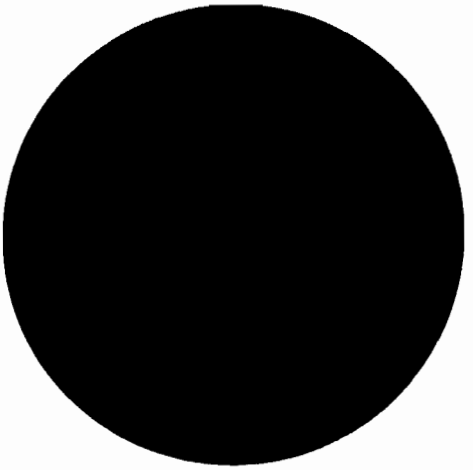
Circle Pattern

T1

Here is a developing circle pattern.

Here is one black circle.



Two white circles of half the radius have been added to the diagram.

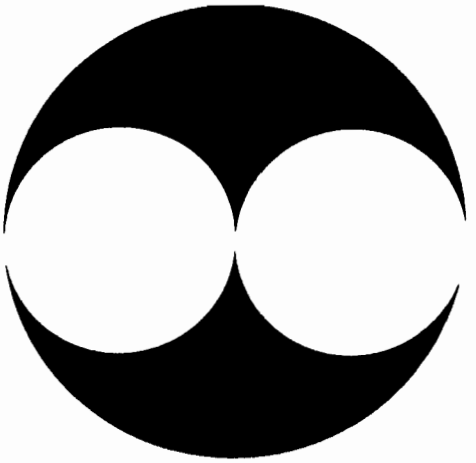
1. Show that the fraction of the diagram that is now black is one half.

r of white circle; r of black circle 2r

area black circle $\pi(2r)^2$: area white $= 2\pi r^2$

black circle: $4r^2\pi$, white circles: $2r^2\pi$

black - white $= 2r^2\pi$, exactly $\frac{1}{2}$ of $4\pi r^2$

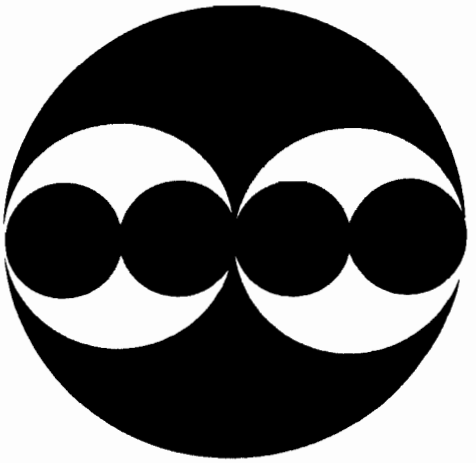


Four black circles have now been added.

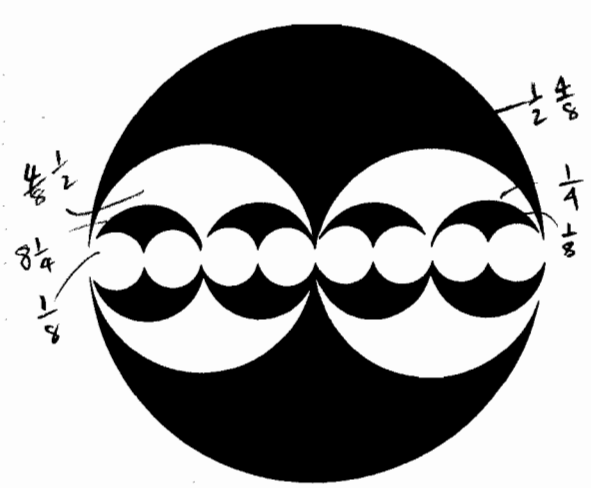
2. What fraction of the diagram is now black?

