Temple Geometry

During the Edo period (1603-1867) of Japanese history, geometrical puzzles were hung in the holy temples as offerings to the gods and as challenges to worshippers.

This is one such problem.

Inside a large circle with radius 2r, two circles of radius r are drawn.

Four smaller circles, of radius p, are drawn to touch the large circle and the circles of radius r.

The following questions will help you to find the relationship between r and p

1. In the right triangle DOB, explain why the length of OD is 2r - p

\[ \overline{OD} = \overline{OF} - \overline{FD} = 2r - p \]
2. Use the Pythagorean theorem in triangle DOB to find an expression for $OB^2$.

$$\overrightarrow{OB}^2 + \rho^2 = (2r-p)^2$$

$$\overrightarrow{OB}^2 + \rho^2 = 4r^2 - 4rp + p^2$$

$$OB^2 = 4r^2 - 4rp$$

3. In the right triangle ADE, explain why the length of AE is $r - p$.

$$\overrightarrow{AO} = r \quad \overrightarrow{EO} = \rho \quad \overrightarrow{AE} = \overrightarrow{AO} - \overrightarrow{EO} = (r - \rho)$$

4. Use the Pythagorean theorem in triangle ADE to find an expression for $ED^2$.

$$\overrightarrow{DE}^2 = \overrightarrow{AD}^2 - \overrightarrow{AB}^2 = (r + p)^2 - (r - p)^2 = 4rp + p^2 - (r^2 - 2rp + p^2)$$

$$ED^2 = 4rp$$

5. Use your results from questions 2 and 4, and the fact that $OB = ED$ to show that $r = 2p$.

$$\overrightarrow{OB}^2 = \overrightarrow{ED}^2 \quad 4rp = 4r^2 \quad 8rp = 4r^2 \quad 2rp = r^2 \quad (2p = r)$$

6. Show that the shaded area of the diagram has area $\pi r^2$.

$$\text{Shaded Area} = \pi (2r)^2 - 2 \pi (r)^2 - 4 \pi \left(\frac{3}{4} r\right)^2$$

$$4 \pi r^2 - 2 \pi r^2 - \pi r^2$$

$$= \pi r^2$$
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The following questions will help you to find the relationship between r and p

1. In the right triangle DOB, explain why the length of OD is 2r - p

   OF is the radius of the large circle and DF is the radius of the smallest. The length of OD must be OF - FB = 2r - p.
2. Use the Pythagorean theorem in triangle DOB to find an expression for $OB^2$.

$$OB^2 + DB^2 = DO^2$$
$$OB^2 + (r-p)^2 = (2r-p)^2$$
$$OB^2 = 4r^2 - 4rp + r^2 + p^2 - 2rp$$
$$OB^2 = 4r^2 - 4rp$$

or

$$OB = 2\sqrt{r^2 - rp}$$

3. In the right triangle ADE, explain why the length of AE is $r - p$.

$$AE^2 + DE^2 = AD^2$$
$$ED^2 = (r-p)^2$$
$$ED^2 = r^2 - 2rp + p^2$$
$$ED^2 = 4rp$$

or

$$ED = 2\sqrt{rp}$$

4. Use the Pythagorean theorem in triangle ADE to find an expression for $ED^2$.

$$ED^2 = 4rp$$

5. Use your results from questions 2 and 4, and the fact that $OB = ED$ to show that $r = 2p$

By substitution in the equations, the end result is

$$r = 2p$$

Because $\triangle OEB$ is a rectangle, $OB = ED$, by substitution,

$$OB = ED$$

(square both sides)

$$OB^2 = ED^2$$

$$4r^2 - 4rp = 4rp$$

$$r^2 - 2rp = rp$$

and

$$r - p = \frac{p}{2}$$

6. Show that the shaded area of the diagram has area $\pi r^2$.
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Inside a large circle with radius \(2r\), two circles of radius \(r\) are drawn.

Four smaller circles, of radius \(p\), are drawn to touch the large circle and the circles of radius \(r\).

The following questions will help you to find the relationship between \(r\) and \(p\).

1. In the right triangle \(DOB\), explain why the length of \(OD\) is \(2r - p\).

\[
\overline{OD} = \overline{OF} - \overline{FD}
\]

\[
\overline{OD} = 2r - p
\]
2. Use the Pythagorean theorem in triangle DOB to find an expression for $OB^2$.

