Temple Geometry

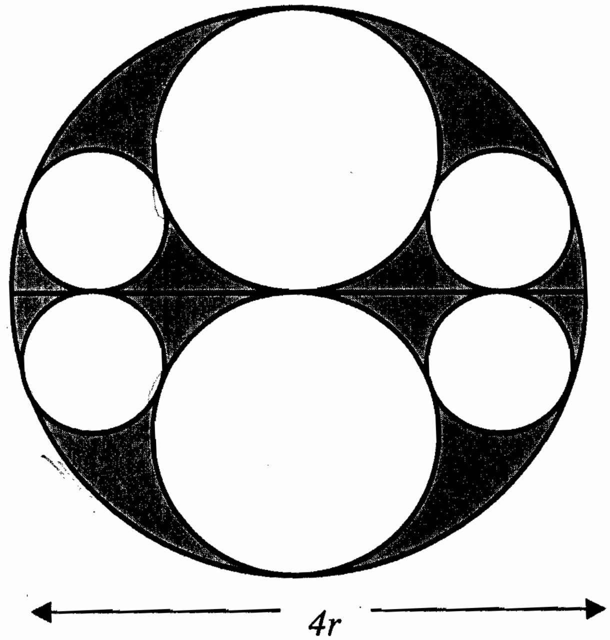
During the Edo period (1603-1867) of Japanese history, geometrical puzzles were hung in the holy temples as offerings to the gods and as challenges to worshippers.

This is one such problem.

Inside a large circle with radius 2r, two circles of radius r are drawn.

Four smaller circles, of radius p, are drawn to touch the large circle and the circles of radius r.

The following questions will help you to find the relationship between r and p



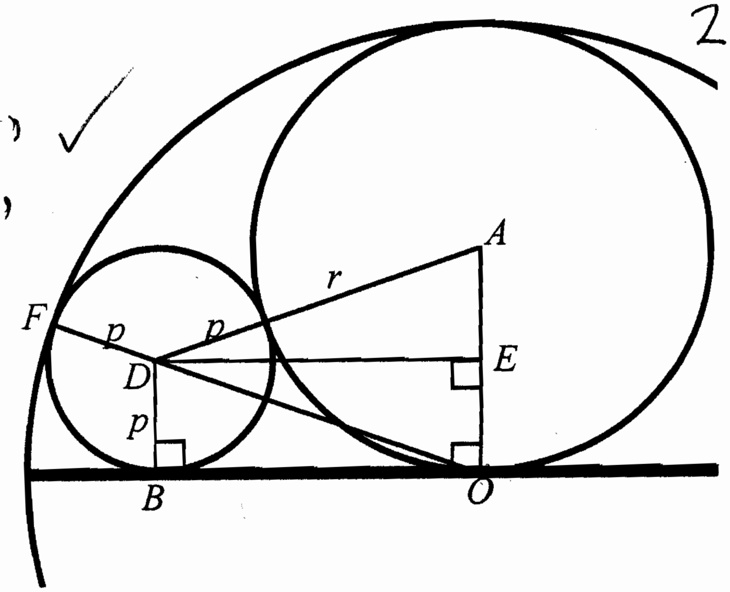
1. In the right triangle DOB, explain why the length of OD is 2r – p

$\overline{FO}$ is radius of Large circle (2r) ✓

$\overline{FD}$ is radius of small circle (p)

$\overline{OD} = \overline{FO} - \overline{FD} = 2r - p$

2



T1

2. Use the Pythagorean theorem in triangle DOB to find an expression for OB^2.

$$\overline{OB}^2 + p^2 = (2r-p)^2$$

$$\overline{OB}^2 + p^2 = 4r^2 - 4rp + p^2$$

$$OB^2 = 4r^2 - 4rp$$

3. In the right triangle ADE, explain why the length of AE is r – p.

$\overline{AO} = r \quad \overline{EO} = \overline{DB} = p \quad \overline{AE} = \overline{AO} - \overline{EO} = r - p$

✓ 2

✓ 1

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED^2.

