Students working collaboratively
How can students learn from discussing mathematics?
A PROFESSIONAL DEVELOPMENT MODULE

Introduction

If students are to make sense of mathematical concepts, then they will need opportunities to share, discuss and work together. Research has shown that cooperative small group work has positive effects on learning, but that this is dependent on the existence of shared goals for the group and individual accountability for the attainment of these goals. It has also been seen to have a positive effect on social skills and self-esteem (Askew & Wiliam, 1995).1

In many classrooms, however, traditional transmission teaching styles have reduced both the quantity and quality of student-student discussion. In others, students do work and talk together, but this talk does not always profit learning.

This unit is designed to offer the professional development provider some resources that will help teachers to:
- consider the characteristics of student-student discussion that benefit learning;
- recognize and face their own worries about introducing collaborative discussion;
- explore techniques for promoting effective student-student discussion;
- consider their own role in managing student-student discussion;
- plan discussion based lessons.

The activities described below are given here as a ‘menu’ of suggestions to help the provider plan. They are presented in a logical order, building up knowledge and expertise. Any planned professional development program should offer opportunities for teachers to take risks and try new pedagogies in the classroom and then report back and reflect on their experiences. Activity G is therefore essential in the program and should not be missed!

Activities

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Acknowledgement:
In preparing this material, we acknowledge the permissions given by the Bowland Charitable Trust to adapt the professional development resources, Bowland Maths, that they had previously funded us to produce for the UK. This includes many of the handouts and most of the video extracts. Additional resources were also adapted from Improving Learning in Mathematics, a government funded program in the UK. The original sources are:

Activity A: Experiencing a discussion

Time needed: 20 minutes.

Teachers find it helpful, and enjoyable, to engage in a collaborative activity themselves before discussing classroom issues. This activity will give a taste of this experience. It also offers an opportunity to reflect on the pedagogical implications for the classroom.

We suggest that you choose just one of the problems shown on Handout 1 for this activity, or substitute a similar problem of your own.

- On your own, write down a response to one of the problems on Handout 1.
- Together with your colleagues, compare your responses and try to refine your answers until you feel that you have reached a consensus.
- Take a few moments to reflect on the experience you have just had.
  - Did you find it helpful to have a chance to think about the question yourself before it was discussed in your group?
  - How far did you really think together, or did you tend to follow independent lines of thought?
  - Did someone ‘take over’? Was someone a ‘passenger’?
  - Did you listen to, share ideas with and consider the alternative views of everyone in the group?
  - Did you build on each others’ ideas to construct chains of coherent reasoning?
  - Did you feel able to share your ideas without fear of embarrassment of being wrong? Did anyone feel uncomfortable or threatened? If so, why?
  - Did your discussion stay ‘on task’ or were you ‘wandering’?
- What are the implications of this activity for your classroom?

Teachers may not complete the problem in the time you have allowed, but do not worry too much about this. What is important is that they have had time to discuss and explore their ideas about one problem together. They may also want to know ‘the answer’. It is interesting to ask different groups to compare answers, but resist giving them a ‘definitive’ answer - in fact most of the problems do not have one definitive answer. That is what makes them good discussion questions!

The initial time for individual work is important. When students are put straight into groups before they have had time to think for themselves, the ‘quick thinking’, confident students are more likely to take over and dominate the group. When students prepare something to share, and then take it in turns to make a contribution, more thoughtful and engaging discussions will result.

The questions asked here reflect the characteristics of powerful discussions that emerge from research literature (Alexander, 2006, 2008; Mercer, 1995, 2000)². This is discussed more explicitly in the next activity.

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## Handout 1: Experiencing a discussion

### Golf shot

How does the speed of the golf ball change as it flies through the air in this amazing golf shot?

Sketch a speed v time graph to illustrate your answer.

### Teachers

About how many teachers are there in your country?

Try to make a reasonable estimate based on facts that you already know.

### Which Sport?

Which sport could this graph represent?

### Trees

About how many trees are needed each day to provide newspapers for your country?

Try to make a reasonable estimate based on facts that you already know.

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**Activity B: Analyzing a discussion**

Time needed: 15 minutes.

There is a clear difference between working *in* a group and working *as* a group. It is quite common to see students working independently, even when they are sitting together. Disputational talk, in which students simply disagree and go on to make individual decisions, is not beneficial. Neither is cumulative talk in which students build uncritically on what each other has said. For true collaborative work, students need to develop exploratory talk consisting of critical and constructive exchanges, where challenges are justified and alternative ideas are offered (Mercer, 1995, 2000)\(^3\). It is not enough for students to simply give each other right answers, as this does not produce enhanced understanding (Reynolds and Muijs, 2001)\(^4\). The most helpful talk appears to be that where the participants work on and elaborate each other’s reasoning in a collaborative, rather than competitive atmosphere. Exploratory talk enables reasoning to become audible and ‘publicly’ accountable.

