Fearless Frames **T1**

Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

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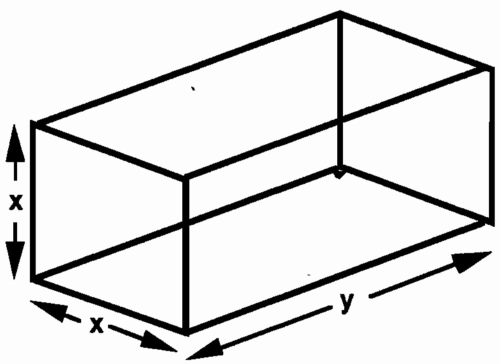
Show how you figured it out.



$P = 4y + 8x = 60$

$y + 2x = 15$

$y = 15 - 2x$

$V = x^2 y = x^2(15 - 2x)$

$x = 4 \quad y = 15 - 8 = 7$

$V = 16 \cdot 7$

$= 112$

$x = 5 \quad y = 15 - 10 = 5$

$V = 25 \cdot 5 =$

$= 125$

$x = 6 \quad y = 15 - 12 = 3$ 3

$V = 36 \cdot 3$

$= 108$

So when $x = 5$ and $y = 5$ the volume is at its highest at $125\ m^3$ 2

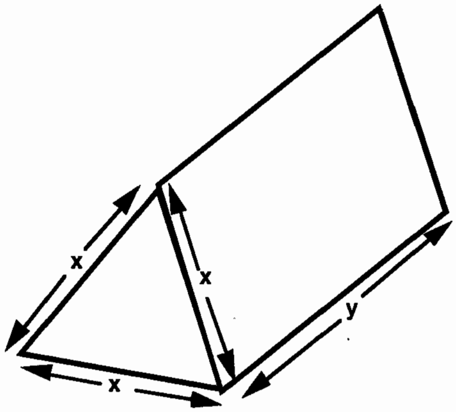
T1

Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

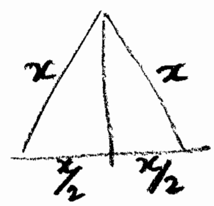
Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.



$P = 60 = 6x + 3y$
$20 = 2x + y$
$y = 20 - 2x$ ✓

$Vol = \frac{x}{2} \cdot \frac{\sqrt{3}(x)}{2} y$

$= \frac{\sqrt{3}x^2}{4}(20 - 2x)$ ✓



$a^2 + b^2 = c^2$
$a^2 + \left(\frac{x}{2}\right)^2 = x^2$
$a^2 = x^2 - \frac{x^2}{4}$
$= \frac{3x^2}{4}$
$a = \frac{\sqrt{3}(x)}{2}$ ✓

• When $x = 5$, $y = 20 - 2x = 10$
$= \frac{\sqrt{3} . 25 . 10}{4} = 108 \cdot 25$

• When $x = 6$, $y = 20 - 12 = 8$
$V = \frac{\sqrt{3} . 36 . 8}{4} = 124 \cdot 7$ bigger.

• when $x = 7$ $y = 20 - 14 = 6$
$V = \frac{\sqrt{3} . 49 . 6}{4} = 127 \cdot 3$ bigger

• when $x = 8$
$\frac{\sqrt{3} . 64 . 4}{4} = 110 \cdot 8$ smaller

• when $x = 6 \cdot 5$
$y = 20 - 13 = 7$
$Vol = \frac{\sqrt{3} . 42.25 . 7}{4} = 128 \cdot 06\ m^3$ highest between $x = 6$ $x = 7$

When $P = 60$
$V = 128\ m^3$. ✓

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3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.

There's not much in it. When $x = y$, the rectangle holds $125m^3$ and the prism holds $128 \cdot 06\ m^3$. The prism holds a little more as it would take a long time and patience to work out the correct dimensions some where between 6 and 7. ✓

1

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Fearless Frames

T2

Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

Max volume = 125 m^3

The company has only 60 meters of suitable metal tubing in stock. Dimensions = 5 x 5 x 5

m m m

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

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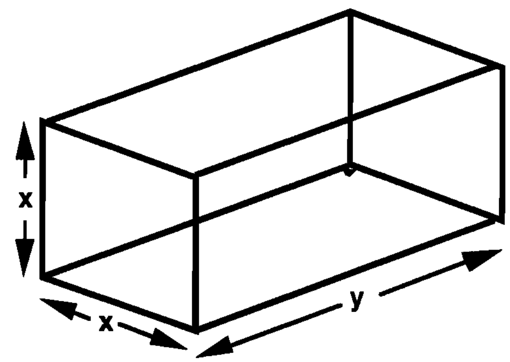
Show how you figured it out.