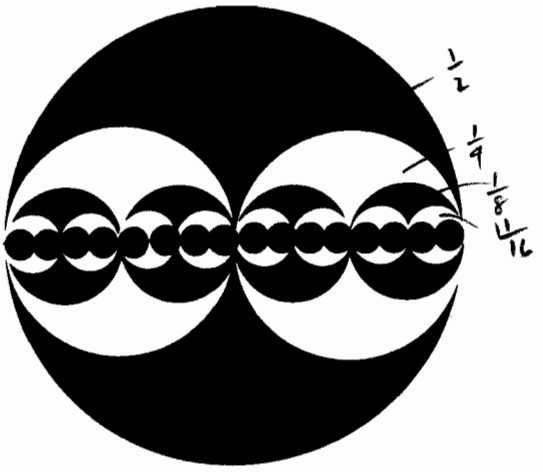
$\frac{1}{2} + \frac{1}{2}(\frac{1}{2}) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4}$



3. Fill in the table to show what happens as the pattern continues.





Pattern	Black fraction	White fraction
One black circle	1	0
Two white circles	$\frac{1}{2}$ (−$\frac{1}{2}$)	$\frac{1}{2}$ (+$\frac{1}{2}$)
Four black circles	$\frac{3}{4}$ (+$\frac{1}{4}$)	$\frac{1}{4}$ ($\frac{1}{4}$)
Eight white circles	$\frac{5}{8}$ (−$\frac{1}{8}$)	$\frac{3}{8}$ (+$\frac{1}{8}$)
Sixteen black circles	$\frac{11}{16}$ (+$\frac{1}{16}$)	$\frac{5}{16}$ (−$\frac{1}{16}$)

4. Write a description of what is happening to the black and white fractions as the pattern continues.

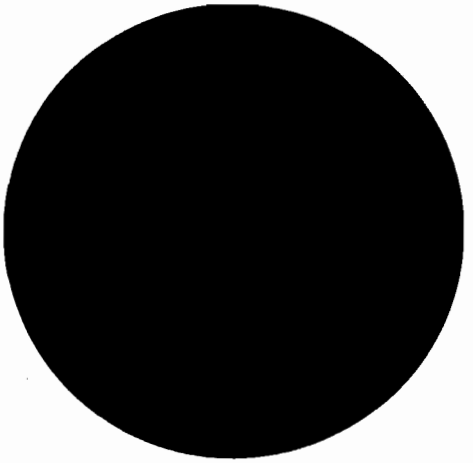
The fractions alternate between adding and subtracting $\frac{1}{2(\text{denominator})}$ to the previous fraction, the denominator corresponds w/ how many of the smallest circles Ex. there smallest ones are 4 black circles, previously $\frac{1}{2}$ of the patter was black, since the smallest circles are the same colour, it will increase, so $\frac{1}{2} + \frac{1}{2(2)} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, if the color was different, then we'd subtract.

Circle Pattern

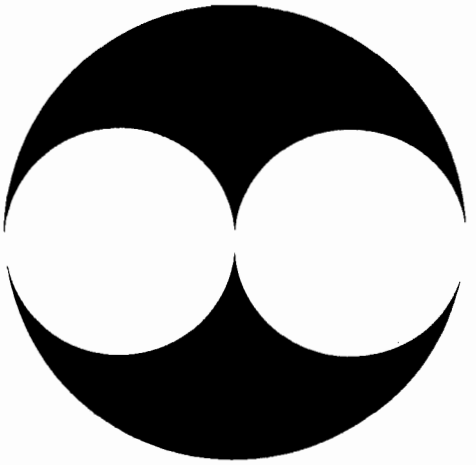
T2

Here is a developing circle pattern.

Here is one black circle.



Two white circles of half the radius have been added to the diagram.



