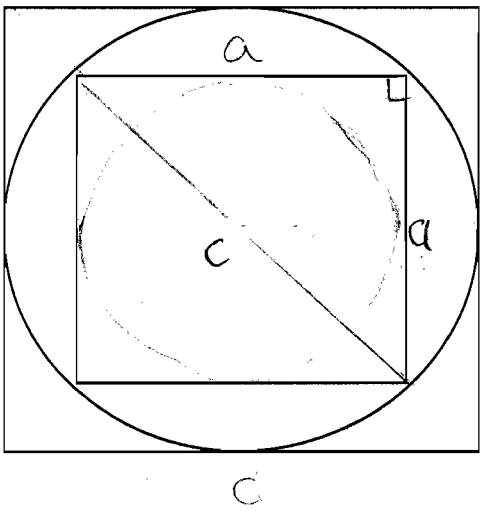
Circles and Squares

T1

This diagram shows a circle with one square inside and one square outside.



1. What is the ratio of the areas of the two squares? Show your work

1:2

$c^2 = a^2 + a^2$ pythagorean theorem

$c^2 = 2a^2$

→ Smaller square $= a^2$
larger square $= c^2$

$\frac{1}{2}$

2. If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

1:2

$c^2 = 2a^2$

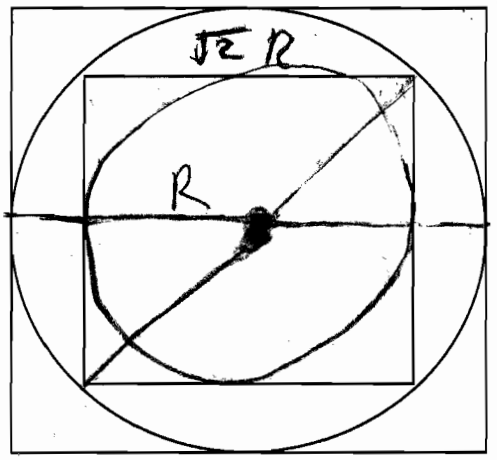
Smaller circle $= \pi(\frac{1}{2}a)^2$
larger circle $= \pi(\frac{1}{2}c)^2$

$\frac{\pi(0.25)(a^2)}{\pi(0.25)(c^2)} = \frac{a^2}{c^2} = \frac{1}{2}$

Circles and Squares

T2

This diagram shows a circle with one square inside and one square outside.



1. What is the ratio of the areas of the two squares? Show your work

2:1

$(2R)^2 = a^2$

$4R^2 = 2a^2$

$2R^2 = a^2$

$\sqrt{2}R = a$

$(\sqrt{2}R)^2 : 4R^2$

$2R^2 : 4R^2$

2. If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

2:1

$\left(\frac{\sqrt{2}R}{2}\right)^2 \pi : R^2\pi$

$\frac{2R^2}{4}\pi : R^2\pi$

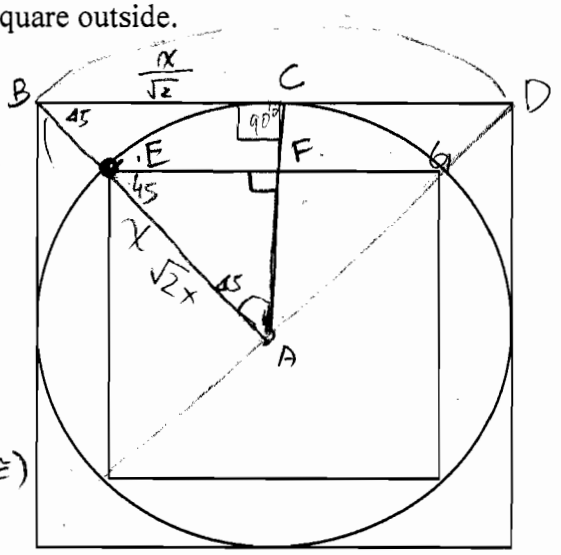
$\frac{1}{2}R^2\pi : 4R^2\pi$

$1:2$

Circles and Squares

T3

This diagram shows a circle with one square inside and one square outside.



1. What is the ratio of the areas of the two squares?
   Show your work

2:1

Suppose AB = x

draw AC ⊥ BD so ∠CAB is 45°

therefore ∠ABC is 45°

△ABC is special 45-45-90 △

$BC = \frac{x}{\sqrt{2}}$; as is CA (square) and AE (radii ≅)

$BD = \frac{2x}{\sqrt{2}}$

area big square $= \left(\frac{2x}{\sqrt{2}}\right)^2 = \frac{4x^2}{2} = 2x^2$

since $AE = \frac{x}{\sqrt{2}}$, EF and FA $= \frac{x}{2}$ (special 45-45-90 △)

$EG = 2\left(\frac{x}{2}\right) = x$

area small square $= (x)^2$

$= x^2$

$\frac{2x^2}{x^2} = \frac{2}{1}$

2. If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

2:1

πr^2 ← area of a circle

Big circle → radius

$\pi\left(\frac{x}{\sqrt{2}}\right)^2$

$\pi\left(\frac{x^2}{2}\right)$

Small circle

AF ≅ FF (square)