x	volume
1	13
2	44
3	81
4	112
5	125
6	108

volume = $x^2 \times y$

$x^2 \times (-2x+15)$

$-2x^3 + 15x^2$

5

$-2(5)+15$

$-10 + 15$

$y = 5$

$4x + 4x + 4y - 60$

$60x - 4y - 60 = 4y$

$\frac{8x - 60}{4} = \frac{4y}{4}$

$y = -2x + 15$

T2

Fearless Frames: (continued)

2. The client changes his mind!

   He asks for a container that is a prism with a cross section which is an equilateral triangle.

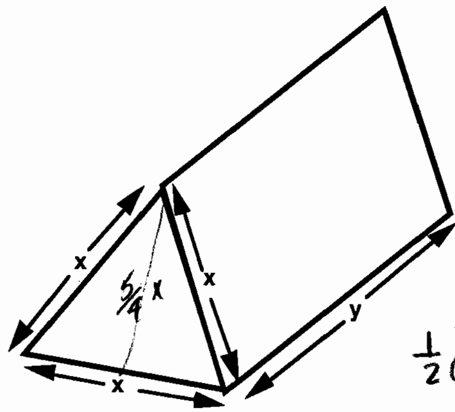
   Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

$x^2+\left(\frac{1}{2}x\right)^2=h^2$

$x^2+\frac{1}{4}x^2=h^2$

$\sqrt{\frac{5}{4}x^2}=\sqrt{h^2}$

$\frac{5}{4}x=h$



length

$2(3x)+3y=60$

$6x+3y=60$

$2x+y=20$

$y=20-2x$

Volume $=\frac{1}{2}bh\cdot y$ ✓

$\frac{1}{2}x\left(\frac{5}{4}x\right)\cdot y$

If $x=5, y=10$

$\frac{1}{2}(5)[\frac{5}{4}5]\cdot 10$

$=15.6\ 25\cdot 10$

$=156.25m^3$

If $x=6, y=8$

$\frac{1}{2}(6)[\frac{5}{4}6]\cdot 8=180m^3$

If $x=4$ $y=12$

Vol $\frac{1}{2}(4)\left(\frac{5}{4}(4)\right)\cdot 12=\frac{1}{2}(20)\cdot 12$

$=120m^3$

If $x=7$ $y=6$

$\frac{1}{2}(7)[(\frac{5}{4})7]\cdot 6$

$=30.625\cdot 6$

$=\boxed{183.75m^3}$

(2)

3. What advice do you think Fearless Frames should offer to this customer?
   Show all your calculations.

The prism container has more volume. If the customer wants the maximum volume, the dimensions (x, y) would be very close to give the most space. If the customer likes larger containers, he should go with the rectangular prism ✓

Fearless Frames

T3

Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

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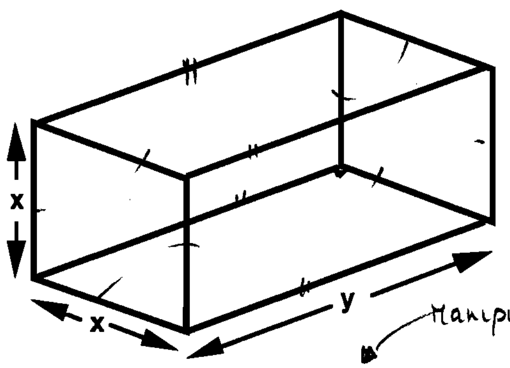
Show how you figured it out.



Find largest if x is a whole number.