1. Show that the fraction of the diagram that is now black is one half.

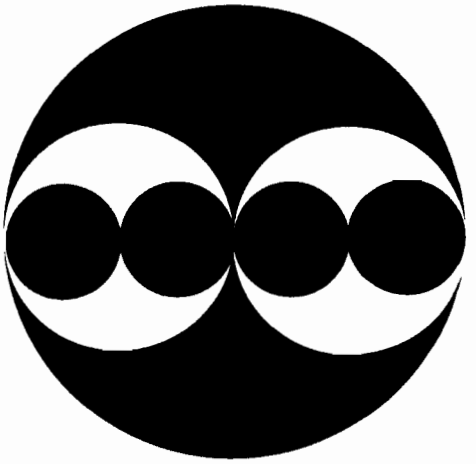
black circle πr^2

one white circle: $\pi(\frac{1}{2}r)^2$

2 white circles: $\pi\frac{1}{2}r^2$

$\pi\frac{1}{2}r^2$ is $\frac{1}{2}$ of πr^2 so black is the other half of the figure.

Four black circles have now been added.



2. What fraction of the diagram is now black?

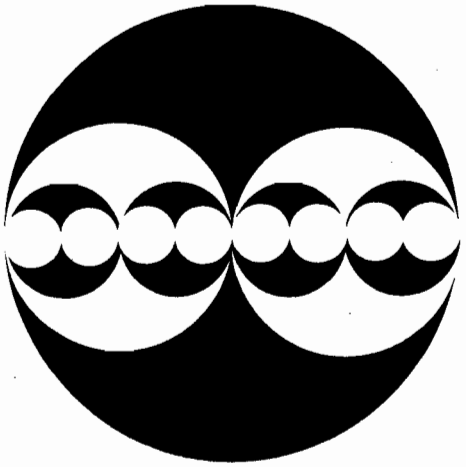
one white circle: $\pi\frac{1}{4}r^2$

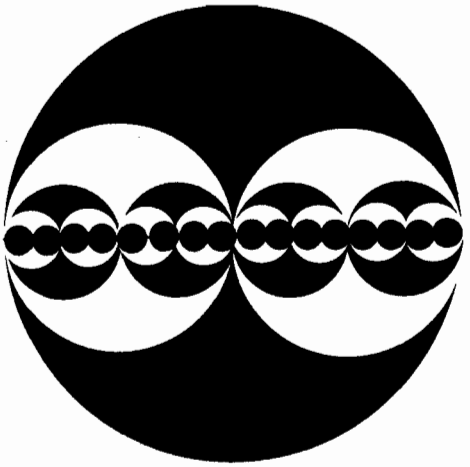
two small black circles: $\pi\frac{1}{8}r^2$

4 small black circles: $\pi\frac{1}{4}r^2$

Black: $\frac{3}{4}$ of figure

3. Fill in the table to show what happens as the pattern continues.





Pattern	Black fraction	White fraction
One black circle	1	0
Two white circles	$\frac{1}{2}$	$\frac{1}{2}$
Four black circles	$\frac{3}{4}$	$\frac{1}{4}$
Eight white circles	$\frac{5}{8}$	$\frac{3}{8}$
Sixteen black circles	$\frac{11}{16}$	$\frac{5}{16}$

4. Write a description of what is happening to the black and white fractions as the pattern continues.

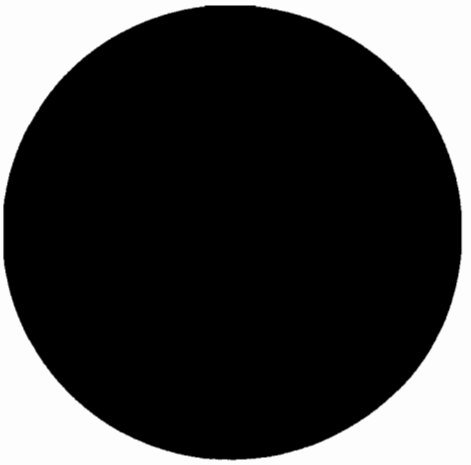
Whenever black circles are added, the black fraction increases by half of what was already there, and the white fraction decreases by half of what was already there. When white circles are added, the black fraction decreases by half of what was already there, and the white fraction increases by half of what was there.