In this activity, the teachers are given transcripts from classroom discussions and are invited to discuss how the discussions either help or hinder learning. Instead of transcripts, you may prefer to show some video clips of student-student discussions for this activity.

- Role-play each of the two transcripts on Handout 2.
- Look at the characteristics of helpful and unhelpful talk shown on Handout 3.
- Which of the characteristics in Handout 3 do you recognize in these transcripts?
- Would you describe the discussions as **Disputational**, **Cumulative** or **Exploratory**?
- What strategies could you use that would help students to discuss more profitably?

The *Find the elephant* transcript is **disputational**. The two boys are treating the computer software as a competitive game, making random guesses. They laugh or make derisory comments when their partner makes an incorrect guess. There is little evidence of collaborative thinking.

The *Rail prices* transcript is more **exploratory**. These students are clearly listening to and engaging with each others’ reasoning. Dan and Harriet are both working together to convince Andy and are backing up their argument with an example.

The *Always, sometimes, or never true* transcript is more cumulative. The students are trying to get through the task as quickly as possible. They do not disagree or challenge each other, they simply reinforce each others’ unqualified assertions.

Clearly, the only dialogue that shows evidence of learning here is the one on *Rail prices*. The final question challenges teachers to think of ways of encouraging this kind of talk. One possible suggestion is that students themselves may role play different ways of working together and discuss how learning takes place.

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Handouts 2 & 3

Handout 2: Analyzing a discussion

**Find the elephant**

Two students are trying to find an elephant on a computer screen by being in coordinates. The computer gives feedback on how close they get.

They take turns, and Harriet wins.

**Rail Prices**

In August, they went down by 20%.

Four students are discussing the Rail prices.

**Students Working Collaboratively**

(1995, p. 100)

- **Exploratory talk**
  - Dan: I think it's 12.
  - Lester: I think that one is an add.
  - Sean: (eventually takes his turn, but fails to find the elephant)
  - Lester: (He then takes his turn, without success)

**Handout 3: Characteristics of helpful and unhelpful talk**

What types of talk engages students, develops understanding and promotes deeper thinking? Robin Alexander (2006) identified the following five principles of helpful classroom talk - which he terms dialogic.

**Dialogic talk is:**

- **Collective**: teachers and children address learning tasks together, as a group or as a class, rather than in isolation
- **Reciprocal**: teachers and children listen to each other, share ideas and consider alternative viewpoints
- **Cumulative**: teachers and children build on their own and each others’ ideas and chain them into coherent lines of thinking and enquiry
- **Supportive**: children articulate their ideas freely, without fear of embarrassment over ‘wrong’ answers and they help each other to reach common understandings
- **Purposeful**: teachers plan and facilitate dialogic teaching with particular educational goals in view

Neil Mercer (1995, 2000) identifies the following three types of student-student talk. It is the third type, exploratory talk, that is most helpful for learning:

**Cumulative talk**

Speakers build positively, but uncritically on what each other has said. This is typically characterised by repetitions, confirmations and elaborations.

**Disputational talk**

This consists of disagreement and individualised decision making. It is characterised by short exchanges consisting of assertions and counter-assertions.

**Exploratory talk**

Speakers work on and elaborate each other’s reasoning in a collaborative, rather than competitive atmosphere. Exploratory talk enables reasoning to become audible and knowledge becomes publicly accountable. It is characterised by critical and constructive exchanges.

\[ \text{Handouts for Teachers} \quad \text{Students Working Collaboratively} \]

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**Activity C: Recognizing the concerns of teachers**

**Time needed: 10 minutes.**

This activity is designed to help teachers recognize and articulate their own concerns relating to collaborative work. It is important to address these explicitly, as they are often serious obstacles to them.

**Handout 4** contains a number of concerns and opinions that are commonly expressed by teachers.

- Which of these do you share?
- In pairs, choose one of these comments and imagine that it was expressed by a teacher from your school. Take a few minutes to prepare a response. In the whole group, share your choice together with your response.

It is important to recognize that group work may not always be appropriate. When the purpose of the lesson is to develop fluency in a particular skill, then individual practice may be more suitable. This should not constitute the whole diet, however. Collaborative group work is necessary when the purpose of the session is to develop conceptual understanding or strategies for solving more challenging problems. In these cases, students need to share alternative views, interpretations or approaches.

