The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm³.

In order to keep costs low, the company wants to use the smallest amount of aluminum.

Find the radius and height of a cylindrical can which holds 200 cm³ and uses the smallest amount of aluminum. \( \text{radius} = 3 \text{ cm}, \text{ height} = 7.09 \text{ cm} \)

Explain your reasons and show all your calculations.

The volume of the drink's formula is

\[
(\pi r^2)h = 200 \text{ cm}^3
\]

The area of aluminum is \(2\pi r^2 + 2\pi rh\). The heights formula is \(\frac{200}{\pi r^2}\).

By using the formula of \(2\pi r^2 + \frac{100}{r}\), I tried different variables for \(r\) or the radius. Then found when the areas were at the smallest amount.
Bestsize Cans (continued)

\[ h = \frac{200}{\pi r^2} \]

\[ X = \text{area of aluminum can} \]

\[ 2\pi r^2 + 2\pi r \times \left( \frac{200}{4\pi r^2} \right) = X \]

\[ 2\pi r^2 \times 2 (\frac{200}{r}) = X \]

\[ 2\pi r^2 \times \frac{400}{r} = X \]

\[ h = \frac{200}{\pi 9} \]

\[ = 7.07 \]

\[ r \quad X \]

\[ 1 \quad 6.28 + 400 = 406.28 \]

\[ 2 \quad 25.12 + 200 = 225.12 \checkmark \]

\[ 3 \quad 56.52 + 133 = 189.85 \checkmark \]

\[ 4 \quad 100.48 + 100 = 200.48 \checkmark \]

\[ 5 \quad 157 + 80 = 237 \]

\[ 2.5 \quad 39.27 + 160 = 199.27 \checkmark \]

\[ 3.5 \quad 76.97 + 114 = 191.25 \]

10
The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm³.

In order to keep costs low, the company wants to use the smallest amount of aluminum.

Find the radius and height of a cylindrical can which holds 200 cm³ and uses the smallest amount of aluminum.

Explain your reasons and show all your calculations

\[ V = \pi r^2 h \]
\[ 200 = \pi r^2 h \]
\[ h = \frac{200}{\pi r^2} \]  

\[ S = 2\pi r^2 + 2\pi rh \]
\[ S = 2\pi r^2 + 2\pi r \left( \frac{200}{\pi r^2} \right) \]
\[ S = 2\pi r^2 + \frac{400\pi}{r} \]

\[ r = 3 \text{ cm} \]
\[ h = \frac{200}{\pi \cdot 3} = \frac{200}{9.42} = 7 \text{ cm} \]

39.25
Bestsize Cans (continued)

<table>
<thead>
<tr>
<th>s</th>
<th>406</th>
<th>225</th>
<th>190</th>
<th>200</th>
<th>237</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm³.

In order to keep costs low, the company wants to use the smallest amount of aluminum.

Find the radius and height of a cylindrical can which holds 200 cm³ and uses the smallest amount of aluminum.

Explain your reasons and show all your calculations.

**Calculations:**

\[
SA = 2\pi r^2 + 2\pi rh
\]

\[
2\pi r^2 + 2\pi h 200 = 2\pi r^2 + 400
\]

\[
\frac{400}{h} = 225.13 \checkmark \quad h = \frac{200}{\pi r^2}
\]

\[
r = 2 \quad SA = 8\pi + 400 = 225.13 \checkmark
\]

\[
r = 2.5 \quad SA = 20.78 + 400 = 199.27 \checkmark
\]

\[
r = 3 \quad SA = 18\pi + 400 = 189.88 \checkmark \quad h = \frac{200}{\pi r^2} = 7.07
\]

\[
r = 3.5 \quad SA = 24.6\pi + \frac{400}{3.5} = 191.25 \checkmark
\]

Smallest SA is for \( r = 3 \) and \( h = 7.07 \).
The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm$^3$.

In order to keep costs low, the company wants to use the smallest amount of aluminum.

Find the radius and height of a cylindrical can which holds 200 cm$^3$ and uses the smallest amount of aluminum.