Circle Pattern

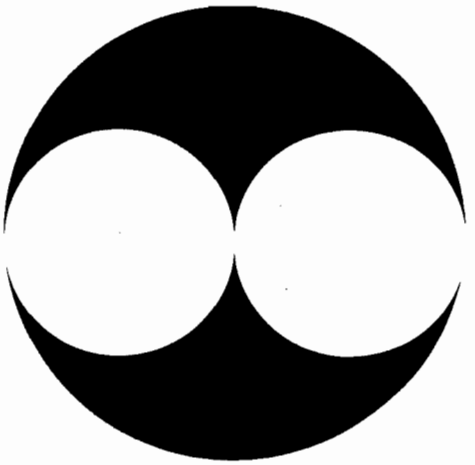
T3

Here is a developing circle pattern.

Here is one black circle.



Two white circles of half the radius have been added to the diagram.



1. Show that the fraction of the diagram that is now black is one half.

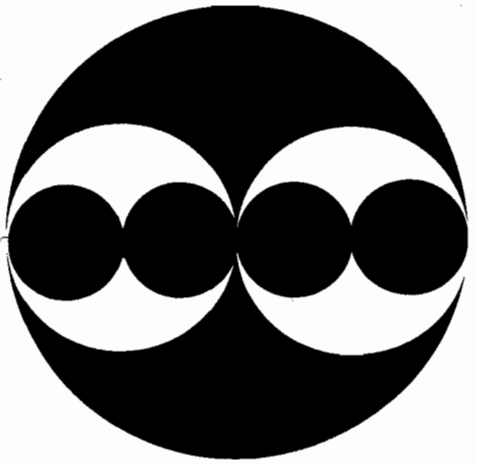
If radius of large circle: 2 small: 1

$(2\times2)\pi = 2(1^2\pi)$

$4\pi = 2(2\pi)$

$4\pi = 4\pi$

Four black circles have now been added.



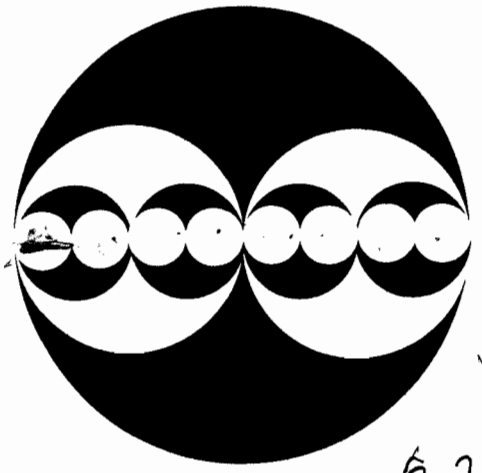
2. What fraction of the diagram is now black?

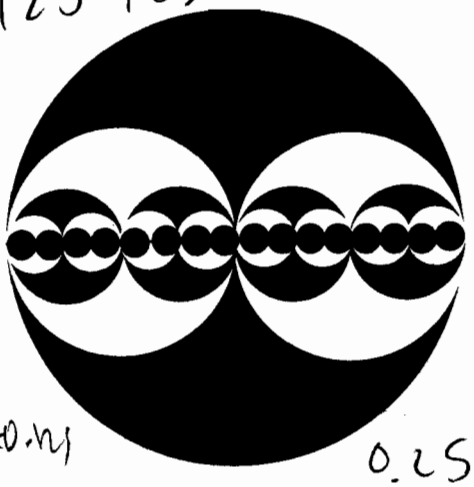
1 small circle: 0.25π

$0.25\pi \times 4 = \pi$

$\frac{3}{4}\pi$: black 75%

3. Fill in the table to show what happens as the pattern continues.





2.5π

.5π

0.25×0.25

0.125×0.125

0.25π

1.25π 2.75π 0.25×0.25 0.015625 0.0625 × 8 = 0.5

Pattern			Black fraction	White fraction
One black circle	4π	2π	1	0
Two white circles	2π	π	$\frac{1}{2}$	$\frac{1}{2}$
Four black circles	π	0.5π	3/4	1/4
Eight white circles	0.5π	0.25π	5/8	3/8
Sixteen black circles	0.25π	0.25π	11/16	5/16

