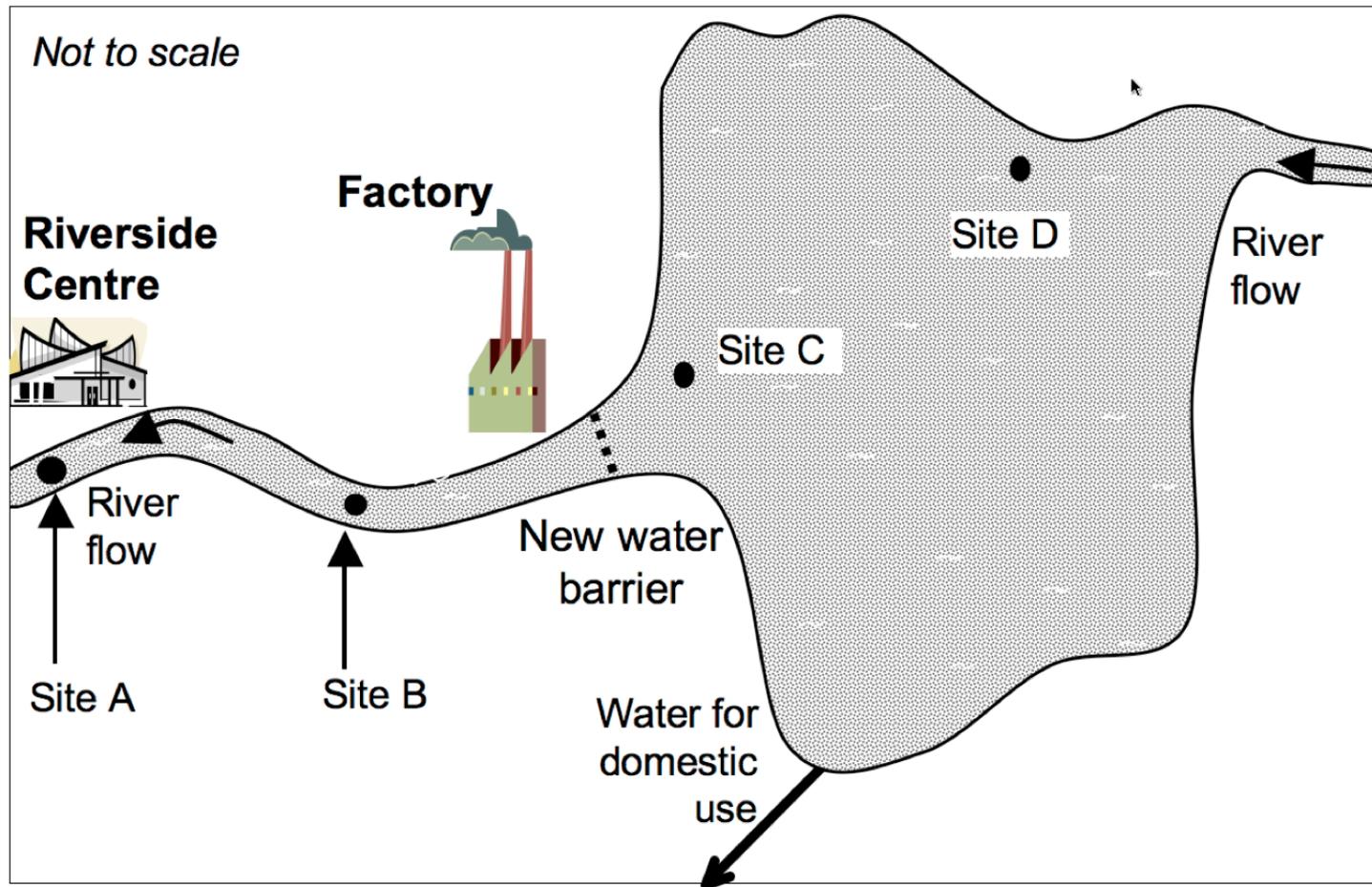


Exhibit 2: Riverside Manager's Letter

Dear Environmental Officer,
Please find enclosed two charts.

The scatter plot clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.

The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.

The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business. Please let me know what action you intend to take.