$\overline{DE}^2 = \overline{AD}^2 - \overline{AE}^2 = (r+p)^2 - (r-p)^2 = r^2 + 2rp + p^2 - (r^2 - 2rp + p^2)$

$ED^2 = 4rp$ ✓

2

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that r = 2p

$\overline{OB}^2 = \overline{ED}^2 \quad 4rp = 4r^2 - 4rp \quad 8rp = 4r^2 \quad 2rp = r^2 \quad 2p = r$

✓ 2

6. Show that the shaded area of the diagram has area πr^2.

Shaded Area $= \pi(2r)^2 - 2\pi(r)^2 - 4\pi(\tfrac{1}{2}r)^2$

$4\pi r^2 - 2\pi r^2 - \pi r^2$ 1

$= \pi r^2$ ✓

CCR 1

10

Temple Geometry

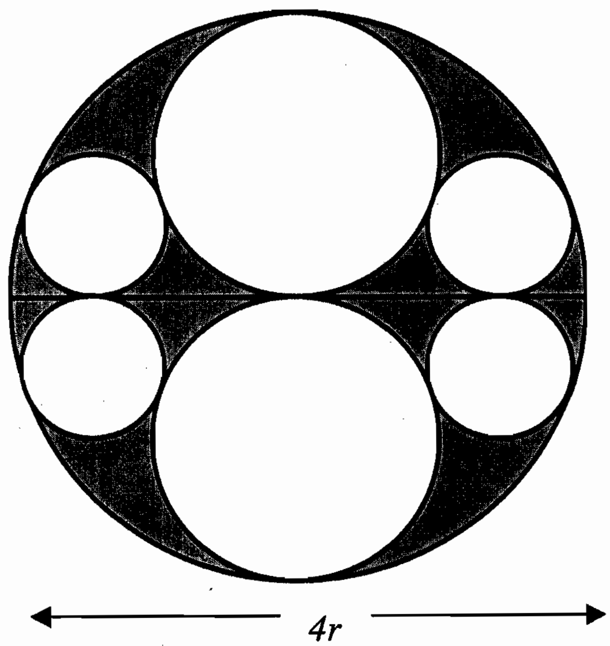
During the Edo period (1603-1867) of Japanese history, geometrical puzzles were hung in the holy temples as offerings to the gods and as challenges to worshippers.

This is one such problem.

Inside a large circle with radius 2r, two circles of radius r are drawn.

Four smaller circles, of radius p, are drawn to touch the large circle and the circles of radius r.

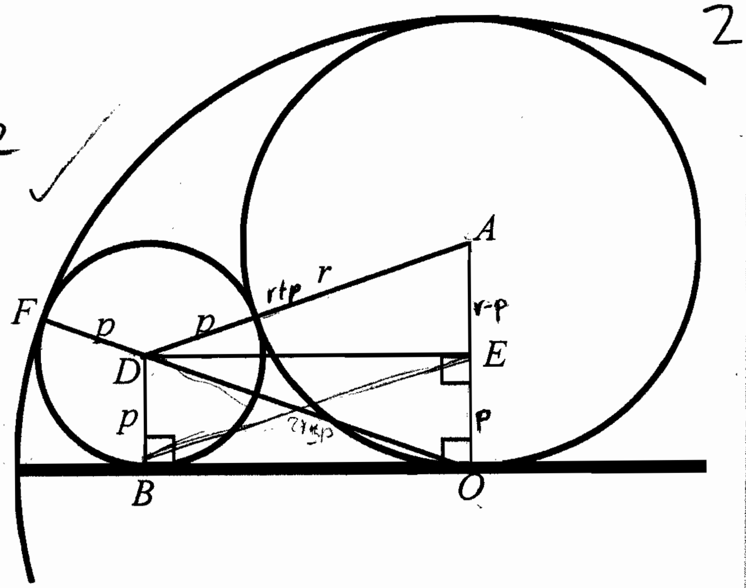
The following questions will help you to find the relationship between r and p



1. In the right triangle DOB, explain why the length of OD is $2r - p$

OF is the radius of the large circle and DF is the radius of the smallest. The length of OD must be OF — FD = $2r-p$.