5

Manipulative variable

x	1	2	3	4	5	6	7
Area of Base	1	4	9	16	25	36	49
y	13	11	9	7	5	3	1
Volume	13	44	81	112	125	108	49

Area of base $= x^2$

$8x + 4y = 60$

$\div 4$

$2x + y = 15$

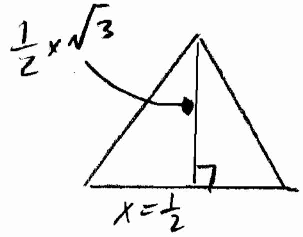
$y = 15 - 2x$

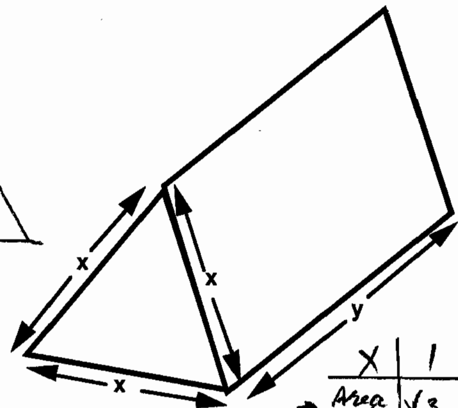
Fearless Frames: (continued)

2. The client changes his mind!

   He asks for a container that is a prism with a cross section which is an equilateral triangle.

   Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.





X	1	2	3	4	5	6	7	8
Area of base	$\frac{\sqrt{3}}{4}$	$\sqrt{3}$	$\frac{9}{4}\sqrt{3}$	$\frac{25}{4}\sqrt{3}$	$16\sqrt{3}$	$9\sqrt{3}$	$\frac{49}{4}\sqrt{3}$	$16\sqrt{3}$
y	18	16	14	12	10	8	6	4
	$\frac{9}{2}\sqrt{3}$	$16\sqrt{3}$	$\frac{63}{2}\sqrt{3}$	$48\sqrt{3}$	$\frac{125}{2}\sqrt{3}$	$72\sqrt{3}$	$\frac{47}{4}\sqrt{3}$	$\frac{81}{2}\sqrt{3}$
	$4.5\sqrt{3}$		$31.5\sqrt{3}$		$62.5\sqrt{3}$		$73.5\sqrt{3}$	

4

$$\text{Area of base} = \frac{1}{2}bh$$
$$= \frac{1}{2}(x)(\tfrac{1}{2}x\sqrt{3})$$
$$= \frac{1}{4}x^2\sqrt{3}$$

$$\text{Area of base} = \frac{\sqrt{3}}{4}x^2$$

$$\frac{6x+3y}{3} = 60$$
$$2x+y = 20$$
$$y = 20-2x$$
$$V = (\text{area})(y)$$

$$\sqrt{3}\,\frac{9}{4}\cdot 14 = 63\sqrt{3}$$

3. What advice do you think Fearless Frames should offer to this customer?
   Show all your calculations.

For both containers, it's best they choose the highest value for x, while y is still a large number.

A 1 0

(9)

Fearless Frames

T4

Fearless Frames Inc makes metal frames for containers.

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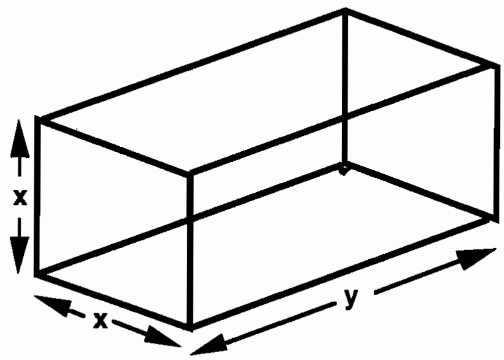
The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

____

Show how you figured it out.

length:



$8x + 4y = 60$

$2(4x) + 4y = 60$

$2x + y = 15$

$y = 15 - 2x$

Volume $= x^2 \cdot y$ ✓ $V = x^2(15-2x)$ ✓

3

If $x = 4$ $y = 7$

Volume $= 4^2 \cdot 7 = \boxed{112\ m^3}$ ✓

if $x = 5,\ y = 5$ but not sure if x can $= y$

$5^2 \cdot 5 = \boxed{125\ m^3}$ ✓

2

If $x = 6,\ y = 3$

$6^2 \cdot 3 = \boxed{108\ m^3}$ ✓

T4

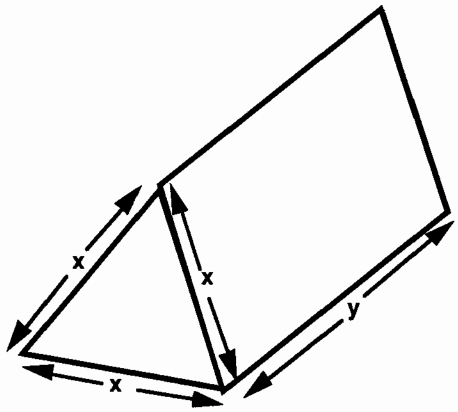
Fearless Frames: (continued)

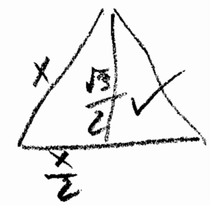
2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.

