Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.

\[ \begin{align*}
\text{Volume:} & \quad V = x^2 y \\
\text{Perimeter:} & \quad P = 4y + 8x = 60 \\
\text{Constraint:} & \quad y = \frac{15 - 2x}{2} \\
\end{align*} \]

So when \( x = 5 \) and \( y = 5 \) the volume is at its highest at 125 m³.
2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

\[
\begin{align*}
\rho &= 60 = 6x + 3y \\
20 &= 2x + y \\
y &= 20 - 2x
\end{align*}
\]

Vol = \( \frac{x}{2} \cdot \frac{\sqrt{3}}{2} (20 - 2x) \)

- When \( x = 5 \), \( y = 20 - 2x = 10 \)
  \[ V = \frac{\sqrt{3} \cdot 10 \cdot 5}{4} = 108.25 \]
- When \( x = 8 \), \( y = 20 - 12 = 8 \)
  \[ V = \frac{\sqrt{3} \cdot 8 \cdot 8}{4} = 124.7 \text{ bigger} \]
- When \( x = 7 \), \( y = 20 - 14 = 6 \)
  \[ V = \frac{\sqrt{3} \cdot 49.6}{4} = 127.3 \text{ bigger} \]
- When \( x = 5.5 \), \( y = 20 - 11 = 9 \)
  \[ V = \frac{\sqrt{3} \cdot 42.5}{4} = 110.8 \text{ smaller} \]

3. What advice do you think Fearless Frames should offer to this customer? Show all your calculations.

There's not much in it. When \( x = y \), the triangle holds 123 m\(^3\) and the prism holds 128.06 m\(^3\). The prism holds a little more as it would take a long time and patience to work out the correct dimensions some where between 6 and 7.
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1. A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section. The company has only 60 meters of suitable metal tubing in stock. Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.

\[
\text{Max volume } = 125 \text{ m}^3
\]

\[
\text{Dimensions } = 5 \times 5 \times \text{？}
\]

\[
\text{Volume } = x^2 \times y
\]

\[
-x^2 \times (-2x+15)
\]

\[-2x^3 + 15x^2
\]

\[-265 + 15
\]

\[-10 + 15
\]

\[y = 5
\]

\[y = -2x + 15
\]
Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

\[
\begin{align*}
X^2 + \left(\frac{5x}{2}\right)^2 &= h^2 \\
X^2 + \frac{1}{3}X^2 &= h^2 \\
\frac{\sqrt{3}}{2}x &= h
\end{align*}
\]

\[
\begin{align*}
4x &= 5y = 10 \\
\frac{1}{2}(5)[\left(\frac{5}{4}\right)] \cdot 10 \\
&= 25 \text{ m} \cdot \text{m} \\
&= 156.25 \text{ m}^3
\end{align*}
\]

\[
\begin{align*}
4x &= 6, y = 8 \\
\frac{1}{2}(6)[\left(\frac{5}{4}\right)] \cdot 8 &= 180 \text{ m}^3
\end{align*}
\]

\[
\begin{align*}
2(3x) + 3y &= 60 \\
6x + 3y &= 60 \\
2x + y &= 20 \\
y &= 20 - 2x
\end{align*}
\]

Volume:
\[
V = \frac{1}{2}bh - y
\]

\[
= \frac{1}{2}x \left(\frac{5x}{2}\right) \cdot y
\]

3. What advice do you think Fearless Frames should offer to this customer?

Show all your calculations.

The prism container has more volume, if the customer wants the maximum volume, the dimensions \((x, y, z)\) need to very close to give the most space. Since customer likes larger containers, he should go with the rectangular prism.
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Show how you figured it out.
Fearless Frames: (continued)

2. The client changes his mind!

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Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

\[
\frac{1}{2} \times \sqrt{3}
\]

\[x = \frac{1}{2}
\]

\[\text{Area of base} = \frac{1}{2} \times \sqrt{3}
\]

\[\frac{3}{4} x^2
\]

\[6x + 3y = 60
\]

\[2x + y = 20
\]

\[y = 20 - 2x
\]

\[\sqrt{\text{(area)xy}}
\]

3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.

For both containers, it's best they choose the highest value for x, while y is still a Cmetic number.

\[
\sqrt{32\sqrt{3} + 63\sqrt{3}}
\]
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The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.

\[8x + 4y = 60\]
\[2(4x) + 4y = 60\]
\[2x + y = 15\]
\[y = 15 - 2x\]

Volume = \(x^2 \cdot y = x^2 (15 - 2x)\)

If \(x = 4, y = 7\)
\[
\text{Volume} = 4^2 \cdot 7 = 112 \text{ m}^3
\]

If \(x = 5, y = 5\) but not sure if \(x = y\)
\[
5^2 \cdot 5 = 125 \text{ m}^3
\]

If \(x = 6, y = 3\)
\[
6^2 \cdot 3 = 108 \text{ m}^3
\]
Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

\[ V = \frac{\sqrt{3}}{2} (x \frac{\sqrt{3}}{2} + (\frac{3}{2}x)) \cdot (-2x + 15) \]

3. What advice do you think Fearless Frames should offer to this customer? Show all your calculations.
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Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.

\[
\begin{align*}
8x + 4y &= 60 \\
2x + y &= 15 \\
y &= 15 - 2x \\
V &= x^2 y \\
V &= x^2 (15 - 2x) = 15x^2 - 2x^3 \\
x = 5 \text{ m} & \quad y = 15 - 10 \\
V &= 125 \text{ m}^3
\end{align*}
\]
Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

\[
\begin{align*}
6x + 3y &= 60 \\
2x + y &= 20 \\
x &= 20 - 2y \\
V &= \frac{\sqrt{3}}{4} x^2 y - \frac{\sqrt{3}}{2} x^2 (10 - x) \\
\text{Maximize} & \quad x^2 (10 - x) \\
x &= 6.7 \text{ m}, \\
y &= 20 - 13.4 \\
V &= \frac{\sqrt{3}}{2} (4.89)(3.9) \\
V &= 128.29 \text{ m}^3
\end{align*}
\]

3. What advice do you think Fearless Frames should offer to this customer? Show all your calculations.

It is better to use a triangular frame than a squareish frame, because a triangular frame gives you a bigger volume while using the same amount of wiring.