4. Write a description of what is happening to the black and white fractions as the pattern continues.

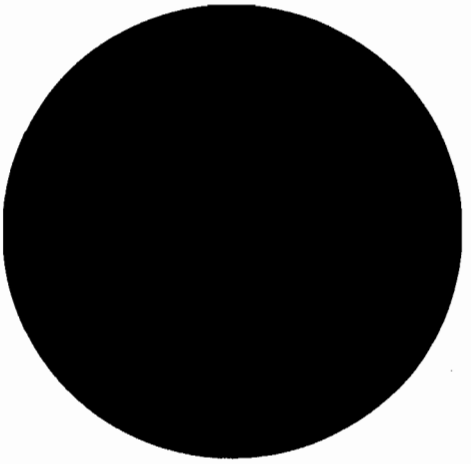
The denominators multiply by 2 each time, while when you put in x# of circles, you get 1/2x of the original circles. Every time black circles are added 1/2x black fraction is increased, while 1/2x is deleted after adding white circles. Same for white circles.

Circle Pattern

T4

Here is a developing circle pattern.

Here is one black circle.



Two white circles of half the radius have been added to the diagram.

1. Show that the fraction of the diagram that is now black is one half.

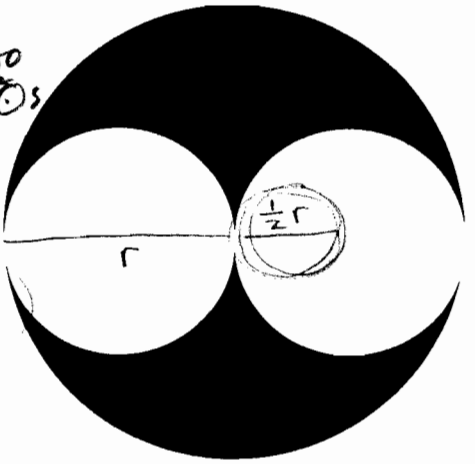
area of one black ⊙ — area of two white ⊙s

πr^2, $\pi(2r)^2$, $2(\pi r^2)$, $2\pi r^2$, $4(\pi \frac{1}{2} r^2)$,

$2\pi r^2$, ~~$4\pi r^2 - 2\pi r^2$~~ $4\pi r^2 - 2\pi r^2 =$

$2\pi r^2 = \frac{1}{2}$ black circle

1/2 over black



Four black circles have now been added.

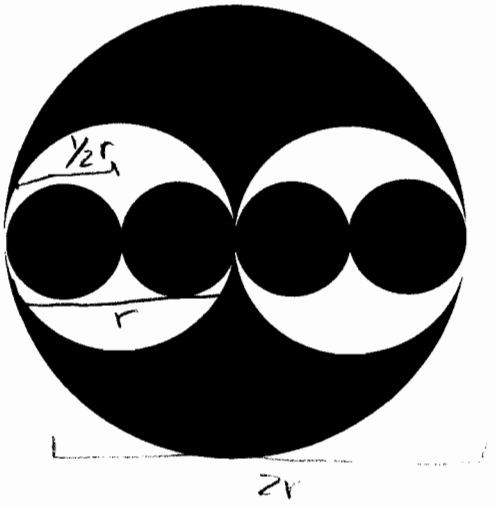
2. What fraction of the diagram is now black?