Max Volume
Dimensions





(2)

$3x + 3x + 3y = 60$

$6x + 3y = 60$

$\frac{6x - 60}{-3} = \frac{+3y}{+3}$

$30 - 2y = y$

Volume $= \frac{x \times h}{2} \times y =$

$V = \frac{x\left(\frac{\sqrt{3}\cdot x}{2}\right)}{2} \cdot (-2x + 30)$

$x\left(\frac{\sqrt{3}}{2}\right) \cdot 2(-x + 15)$

$V = \frac{\sqrt{3}\, x^2 - (-x+5)}{2}$

$-\sqrt{3}x^3 + 15\sqrt{3} \cdot x^2 = 2V$

$-\sqrt{3}(10)^3 + 15\sqrt{3} \cdot (10)^2 = 2V$

$\cdot 100 = 2V$

3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.

$\wedge$

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Fearless Frames

Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

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Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

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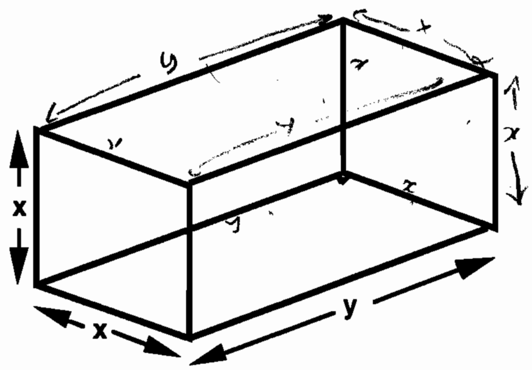
Show how you figured it out.



$8x + 4y = 60$

$2x + y = 15$

$y = 15 - 2x$

$V = x^2 y$ = maximum

$V = x^2(15-2x) = 15x^2 - 2x^3$

$\boxed{x = 5m}$ $y = 15 - 10$

$\boxed{y = 5m}$

$\boxed{V = 125 m^3}$

X	V
1	13
2	44
3	81
4	112
5	125
6	108

x	V
4.5	121.5

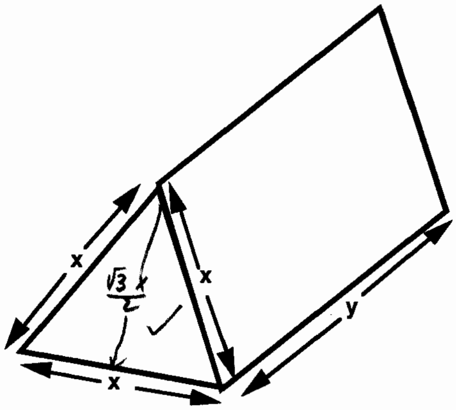
5

Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.



$0 < x < 10$

x	$x^2(10-x)$
2	32
3	63
4	96
5	125
6	144
7	147
8	128
6.5	147,875
6.7	148,137
6.8	147,968

4

$6x + 3y = 60$
$2x + y = 20 \quad y = 20 - 2x)$
$V = \frac{\sqrt{3}}{4}x^2y = \frac{\sqrt{3}}{2}x^2(10-x)$

Maximize $x^2(10-x)$
$x = 6{,}7\,m \quad y = 20 - 13{,}4$
$y = 6{,}6\,m$

$V = \frac{\sqrt{3}}{2}(6{,}7)^2(10-6{,}7)$
$V = \frac{\sqrt{3}}{2}(44{,}89)(3{,}3)$
$\boxed{V = 128{,}29\,m^3}$

3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.

It is better to use a triangular frame than a squarish frame, because a triangular frame gives you a bigger volume while using the same amount of wiring. 1

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