2



T2

2. Use the Pythagorean theorem in triangle DOB to find an expression for OB^2.

$OB^2 + DB^2 = DO^2$ or $OB = 2\sqrt{r^2 - rp}$

$OB^2 + p^2 = (2r-p)^2$

$OB^2 = 4r^2 + p^2 - 4rp) - p^2$

$OB^2 \quad 4r^2 - 4rp$ ✓

2

3. In the right triangle ADE, explain why the length of AE is r – p.

0

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED^2.

$ED^2 = 4rp$ ✓

2

$AE^2 + DE^2 = AD^2$

$ED^2 + (r-p)^2 = (r+p)^2$

$ED^2 = r^2 + p^2 + 2rp - (r^2 + p^2 - 2rp$ or $ED = 2\sqrt{rp}$

$ED^2 = 4rp$

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that r = 2p

By substituting in the equations, the end results $r = 2p$

Becaus DEOB is a rectangle, OB = ED, by substituting,

$OB = ED$ (square both sides) $\sqrt{r^2 - rp} = rp$

$OB^2 = ED^2$ $\quad r - p = p$

$4r^2 - 4rp = 4rp$ ✓ $\quad r = 2p$ ✓

2

6. Show that the shaded area of the diagram has area πr^2.

0

CCR 1

8

Temple Geometry

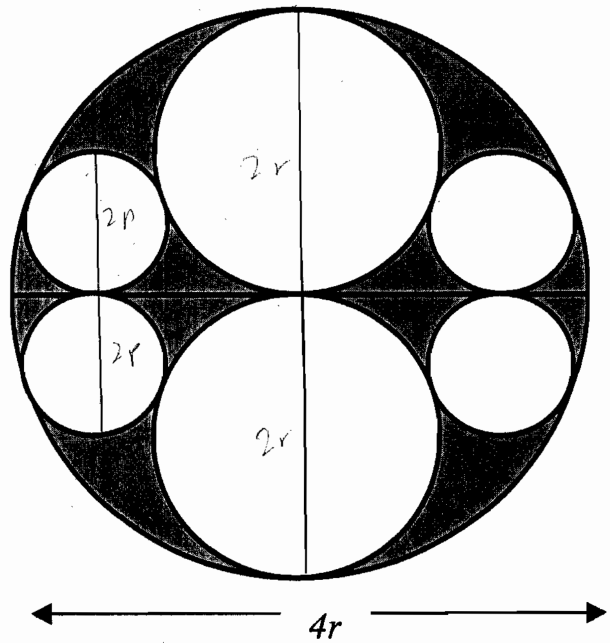
During the Edo period (1603-1867) of Japanese history, geometrical puzzles were hung in the holy temples as offerings to the gods and as challenges to worshippers.

This is one such problem.

Inside a large circle with radius 2r, two circles of radius r are drawn.

Four smaller circles, of radius p, are drawn to touch the large circle and the circles of radius r.

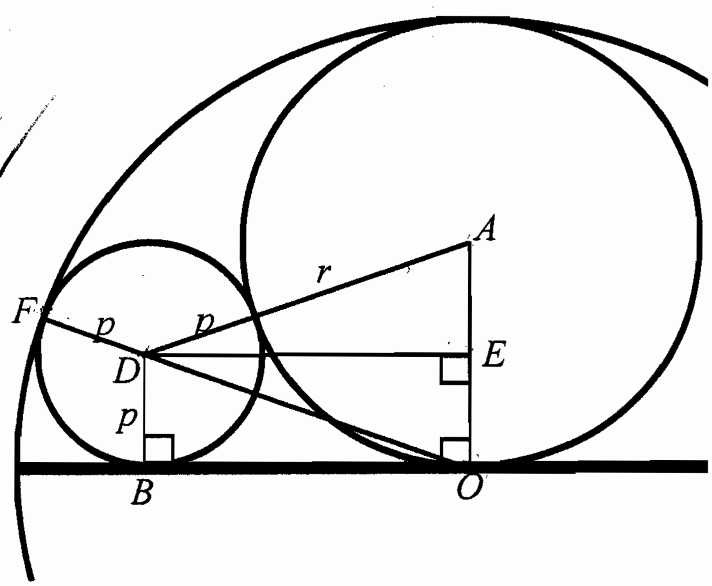
The following questions will help you to find the relationship between r and p