**Bestsize Cans**

![Cylinder Diagram]

**Calculations:**

Volume = $200 \text{ cm}^3 = \pi r^2 h$

Surface Area = $2\pi r^2 + 2\pi rh$

\[
\frac{200}{\pi} = r^2 h \Rightarrow \frac{200\pi}{r^2} = h \approx 63.66197724
\]

\[
\begin{align*}
\text{r=2, } h=16 & \quad 2\times2=4 \quad 9\times16=64 \\
\text{r=1, } h=64 & \quad 1\times1=1 \quad 1\times64=64 \\
\text{r=3, } h=7\frac{1}{3} & \quad 3\times9=27 \quad 9\times9\frac{1}{3}=64
\end{align*}
\]

\[
2\pi(2)^2 + 2\pi(32) \approx 25.1 + 201.1 \approx 226.2
\]

\[
2\pi(1)^2 + 2\pi(64) \approx 6.3 + 402.1 \approx 408.4
\]

\[
2\pi(3)^2 + 2\pi\left(\frac{69}{3}\right) \approx 56.5 + 134.0 \approx 190.5
\]

So, radius larger $\Rightarrow$ surface area smaller! Now, try find largest

radius (smallest surface area)

\[
\begin{align*}
\text{r=8, } h=1 & \quad 8\times8=64 \quad 64\times1=64 \\
\text{r=5, } h=256 & \quad 5\times5=25 \quad 25\times25=64 \\
\text{r=4, } h=4 & \quad 4\times4=16 \quad 16\times4=64 \\
\text{r=6, } h=\frac{16}{1} & \quad 6\times6=36 \quad 36\times\frac{16}{1}=64
\end{align*}
\]

\[
\begin{align*}
2\pi(8)^2 + 2\pi(8) & \approx 402.1 + 50.2 \approx 452.3 \\
2\pi(5)^2 + 2\pi(12.8) & \approx 157.1 + 80.4 \approx 237.5 \\
2\pi(4)^2 + 2\pi(16) & \approx 100.5 + 100.5 \approx 201
\end{align*}
\]
Bestsize Cans (continued)

I kind of did a "guess and check" problem solving method. At first, I thought that the larger the radius, the smaller the surface area. The goal of this task is to find the smallest surface area for a can that can hold 200 cm$^3$ in volume. After many "guess and check" trials, I came to a conclusion that a radius of $3\sqrt{2}$ and a height of $7\sqrt{2}$ in a can can have a volume of about 201. That means it can hold 200 cm$^3$ of liquid. And, it uses the smallest amount of aluminum possible, which is about 190.5 cm$^2$. 
The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm\(^3\).

In order to keep costs low, the company wants to use the smallest amount of aluminum.

Find the radius and height of a cylindrical can which holds 200 cm\(^3\) and uses the smallest amount of aluminum.

Explain your reasons and show all your calculations

1. If \( r = 2 \)
   \[
   \text{SA} = 2\pi r^2 + \frac{400}{r} = 8\pi + 200 \approx 225.13274, \quad \text{SA} = 2\pi r^2 + 2\pi rh
   \]
   \[200 = \pi r^2 h \Rightarrow h = \frac{200}{\pi r^2} \]

2. If \( r = 3 \)
   \[
   \text{SA} = 2\pi (3)^2 + \frac{400}{3} = 18\pi + \frac{400}{3} \approx 189.882, \quad \text{SA} = 2\pi r^2 + \frac{400}{r}
   \]

3. If \( r = 4 \)
   \[
   \text{SA} = 2\pi (4)^2 + \frac{400}{4} > 32\pi + 100 \approx 200.530
   \]

4. If \( r = 3.1 \)
   \[
   \text{SA} = 2\pi (3.1)^2 + \frac{400}{3.1} \approx 189.4114
   \]
Bestsize Cans  (continued)

\[ r = 3.2 \quad \text{SA} = 2\pi(3.2)^2 + 400/3.2 = 189.34 \]

\[ r = 3.3 \quad \text{SA} = 2\pi(3.3)^2 + 400/3.3 = 189.636 \]

\[ h = \frac{100}{\pi r^2} \approx 6.217 \]

Radius = 3.2 \quad \checkmark

Height = 6.217 \quad \checkmark