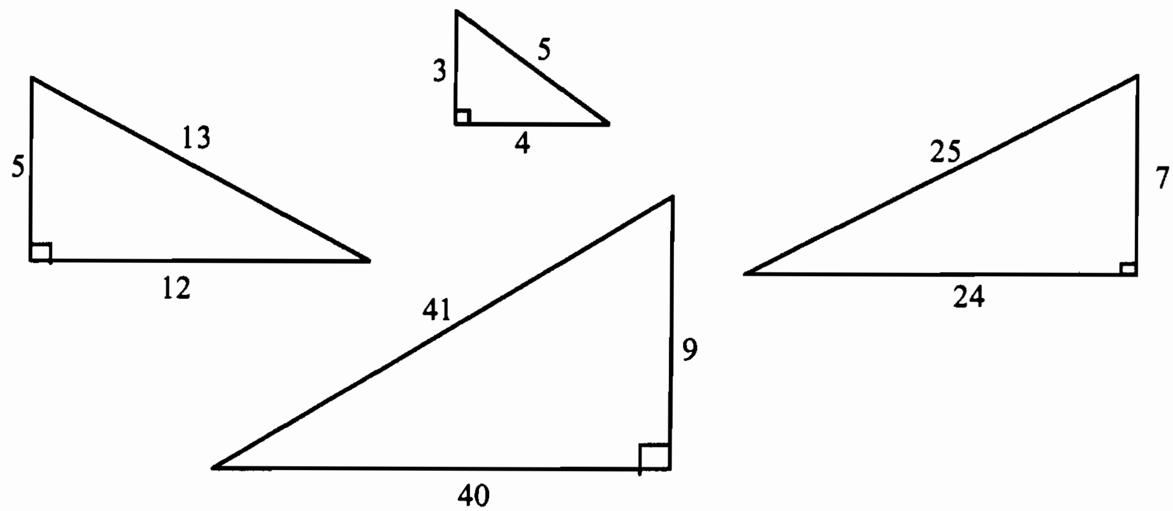
Pythagorean Triples

T1



(3, 4, 5), (5, 12, 13), (7, 24, 25) and (9, 40, 41) are called Pythagorean Triples because they satisfy the condition

$$c^2 = a^2 + b^2$$

1. Investigate the relationships between the lengths of the sides of triangles that belong to this set.

$m > n > 0$ 3 sides of a pythagorean Δ $m^2 - n^2$, $m^2 + n^2$, $2mn$

m	n				
2	1	2^2-1^2,	2^2+1^2	2(2)(1)	3, 4, 5
3	2	3^2-2^2,	3^2+2^2,	3(2)(2)	5, 12, 13
4	3	4^2-3^2	4^2+3^2	2(4)(3)	7, 24, 25
5	4	5^2-4^2,	5^2+4^2	2(5)(4)	9, 40, 41

Smallest side odd

Next 2 sides are always one more than each other

T1

Pythagorean Triples (continued)

2. Use these relationships to find the numerical values of at least two further Pythagorean Triples that belong to this set.

m	n		
6	5	$6^2-5^2, 6^2+5^2, 2(6)(5)$	11, 60, 61
7	6	$7^2-6^2, 7^2+6^2, 2(7)(6)$	13, 84, 85
8	7	$8^2-7^2, 8^2+7^2, 2(8)(7)$	15, 112, 113

m^2-n^2 m^2+n^2 $2mn$ $m+n$

$(m-n)(m+n)$.

always 1

3. Investigate rules for finding the perimeter and area of triangles that belong to this set when you know the length of the shortest side.

