Circle Pattern

Here is a developing circle pattern.

Here is one black circle.

Two white circles of half the radius have been added to the diagram.

1. Show that the fraction of the diagram that is now black is one half.

\[ \text{of white circle} : \frac{1}{2} \text{ of black circle} = \frac{2r}{2r} \]

area black circle 4πr², white circle 2πr²

black circle: 4πr², white circles 2πr²

black: white = 2πr², exactly \( \frac{1}{2} \) of 4πr²

Four black circles have now been added.

2. What fraction of the diagram is now black?

\[ \frac{1}{2} + \frac{1}{4}(\frac{3}{4}) - \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \]

\[ \frac{3}{4} \checkmark \]

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3. Fill in the table to show what happens as the pattern continues.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Black fraction</th>
<th>White fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>One black circle</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
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<tr>
<td>Two white circles</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>Four black circles</td>
<td>$\frac{5}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>Eight white circles</td>
<td>$\frac{11}{16}$</td>
<td>$\frac{5}{16}$</td>
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<tr>
<td>Sixteen black circles</td>
<td></td>
<td></td>
</tr>
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4. Write a description of what is happening to the black and white fractions as the pattern continues.

The fractions alternate between adding and subtracting $\frac{1}{2n}$ to the previous fraction, where $n$ is the denominator and corresponds to how many of the smallest circles are considered. For example, if there are $4$ black circles, previously $\frac{1}{2}$ of the pattern was black, since the smallest circles are the same color, it will increase. If $\frac{1}{2} + \frac{1}{2} = \frac{3}{4}$, if the color was different, then we'd subtract.
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Two white circles of half the radius have been added to the diagram.

1. Show that the fraction of the diagram that is now black is one half.

- **Black circle**: $\pi r^2$
- **One white circle**: $\pi \left(\frac{1}{2}r\right)^2$ ✓
- **Two white circles**: $\pi \left(\frac{1}{2}r\right)^2$
- $\pi \left(\frac{1}{2}r\right)^2$ is $\frac{1}{2}$ of $\pi r^2$ so black is the other half of the figure.

Four black circles have now been added.

2. What fraction of the diagram is now black?

- **One white circle**: $\pi \left(\frac{1}{4}r\right)^2$
- **Two small black circles**: $\pi \left(\frac{1}{8}r\right)^2$
- **Four small black circles**: $\pi \left(\frac{1}{4}r\right)^2$
- Black: $\frac{3}{4}$ of figure ✓
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4. Write a description of what is happening to the black and white fractions as the pattern continues.

Whenever black circles are added, the black fraction increases by half of what was already there, and the white fraction decreases by half of what was already there. When white circles are added, the black fraction decreases by half of what was already there, and the white fraction increases by half of what was there.
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Here is one black circle.

Two white circles of half the radius have been added to the diagram.

1. Show that the fraction of the diagram that is now black is one half.

\[
\text{If radius of large circle: } 2 \quad \text{small: } 1
\]
\[
(2 \times 2) \pi = 2 (1^2 \pi)
\]
\[
4 \pi = 2 (2 \pi)
\]
\[
4 \pi = 4 \pi
\]  

Four black circles have now been added.

2. What fraction of the diagram is now black?

\[
\text{small circle: } 0.25 \pi
\]
\[
0.25 \pi \times 4 = 7 \pi
\]
\[
\frac{3}{4} \pi \text{ black } 75\% \checkmark
\]
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<td>$2\pi$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>Four black circles</td>
<td>$4\pi$</td>
<td>$3/4$</td>
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4. Write a description of what is happening to the black and white fractions as the pattern continues.

The denominators multiply by 2 each time, while when you put in $2^k$ of circles, you get $2^k$ of the original circles. Every time black circles are added, $\pi$ black fraction is increased, while $2^k$ is deleted after adding white circles. Same for white circles.
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1. Show that the fraction of the diagram that is now black is one half.

\[
\frac{\pi r^2}{2 \pi r^2} + 2 \left(\pi r^2 \right) - 4 \left(4 \pi r^2 \right) = \frac{1}{2} \text{ black circle}
\]

Four black circles have now been added.

2. What fraction of the diagram is now black?

\[
\frac{4 \pi r^2}{4 \pi r^2 - 2 \pi r^2 + \pi r^2} = \frac{3}{4} \text{ of diagram}
\]
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4. Write a description of what is happening to the black and white fractions as the pattern continues.

The white fraction = $\frac{1}{2}$ black circle's fraction. The black circle fraction next in the pattern = the previous white circle fraction + $\frac{1}{2}$. Ex. $1 - \frac{1}{2}(1) = \frac{1}{2}, \frac{1}{2} + \frac{1}{2}(\frac{1}{2}) = \frac{3}{4}$. 

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1. Show that the fraction of the diagram that is now black is one half.

\[ a_{\text{black}} = \pi r^2 \]
\[ a_{\text{white}} = \pi \left(\frac{r}{2}\right)^2 \cdot 2 \]
\[ = \pi \left(\frac{r^2}{4}\right) \cdot 2 \]
\[ a_{\text{white}} = \pi \frac{r^2}{2} \]

Four black circles have now been added.

2. What fraction of the diagram is now black?

Outer black = \( \frac{1}{2} \)
\[ 2 \text{ white } = \frac{1}{2}, 1 \text{ white } = \frac{1}{4} \]
Small black = \( \frac{1}{10} \)
\[ \frac{1}{6} \times 4 = \frac{1}{6} = \frac{1}{4}, \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \]
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4. Write a description of what is happening to the black and white fractions as the pattern continues.

The pattern is add, subtract, add, subtract etc.

The number you add/subtract gets smaller by $\frac{1}{2}$ each time. EX. $-\frac{1}{2}, +\frac{1}{4}, -\frac{1}{8}, +\frac{1}{16}$. 2