1. In the right triangle DOB, explain why the length of OD is 2r – p

$\overline{OD} = \overline{OF} - FD$

$\overline{OD} = 2r - p$



2

T3

2. Use the Pythagorean theorem in triangle DOB to find an expression for OB^2.

$DB^2 + OB^2 = DO^2$

$p^2 + OB^2 = (2r-p)^2$

$OB^2 = (2r-p)(2r-p) - p^2$

$OB^2 = 4r^2 - 2pr - 2pr - ^2$

$\boxed{OB^2 = 4r^2 - 4pr - p^2}$

✓ 2

3. In the right triangle ADE, explain why the length of AE is r – p.

$\overline{DE} // \overline{BO}$, $\overline{DB} // \overline{EO}$, because of rt ∠s. thus, $\overline{DB} \cong \overline{EO} = p$

$AO = r$

$AE = AO - EO$

$AE = r - p$ ✓ 1

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED^2.

→ $AD = p+r$ $AE = r-p$ $(r-p)^2 + ED^2 = (p+r)^2$

$ED^2 = p^2 + 2pr + r^2 - (r^2 - 2pr + p^2)$

$\boxed{ED^2 = 4pr}$

✓ 2

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that r = 2p $p = \frac{r}{2}$

→ $4r^2 - 4pr = 4pr$ $4r^2 = 8pr$ $r = 2p$ ✓

2

6. Show that the shaded area of the diagram has area πr^2.

Big circle = $4r^2\pi$ $4\pi r^2$

2 medium circles = $2\pi r^2$ $2\pi r^2$

4 small circles = $4p^2\pi$ $4(\frac{r}{2})^2\pi$ $4\frac{r^2}{4}\pi$ $r^2\pi$ πr^2

πr^2 ✓ 1

10

Temple Geometry

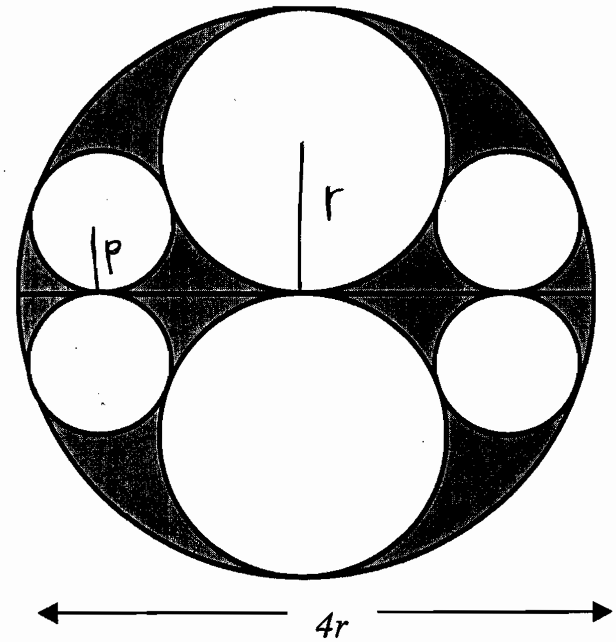
During the Edo period (1603-1867) of Japanese history, geometrical puzzles were hung in the holy temples as offerings to the gods and as challenges to worshippers.

This is one such problem.

Inside a large circle with radius 2r, two circles of radius r are drawn.

Four smaller circles, of radius p, are drawn to touch the large circle and the circles of radius r.