Big black ⊙ = $4\pi r^2$, 2 white ⊙s = $2\pi r^2$,

4 small black = $\left(\frac{1}{2}r\right)^2\pi = \left(\frac{1}{4}r^2\pi\right)4 = \pi r^2$

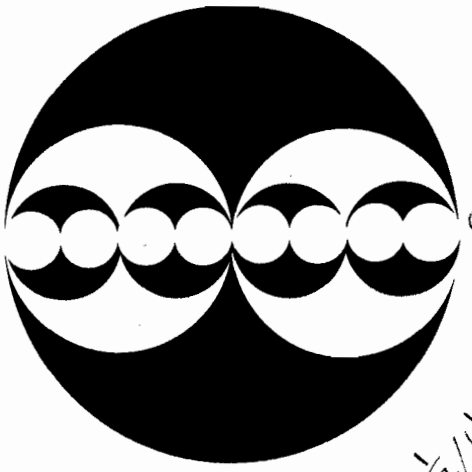
$4\pi r^2 - 2\pi r^2 + \pi r^2 = 2\pi r^2 + \pi r^2 =$

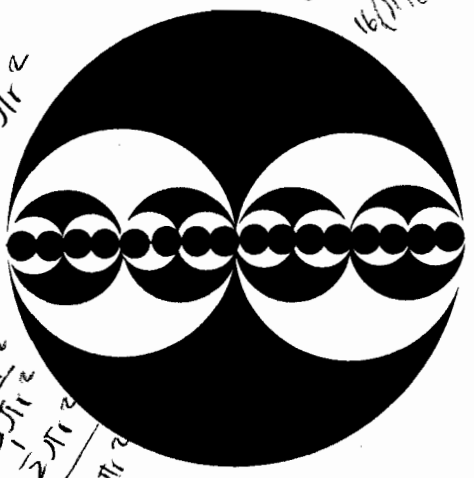
$3\pi r^2 = \frac{3}{4}$ of diagram



T4

3. Fill in the table to show what happens as the pattern continues.





Pattern	Black fraction	White fraction
One black circle	1	0
Two white circles	$\frac{1}{2}$	$\frac{1}{2}$
Four black circles	$\frac{3}{4}$	$\frac{1}{4}$
Eight white circles	$\frac{5}{8}$ ~~$2\frac{1}{2}$~~	$\frac{3}{8}$
Sixteen black circles	$\frac{11}{16}$	$\frac{5}{16}$

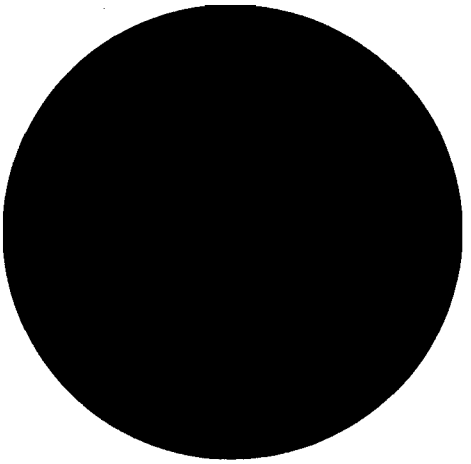
4. Write a description of what is happening to the black and white fractions as the pattern continues.

The white fraction = $\frac{1}{2}$ black circle's fraction. The black circle fraction next in the pattern = the previous white circle fraction + $\frac{1}{2}$. EX. $1 - \frac{1}{2}(1) = \frac{1}{2}$, $\frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{3}{4}$

Circle Pattern

T5

Here is a developing circle pattern.

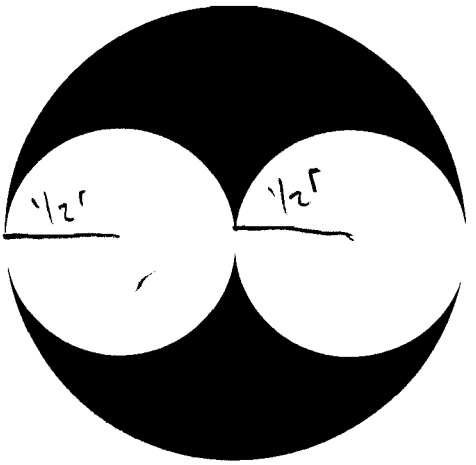


Here is one black circle.