smallest length = $m+n$ where $m-n=1$

The three sides will be $(m+n),(m^2+n^2),(2mn)$

The perimeter is $2mn+m+n+m^2+n^2$

Area = $\frac{(m+n)(2mn)}{2}$

check smallest side = 19

$=9+10$

P $m+n+m^2+n^2+2mn$

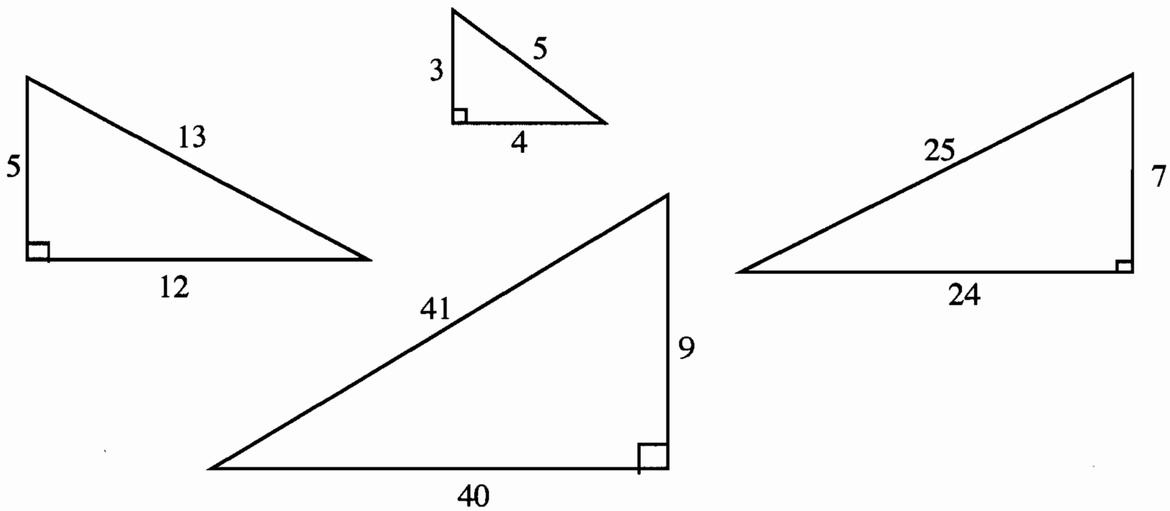
sides $=19, 9^2+10^2, 2\times9\times10$

A $\frac{(m+n)2mn}{2}$

19, 181, 180

$19^2+180^2=32761=181^2$ ✓

Pythagorean Triples **T2**



(3, 4, 5), (5, 12, 13), (7, 24, 25) and (9, 40, 41) are called Pythagorean Triples because they satisfy the condition

$$c^2 = a^2 + b^2$$

1. Investigate the relationships between the lengths of the sides of triangles that belong to this set.

Two Sides of the triangle add up to the smallest side squared.

$5^2 = 25 = 12 + 13$
$3^2 = 9 = 4 + 5$
$7^2 = 49 = 24 + 25$
$9^2 = 81 = 40 + 41$

Pythagorean Triples (continued)

T2

2. Use these relationships to find the numerical values of at least two further Pythagorean Triples that belong to this set.

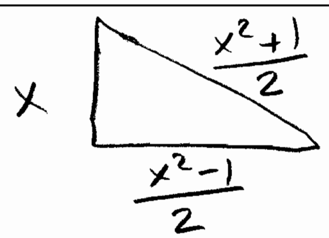
(11, 60, 61) $\quad 11^2 + 60^2 = 61^2 \quad 121 + 3600 = 3721$

(13, 84, 85) $\quad 13^2 + 84^2 = 85^2 \quad 169 + 7056 = 7225$

3. Investigate rules for finding the perimeter and area of triangles that belong to this set when you know the length of the shortest side.