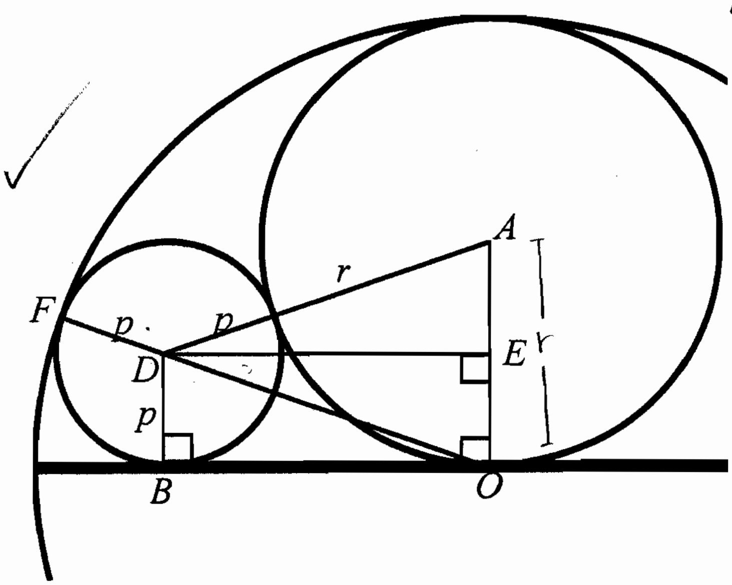
The following questions will help you to find the relationship between r and p



2

1. In the right triangle DOB, explain why the length of OD is 2r – p

OD is 2r-p because the radius of the big circle is 2r, and the radius of small is p. $\overline{OD}$ + p = 2r. (all radii of a ⊙ ≅) To get $\overline{OD}$, $\overline{OD}$ = 2r-p



2. Use the Pythagorean theorem in triangle DOB to find an expression for OB^2.

$OB^2 + p^2 = (2r-p)^2$ $\quad$ $OB^2 = 4r^2 - 4rp + p^2 - p^2$

$OB^2 = 4r^2 - 4rp$ ✓

2

3. In the right triangle ADE, explain why the length of AE is r – p.

^

0

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED^2.

$ED^2 + (r-p)^2 = (p+r)^2$; $ED^2 + r^2 - 2pr + p^2 = p^2 + 2pr + r^2$.

$ED^2 = p^2 + 2pr + r^2 - r^2 + 2pr - p^2$; $ED^2 = 4pr$ ✓

$(r-p)(r-p)$ $\quad$ $(p+r)(p+r)$

2

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that r = 2p

Since OB = ED, $\sqrt{4r^2-4rp} = \sqrt{4pr}$; $4r^2 - 4rp = 4pr$; $4r^2 = 8rp$;

$r^2 = 2rp$; $r = 2p$ ✓

2

6. Show that the shaded area of the diagram has area πr^2.

area of circle = π radius^2; big circle = $\pi(2r)^2 = \pi 4r^2$. medium

circle = πr^2. since $r = 2p$, 2 small circle's area = 1 medium. ✓

so 2 medium + 4 small = $\pi r^2 + \pi r^2 + \pi r^2$, so $\pi 3r^2$; $\pi 4r^2 - \pi 3r^2 = \pi r^2$

1

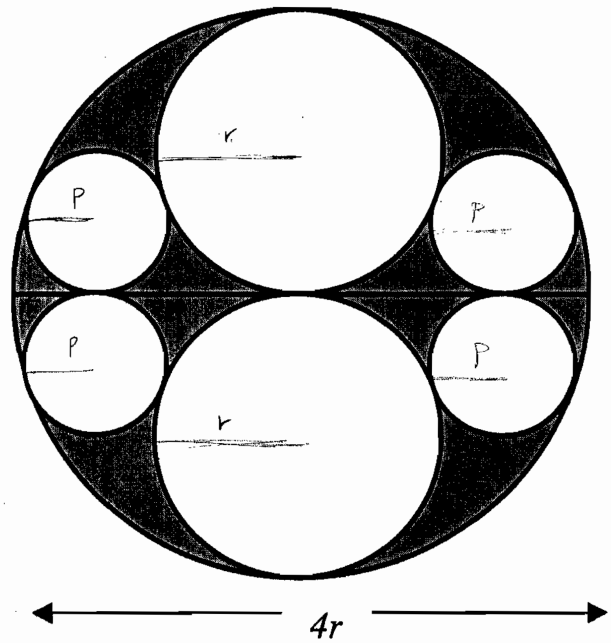
9

Temple Geometry

During the Edo period (1603-1867) of Japanese history, geometrical puzzles were hung in the holy temples as offerings to the gods and as challenges to worshippers.