$a_b = \pi r^2$

$a_w = \pi\left(\frac{r}{2}\right)^2 \cdot 2$

Two white circles of half the radius have been added to the diagram.



1. Show that the fraction of the diagram that is now black is one half.

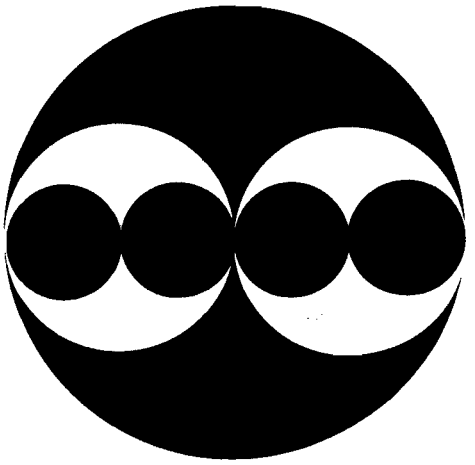
$a_{black} = \pi r^2$

$a_{white} = \pi\left(\frac{r}{2}\right)^2 \cdot 2$

$= \pi\left(\frac{r^2}{4}\right) \cdot 2$

$a_{white} = \frac{\pi r^2}{2}$

Four black circles have now been added.



2. What fraction of the diagram is now black?

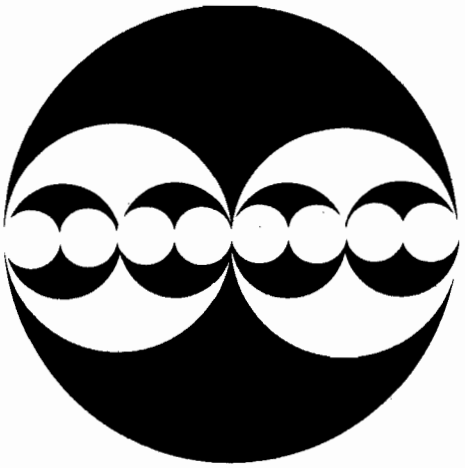
outer black = 1/2

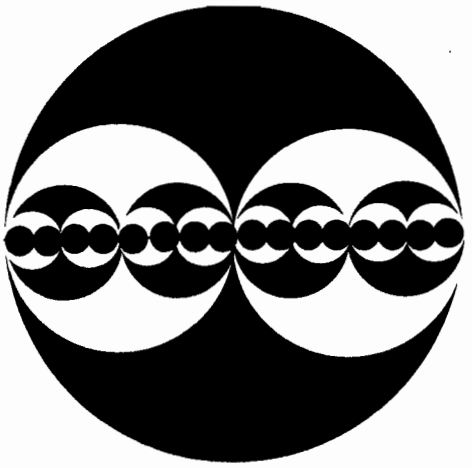
2 white = 1/2, 1 white = 1/4

1 small black = 1/16

1/16 × 4 = 4/16 = 1/4, 1/4 + 1/2 = 2/4 = 3/4

3. Fill in the table to show what happens as the pattern continues.





Pattern	Black fraction	White fraction
One black circle	1	0
Two white circles	$\frac{1}{2}$	$\frac{1}{2}$
Four black circles	$\frac{3}{4}$	$\frac{1}{4}$
Eight white circles	$\frac{5}{8}$	$\frac{3}{8}$
Sixteen black circles	$\frac{11}{16}$	$\frac{5}{16}$

4. Write a description of what is happening to the black and white fractions as the pattern continues.

The pattern is add, subtract, add, subtract etc.
The number you add/subtract gets smaller by ½ each time. ex. −½, +¼, −⅛, +1/16.