perimeter $= x^2 + x$

Area $= \dfrac{x^3 - x}{4}$



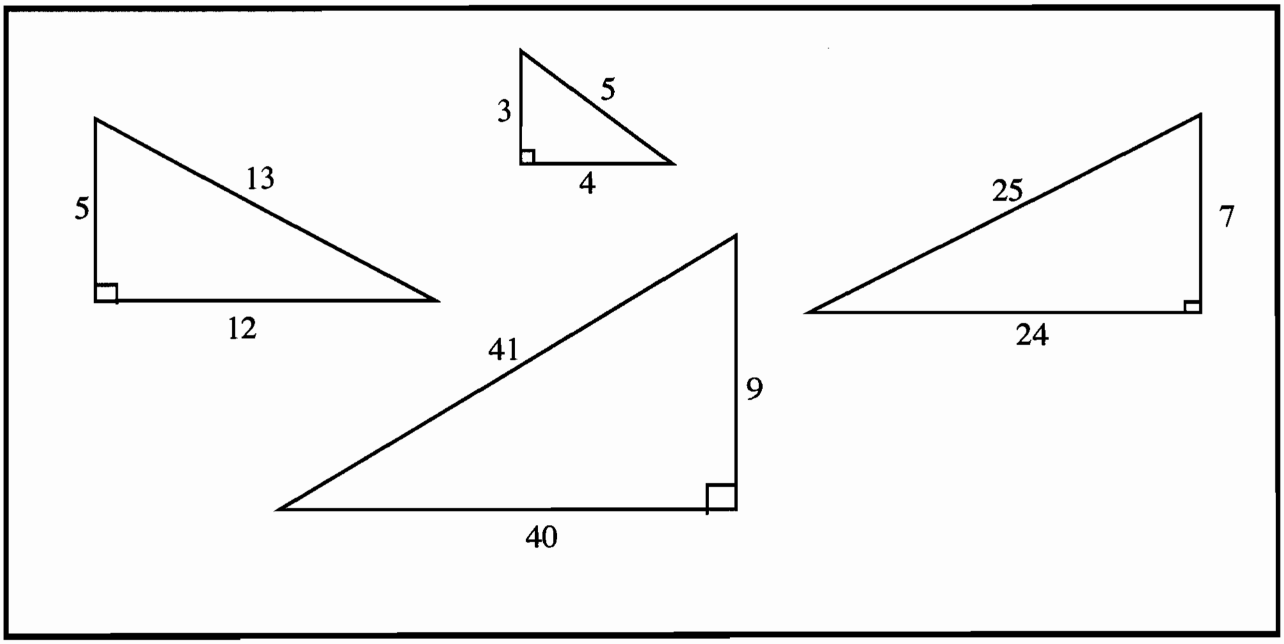
$x^2 + x$

$\frac{1}{2} bh = \frac{1}{2}\left(\frac{x^2-1}{2}\right)x$

$= \dfrac{x^3 - x}{4}$

Pythagorean Triples

T3



(3, 4, 5), (5, 12, 13), (7, 24, 25) and (9, 40, 41) are called Pythagorean Triples because they satisfy the condition

$$c^2 = a^2 + b^2$$

this set of triangles are all unique and cannot be reduced by a common factor.

1. Investigate the relationships between the lengths of the sides of triangles that belong to this set.

If $a<b<c$ then a is always an odd number and $b = \frac{a^2-1}{2}$ and

$c = b+1$

formula applies only to unique Pythagorean triples,

the triangles must not be able to be reduced by common factors.

Pythagorean Triples (continued)

T3

2. Use these relationships to find the numerical values of at least two further Pythagorean Triples that belong to this set.

11 & 60 & 61 ; 13 & 84 & 85 ; 15 & 112 & 113
17 & 144 & 145

3. Investigate rules for finding the perimeter and area of triangles that belong to this set when you know the length of the shortest side.

$$\text{perimeter} = a + \frac{a^2-1}{2} + \left(\frac{a^2-1}{2} + 1\right) = a^2 + a$$