This is one such problem.

Inside a large circle with radius 2r, two circles of radius r are drawn.



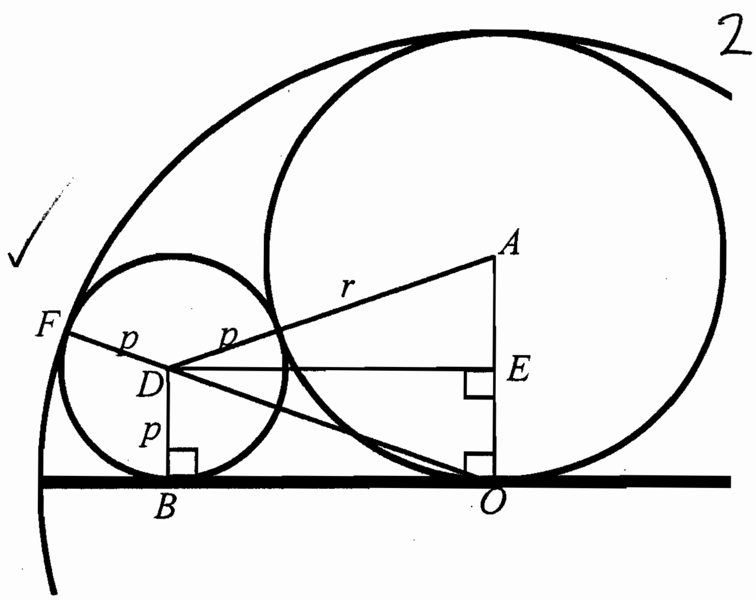
Four smaller circles, of radius p, are drawn to touch the large circle and the circles of radius r.

The following questions will help you to find the relationship between r and p

1. In the right triangle DOB, explain why the length of OD is $2r - p$

$\overline{OD}$ is $2r-p$ because $\overline{OD}+\overline{FD}=2r$ since that lines draws the radius of the largest circle. $\overline{FD}$ is the radius of the smallest circle so $\overline{OF}-\overline{FD}=\overline{OD}$ and by substituting the radii values you get $2r-p$.

✓



T5

2. Use the Pythagorean theorem in triangle DOB to find an expression for OB^2.

$(2r-p)(2r-p)$

$4r^2-2pr-2pr+p$

$(2r-p)^2-(p)^2=OB^2$

$4r^2-4pr+p^2-p^2=OB^2$

$4r(r-p)=OB^2$ ✓

2

3. In the right triangle ADE, explain why the length of AE is r – p.

$\overline{AO}=r$ $\overline{OE}\cong\overline{DB}$ because all ∠s are 90° making it a rectangle. $\overline{DB}\cong p$ so $\overline{OE}\cong p$

$\overline{AO}-\overline{OE}=\overline{AE}$

$r-p=\overline{AE}$ ✓

1

4. Use the Pythagorean theorem in triangle ADE to find an expression for ED^2.

$(r+p)^2-(r-p)^2=ED^2$

$ED^2=4pr$ ✓

2

$(r+p)(r+p)$

$r^2+pr+pr+p^2$

$r^2+2pr+p^2$

$(-r+p)(-r+p)$

$r^2-pr-pr+p^2$

$r^2-2pr+p^2$

5. Use your results from questions 2 and 4, and the fact that OB = ED to show that r = 2p

$\overline{OB}\cong\overline{ED}$ so $4r^2-4pr=4pr$

$\frac{4r^2}{4r}=\frac{8pr}{4r}$ so $r=2p$ ✓

2

6. Show that the shaded area of the diagram has area πr^2.

Area of largest circle $=4\pi r^2$

Area of 2 circles w/radius $r = 2\pi r^2$

Area of 4 circles w/ radius $\frac{r}{2}=\pi r^2$

$2\pi r^2 + \pi r^2 = 3\pi r^2$

$4\pi r^2 - 3\pi r^2 = \pi r^2$ ✓

1

CCR 1

10