$$\frac{OB^2 + OB^2}{p^2} = \frac{DO^2}{(2r-p)^2}$$

$$OB^2 = \frac{(2r-p)(2r-p)}{2} - p^2$$

$$OB^2 = \frac{4r^2 - 4rp - p^2}{2}$$

3. In the right triangle ADE, explain why the length of AE is $r - p$.

$\overline{DE} \parallel \overline{BO}, \overline{OB} \parallel \overline{EO}$ because of radii. Thus, $\overline{OB} = \overline{EO} = p$

$$AO = r$$

$$AE = AO - EO$$

$$AE = r - p$$

4. Use the Pythagorean theorem in triangle ADE to find an expression for $ED^2$.

$$AD = p + r, AE = r - p$$

$$(r-p)^2 + ED^2 = (p+r)^2$$

$$ED^2 = p^2 + 2pr + r^2 - (r^2 - 2pr + p^2)$$

$$ED^2 = 4pr$$

5. Use your results from questions 2 and 4, and the fact that $OB = ED$ to show that $r = 2p$.

$$4r^2 - 4pr = 4pr$$

$$4r^2 = 8pr$$

$$r = 2p$$

6. Show that the shaded area of the diagram has area $\pi r^2$.

Big circle: $4r^2\pi$

2 medium circles: $2\pi r^2$

4 small circles: $4\pi r^2$

Total: $4\pi r^2$
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The following questions will help you to find the relationship between r and p.

1. In the right triangle DOB, explain why the length of OD is 2r - p.

OD is 2r - p because the radius of the big circle is 2r, and the radius of small is p. OD + p = 2r. (All radii of a circle) to get OD, OD = 2r - p.
2. Use the Pythagorean theorem in triangle DOB to find an expression for OB².

\[ OB^2 + p^2 = (2r - p)^2 \]
\[ OB^2 = 4r^2 - 4rp + p^2 - p^2 \]
\[ OB^2 = 4r^2 - 4rp \]

3. In the right triangle ADE, explain why the length of AE is \( r - p \).

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED².

\[ ED^2 + (1- p)^2 = (p + r)^2 \]
\[ ED^2 + 1^2 - 2pr + p^2 = p^2 + 2pr + r^2 \]
\[ ED^2 = p^2 + 2pr + r^2 - 1^2 - 2pr - p^2 \]
\[ ED^2 = 4pr \]
\[ (r - p)(1 - p) \]
\[ (p + r)(p + r) \]

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that \( r = 2p \)

Since OB = ED, \( \sqrt{4r^2 - 4p} = \sqrt{4pr} \)
\[ 4r^2 - 4rp = 4pr; 4r^2 = 8rp \]
\[ r^2 = 2rp; r = 2p \]

6. Show that the shaded area of the diagram has area \( \pi r^2 \).

Area of circle = \( \pi \) radius \( r \)
Big circle = \( \pi (2r)^2 = 4\pi r^2 \) (medium)
Circle = \( \pi r^2 \) since \( r = 2p \), 2 small circle's area = 1 (medium)
So 2 medium + 4 small = \( \pi r^2 + \pi r^2 + \pi r^2 \), so \( \pi \) \( 3r^2 \), \( 4\pi r^2 - \pi 3r^2 = \pi r^2 \)
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The following questions will help you to find the relationship between r and p.

1. In the right triangle DOB, explain why the length of OD is 2r - p.

   \[ \overline{OD} = 2r - p \text{ because } \overline{OD} + \overline{FD} = 2r \]

   since that lines draws the radius of the largest circle. \( \overline{FD} \) is the radius of the smallest circle so \( \overline{OF} = \overline{FD} = \overline{OD} \) and by substituting the radii values you get \( 2r - p \).
2. Use the Pythagorean theorem in triangle DOB to find an expression for OB².

\[
(2r-p)(2r-p) = (2r-p)^2 = OB^2
\]

\[
+r^2 - 2pr - 2pr^2 + p^2
\]

\[
= 4r^2 - 4pr + 2p^2 - r^2 = OB^2
\]

\[
= 4r(r - p) = OB^2
\]

3. In the right triangle ADE, explain why the length of AE is r - p.

\[
\overline{AO} = r \quad \overline{OE} = \overline{DB} \text{ because all } 90^\circ \text{ angles making it a rectangle. } \overline{DE} = p
\]

\[
\overline{AO} - \overline{OE} = \overline{AE}
\]

\[
r - p = \overline{AE}
\]

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED².

\[
(r+p)^2 - (r-p)^2 = ED^2
\]

\[
ED^2 = 4pr
\]

\[
(r+p)(r+p) = (r+p)^2
\]

\[
+r^2 + 2pr + p^2
\]

\[
= r^2 - pr - p^2 + p^2
\]

\[
r^2 - 2pr + p^2
\]

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that r = 2p

\[
\overline{OB} = \overline{ED} \text{ so } 4r^2 - 4pr = 4pr
\]

\[
4r^2 = 8pr \quad \text{so } r = 2p
\]

\[
\frac{4r}{4} = \frac{1}{r}
\]

6. Show that the shaded area of the diagram has area \(\pi r^2\).

- Area of largest circle = \(4 \times r^2\)

- Area of 2 circles with radius \(r = 2 \times \pi r^2\)

- Area of 4 circles with radius \(\frac{r}{2} = \pi r^2\)