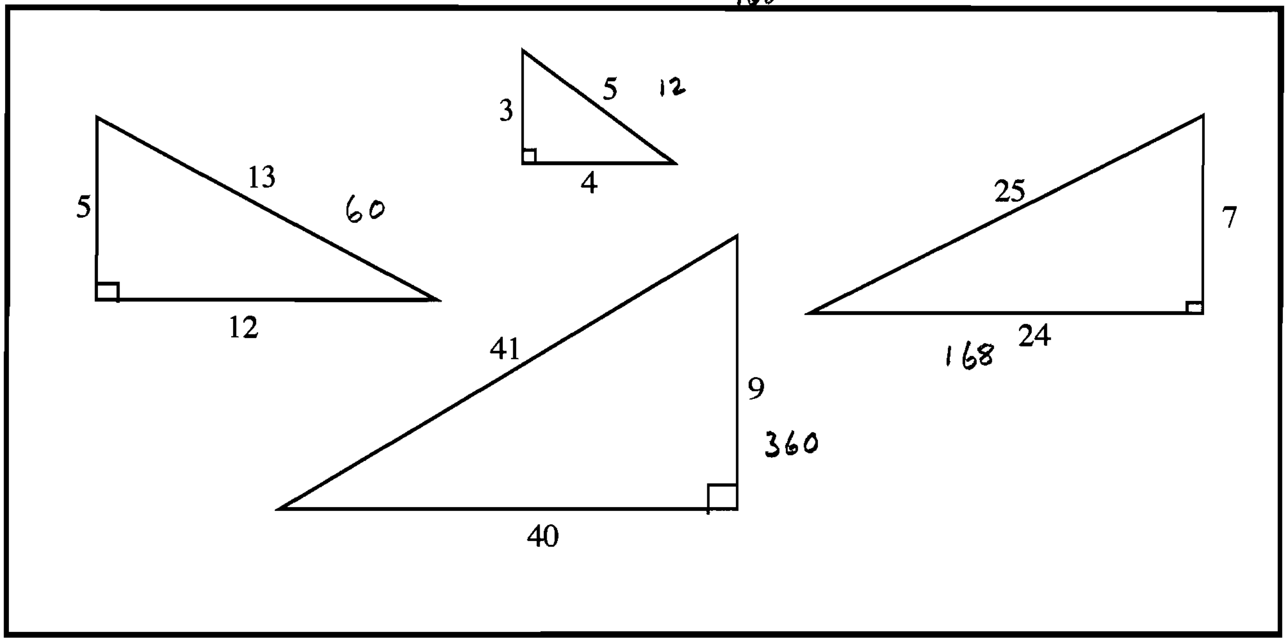
$$\text{Area} = \frac{1}{2}\left(a \cdot \frac{a^2-1}{2}\right) \qquad \frac{a^3-a}{2} = \frac{a(a^2-1)}{2} = \frac{a(a+1)(a-1)}{2}$$

$$= \frac{a^3-a}{4}$$

Pythagorean Triples

T4

24
× 7
168



(3, 4, 5), (5, 12, 13), (7, 24, 25) and (9, 40, 41) are called Pythagorean Triples because they satisfy the condition

$$c^2 = a^2 + b^2$$

1. Investigate the relationships between the lengths of the sides of triangles that belong to this set.

The shortest leg of each Δ are all odd #'s, increasing by 2 each time (3, 5, 7, 9..)
The hypotenuse & the longer of the 2 legs have a difference of exactly one.

2. Use these relationships to find the numerical values of at least two further Pythagorean Triples that belong to this set.

(11, 60, 61) , (13, 84, 85)