**Handout 4**

| Time pressures | “It’s a gallop to the main exam; we don’t have time for discussion”  
|                | “Students will waste time in social talk. They would rather talk about what is on TV than about maths.” |
| Control        | “What will other teachers think of the noise?”  
|                | “How can I possibly monitor what is going on?” |
| Personal insecurity | “What if they start asking questions I cannot answer?”  
|                | “What if they stray off the point of the lesson?” |
| Views of students | “My students cannot discuss.”  
|                | “My students are too afraid of being seen to be wrong.” |
| Views of the subject | “In Mathematics, answers are either right or wrong – there is nothing to discuss.”  
|                | “In math if they understand it there is nothing to discuss, and if they don’t, they are in no position to discuss anything. In fact they may even spread their own misconceptions.” |
| Views of learning | “Mathematics is a subject where you listen and practise.”  
|                | “Learning is a private activity.” |
Activity D: Creating & Establishing ‘Ground Rules’ with students

Time needed: 15 minutes.

As we have seen, students (and adults!) do not always discuss in helpful ways. Some are reluctant to talk at all, while others take over and dominate. Students therefore need to be taught how to discuss. Some teachers have found it helpful to introduce a list of ‘ground rules for discussion’ into their classes. These ground rules should, in appropriate language, give explicit guidance to students on how to talk together profitably.

- Imagine that you are starting with a new class. You want them to begin to work collaboratively. What classroom ‘rules’ would you seek to establish?
- The ground rules on Handout 5 were developed for use with one class of students. Compare your ideas with those offered here.
- How would you encourage your students to follow these rules?
- How could you involve your students in developing such a list?

Handout 5

Here are some suggested ‘ground rules’ for students to use as they work in groups. These could be displayed and reinforced over time. Maybe you could involve your class in drawing up a similar list.

1. Give everyone in your group a chance to speak
   - “Let’s take it in turns to say what we think.”
   - “Claire, you haven’t said anything yet.”

2. Listen to what people say
   - “Don’t interrupt - let Sam finish.”
   - “I think Sam means that ....”

3. Check that everyone else listens
   - “What did Sue just say?”
   - “I just made a deliberate mistake - did you spot it?”

4. Try to understand what is said
   - “I don’t understand. Can you repeat that?”
   - “Can you show me what you mean?”

5. Build on what others have said
   - “I agree with that because ....”
   - “Yes and I also think that ....”

6. Demand good explanations
   - “Why do you say that?”
   - “Go on ... convinced me.”

7. Challenge what is said
   - “That cannot be right, because...”
   - “This explanation isn’t good enough yet.”

8. Treat opinions with respect
   - “That is an interesting point.”
   - “We all make mistakes!”

9. Share responsibility
   - “Let’s make sure that we are all able to report this back to the whole class.”

10. Reach agreement
    - “We’ve got the general idea, but we need to agree on how we will present it.”
Activity E: Managing collaborative discussion

Time needed: 15 minutes.

A well-organized discussion lesson often has a number of distinct phases:

- **Individual ‘thinking time.**
  Students think about the problem before the discussion begins. Without this opportunity, the discussion is likely to be dominated by more confident and assertive students.

- **Small group discussion.**
  Students share and refine their ideas in small groups.

- **Whole class discussion.**
  Students ‘report back’ to the class, and share their ideas with a wider audience.

In this session, participants consider these phases and in particular the teacher’s role in each one.

How would you manage a discussion lesson?

- What is the purpose of ‘thinking time’? What is your role?
- What is the purpose of the small group discussion? What is your role?
- What is the purpose of the final whole class discussion? What is your role?

Compare your thoughts with those given on **Handouts 6 and 7.**

Many teachers, used to transmission methods, appear unsure of their role during discussion lessons. When students are struggling, some quickly intervene and try to ‘ease the path’ by giving strong hints and explanations. Others withdraw and offer little help, as though they now expect students to discover everything by themselves. The most effective teachers take neither of these positions. They challenge students to think more deeply, explain and justify (**Handout 6**):

- Make the purpose of the task clear.
- Keep reinforcing the ‘ground rules’.
- Listen before intervening.
- Join in, don’t judge.
- Ask students to describe, explain and interpret.
- Make students do the thinking.
- Don’t be afraid of leaving discussions unresolved.

Later in the lesson, when students have had an opportunity to think and discuss for themselves, a whole class discussion may be held in order to:

- present and report students’ discussions;
- recognize and value the important ideas that have emerged;
- generalize and link these ideas to other situations.

During this phase of the lesson, the teacher’s role is to chair the discussion and help students to clarify their own thinking.
Handout 6: The teacher’s role during small group discussion

Make the purpose of the task clear
Explain what the task is and how they should work on it. Also, explain why they should work in this way. “Don’t rush, take your time. The answers are not the focus here. It’s the reasons for those answers that are important. You don’t have to finish, but you do have to be able to explain something to the rest of the class.”

Keep reinforcing the ‘ground rules’
Try to ensure that students remember the ground rules that were discussed at the beginning. Encourage students to develop a responsibility for each other’s understanding. “I will pick one of you to explain this to the whole class later – so make sure all of you understand it.”