$\pi\left(\frac{x}{2}\right)^2$ ← radius

$\pi\left(\frac{x^2}{4}\right)$

$\frac{x^2}{2} : \frac{x^2}{4}$

$x^2 : \frac{x^2}{2}$

2:1 the large circles area is 2 time larger than the small circles

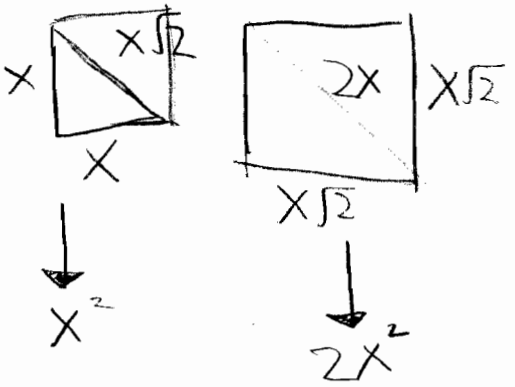
Circles and Squares

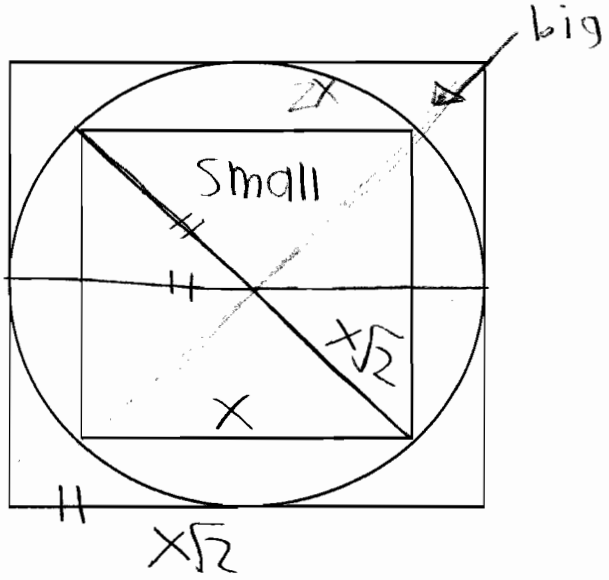
T4

This diagram shows a circle with one square inside and one square outside.

1. What is the ratio of the areas of the two squares? Show your work

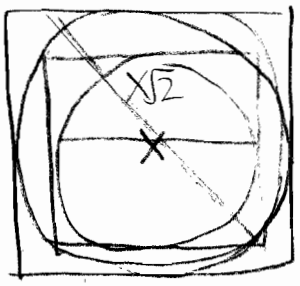
small : big → 1 : 2

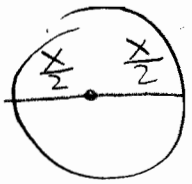


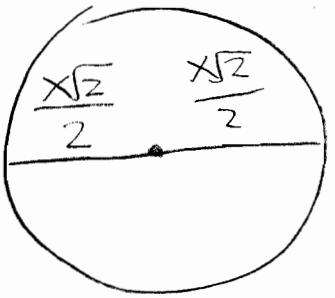


2. If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

small : big → 1 : √2







$\frac{x}{2} \times \frac{2}{x\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

small : big

1 : √2

Circles and Squares

T5

This diagram shows a circle with one square inside and one square outside.

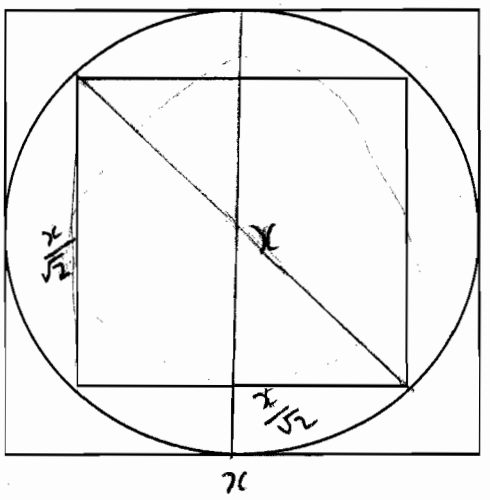
1. What is the ratio of the areas of the two squares? Show your work

2:1

length & width of big square = x

diagonal of square = x

length & width of small square = $\frac{x}{\sqrt{2}}$



$x^2 : \frac{x^2}{2}$ ~~$\frac{x^2 \times 2}{\frac{x^2}{2} \times 2}$~~ $\frac{2x^2}{x^2} : \frac{2}{1}$

$x^2 : \frac{x^2}{2}$ $\frac{x^2 \times 2}{\frac{x^2}{2} \times 2}$ $\frac{2x^2}{x^2} = \frac{2}{1}$

2. If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

2:1

radius of big circle = $\frac{1}{2}x$

$\frac{x}{\sqrt{2}} \times \frac{1}{2} = \frac{x}{2\sqrt{2}}$

area of big circle $\pi\left(\frac{1}{2}x\right)^2$

$\pi \frac{1}{4}x^2 \times 8$

$\pi \frac{x^2}{8} \times 8$

$\pi\left(\frac{x}{2\sqrt{2}}\right)^2$

$\frac{2x^2}{1x^2}$