$a^2 + b^2 = c^2$

$11^2 + b^2 = c^2$

$11^2 + b^2 = (b+1)^2$

$121 + b^2 = b^2 + 2b + 1$

$121 - 2b - 1 = 0$

$-2b = -120$

$b = 60$

$c - b = 1$

$c = b + 1$

$(b+1)(b+1)$

$b^2 + 2b + 1$

$c = b + 1$

$= 60 + 1$

$= 61$

$13^2 + b^2 = c^2$

$13^2 + b^2 = (b+1)^2$

$13^2 + b^2 = b^2 + 2b + 1$

$169 - 2b = 1$

$-2b = -168$

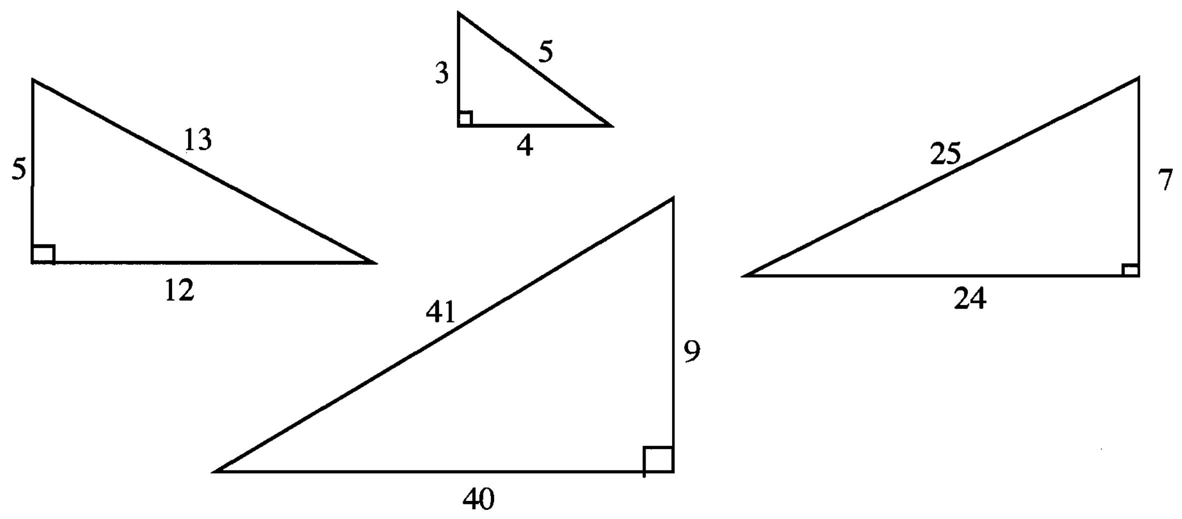
$b = 84$

$168 \div 2 = 84$

3. Investigate rules for finding the perimeter and area of triangles that belong to this set when you know the length of the shortest side.

Perimeter: to find the perimeter you must find the lengths of the remaining 2 sides. If the shortest side = a and c = b+1, then the formula to find b = "$a^2 + b^2 = (b+1)^2$". Once b is found a & b can be used in the Pythagorean formula to find c.

Area: once you've found a & b (you already know a), then you can multiply ab and divide by 2 to find the area of the Δs.



(3, 4, 5), (5, 12, 13), (7, 24, 25) and (9, 40, 41) are called Pythagorean Triples because they satisfy the condition

$$c^2 = a^2 + b^2$$

1. Investigate the relationships between the lengths of the sides of triangles that belong to this set.

$c^2 + a^2 = b^2$, $c = b+1$ (let b = longer leg)

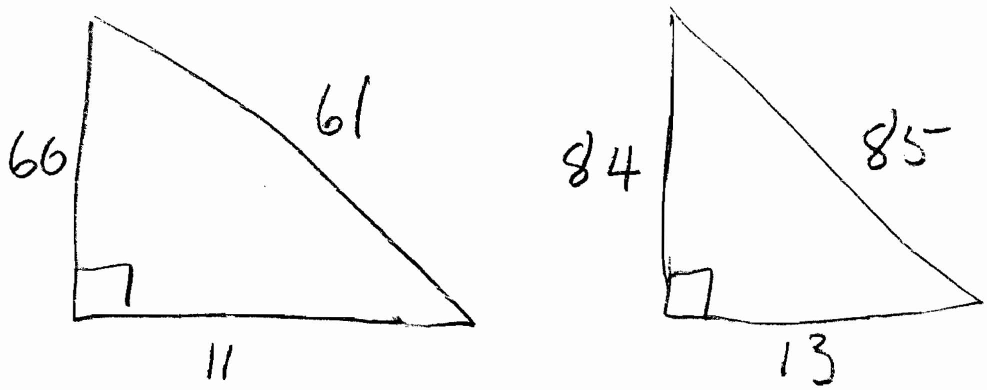
$(b+1)^2 = a^2 + b^2 \rightarrow (b+1)^2 - b^2 = a^2 \rightarrow 2b + 1 = a^2$

(a is shorter leg) a are odd #s, starting with 3.

2. Use these relationships to find the numerical values of at least two further Pythagorean Triples that belong to this set.

$a = 11 \rightarrow 2b+1 = (11)^2 \rightarrow b = 66 \rightarrow c = 61$

$a = 13 \rightarrow 2b+1 = (13)^2 \rightarrow b = 84 \rightarrow c = 85$



3. Investigate rules for finding the perimeter and area of triangles that belong to this set when you know the length of the shortest side.

$\text{area} = (\frac{1}{2})ab$

$2b+1 = a^2 \rightarrow 2b = a^2-1 \rightarrow b = \frac{a^2-1}{2}$

$\boxed{\text{area} = \frac{a^3-a}{4}}$ $\quad \text{perimeter} = a+b+c$

$a + 2\left(\frac{a^2-1}{2}\right)+1 \rightarrow a+(a^2-1)+1$

$\rightarrow \boxed{\text{perimeter} = a^2+a}$