Yours faithfully,
Manager, Riverside Center

Scene 1: The Assistant DA's Office



Scene 2: At the Factory



Exhibit 3

Bar chart showing level of chemical concentration in the river water:

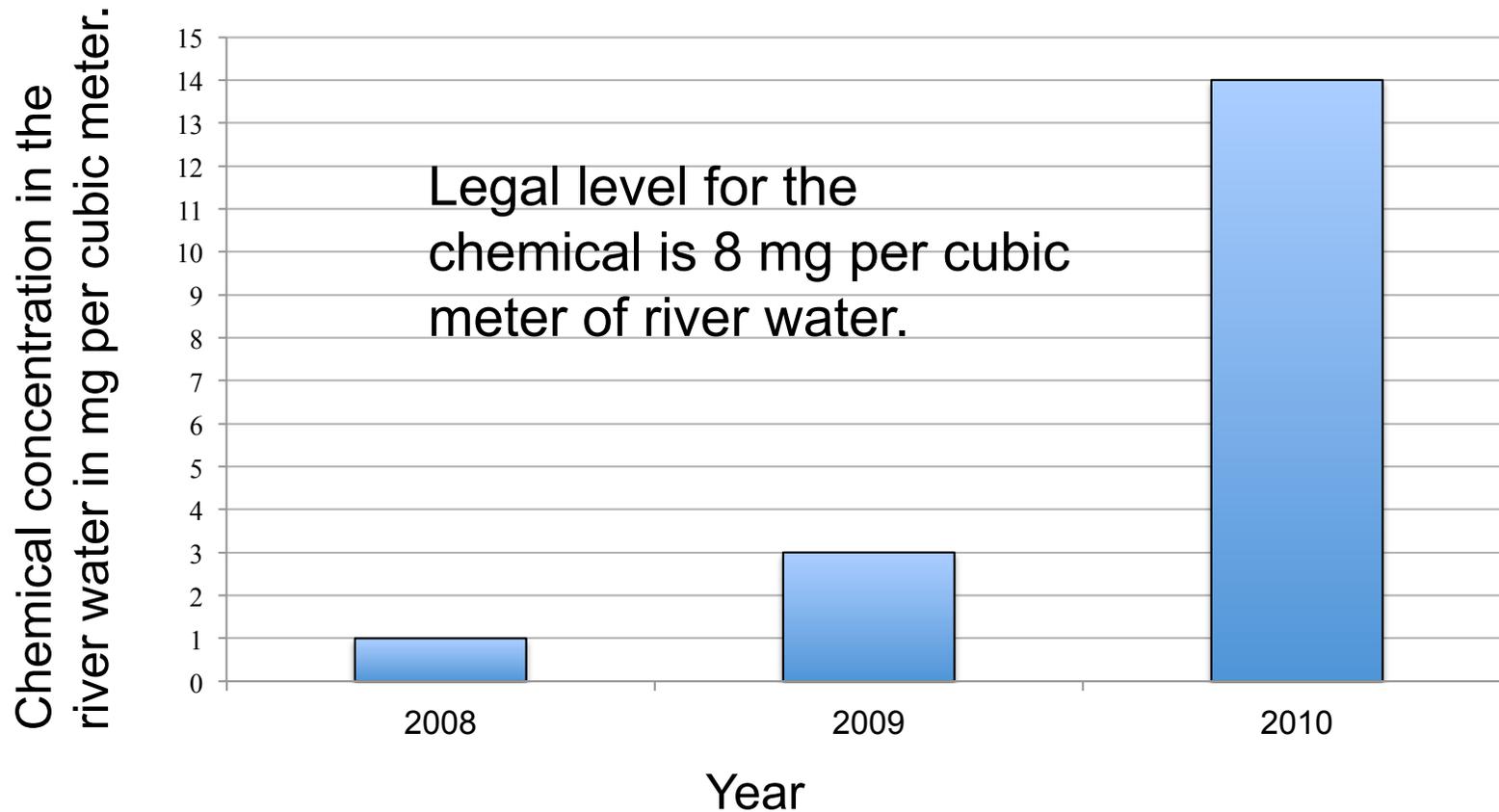
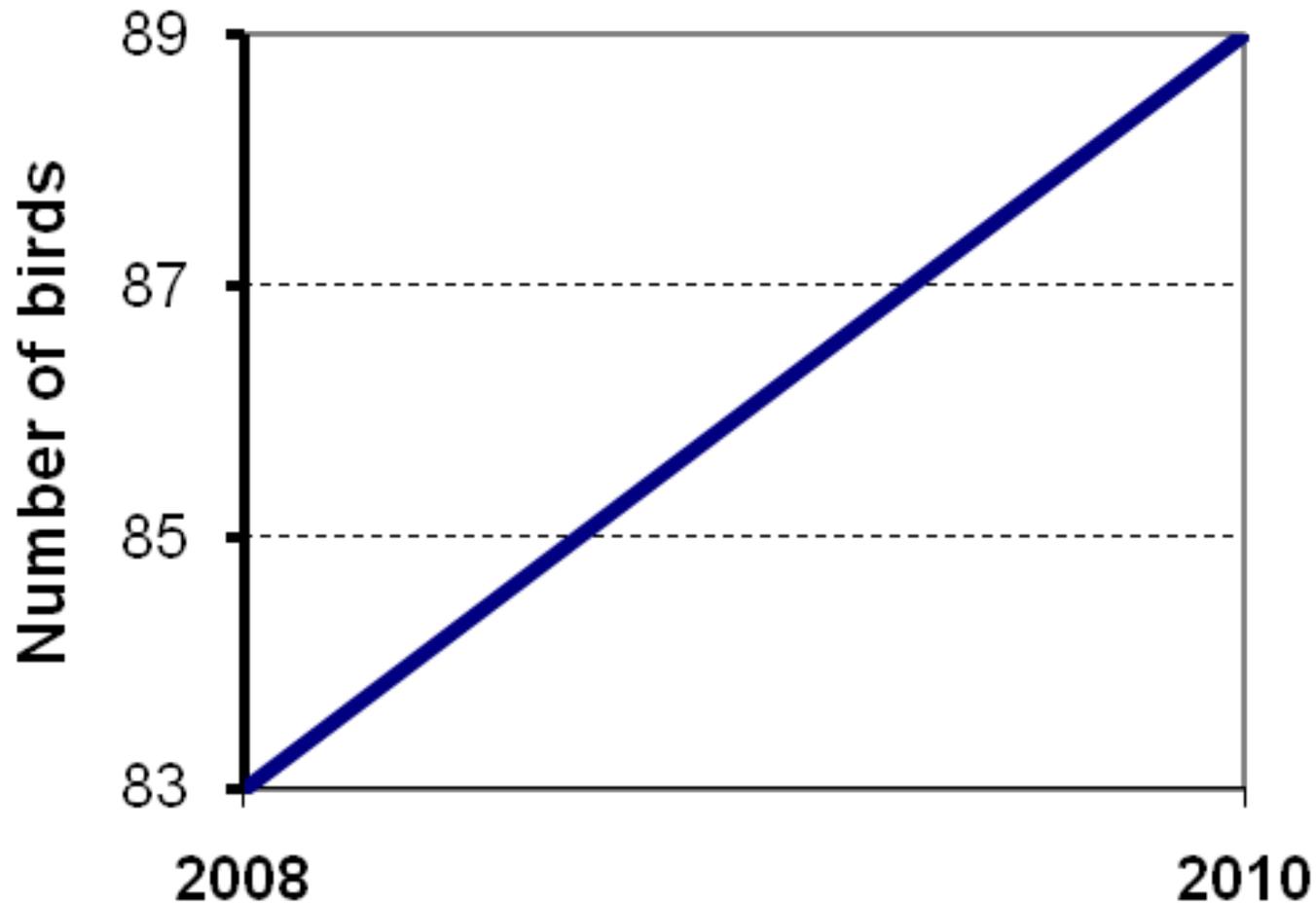


Exhibit 4

Survey of the number of birds seen next to the factory in one afternoon:



Judge's Instructions

You have to reach a fair judgment about who wins the case.

- Read through the information carefully.
- Write notes on what you think the data and statistics show.
- Explain why you agree or disagree with the arguments, using math.
- Look critically at all of the information.
- Do not just accept what people say as *fact*.

Exhibit 5

Survey of diseased fish near site A

Both surveys covered a period of five days.

- Two years ago 6 fish out of 300 were diseased.
- Last week 64 fish out of 1,600 were diseased.

Exhibit 6

Survey of numbers of invertebrates:

	<u>Two years ago</u>	<u>Now</u>
Site A	20	15
Site B	22	9
Site C	19	23
Site D	23	29

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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<http://map.mathshell.org>