Listen before intervening
When approaching a group, stand back and listen to the discussion before intervening. It is all too easy to interrupt a group with a predetermined agenda, diverting their attention from the ideas they are discussing. This is not only annoying and disruptive (for the group), it also prevents students from concentrating.

Join in, don’t judge
Try to join in as an equal member of the group rather than an authority figure. When teachers adopt judgmental roles, students tend to try to ‘guess what’s in the teacher’s head’ rather than try to think for themselves: “Do you want us to say what we think, or what we think you want us to say?”

Ask students to describe, explain and interpret
The purpose of an intervention is to increase the depth of reflective thought. Challenge students to describe what they are doing (quite easy), to interpret something (“Can you show us why you say that?”).  

Make students do the thinking
Many students are experts at making their teachers do the work. They know that if they ‘play dumb’ long enough, then the teacher will eventually take over. They cannot explain something, ask another student in the group to do so. Students Working Collaboratively

Don’t be afraid of leaving discussions unresolved
Some teachers like to resolve discussions before they leave the group. When the teacher leads the group to the right answer, students are left with nothing to think about, or they go on to a new idea. They know there is no point in continuing to think about something (“why do you need to say that?”). Students Working Collaboratively

Do you want us to say what we think, or what we think you want us to say?

The final whole class discussion is for...

<table>
<thead>
<tr>
<th>Presenting and Reporting</th>
<th>Students may be asked to describe something they have done, an answer they have obtained and their method for obtaining it, or to explain something they have learned. Their ideas may be compared and evaluated by the whole class.</th>
</tr>
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<tbody>
<tr>
<td>Recognizing and Valuing</td>
<td>Some of the ideas generated in the discussion will be more important and significant than others. It is the teacher’s role to recognize these ‘big ideas’, make them the focus of attention and give them status and value.</td>
</tr>
<tr>
<td>Generalizing and linking</td>
<td>This involves showing how the ideas generated in the session may be developed and used in other situations. Learning is thus put into a wider context.</td>
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The teacher’s role is to...

Mainly be a ‘Chairperson’ or ‘Facilitator’ who:
- Directs the flow of the discussion and gives everyone a chance to participate.
- Does not interrupt or allow others to interrupt the speaker.
- Values everyone’s opinion and does not push his or her point of view.
- Helps learners to clarify their own ideas in their own words.

Occasionally be a ‘Questioner’ or ‘Challenger’ who:
- Introduces a new idea when the discussion is flagging.
- Follows up a point of view.
- Plays devil’s advocate.
- Focuses on an important concept.
- Asks provocative questions, but not ‘leading’, or ‘closed’ questions.

Don’t be a ‘Judge’ or ‘Evaluator’ who:
- Assesses every response with a ‘yes’, ‘good’ or ‘interesting’, etc.
- This tends to prevent others from contributing alternative ideas, and encourages externally acceptable performances rather than exploratory dialogue.
- Sums up prematurely.

Handouts 6 & 7

Handouts for Teachers

Students Working Collaboratively
Activity F: Observe and analyze a Discussion lesson

Time needed: 20 minutes.

An 11 minute video clip is provided. This shows one class of secondary students engaging in a discussion lesson. Teachers may enjoy watching the lesson and discussing many of the issues raised in this professional development unit.

**Video:** ‘How many school teachers?’ shows one teacher, Eve, teaching a class using the problem:

*Estimate how many teachers there are in the UK. The UK has a population of 60 million people.*

Watch the video clip, and then consider the following issues:

- How does the teacher introduce the problem?
- Which 'ground rules' does she emphasize?
- What different approaches are being used by students?
- How does the teacher help students to discuss productively?
- Can you characterize the types of talk they are using? (Refer again to **Handout 3**)

The video clip shows the three phases of the lesson described in Activity E.

**Handout 3.**

**Handout 3: Characteristics of helpful and unhelpful talk**

What types of talk engages students, develops understanding and promotes deeper thinking? Robin Alexander (2006) identified the following five principles of helpful classroom talk — which he terms dialogic:

*Dialogic talk is:*
- **Collective:** teachers and children address learning tasks together, as a group or as a class, rather than in isolation.
- **Responsive:** teachers and children listen to each other, share ideas and consider alternative viewpoints.
- **Cumulative:** teachers and children build on their own and each others’ ideas and chain them into coherent lines of thinking and enquiry.
- **Supportive:** children articulate their ideas freely, without fear of embarrassment over ‘wrong’ answers and they help each other to reach common understandings.
- **Purposeful:** teachers plan and facilitate dialogic teaching with particular educational goals in view.

Neil Mercer (1995, 2000) identifies the following three types of student-to-student talk. It is the third type, exploratory talk, that is most helpful for learning:

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<td>Speakers work on and elaborate each other’s reasoning in a collaborative, rather than competitive, atmosphere. Exploratory talk enables reasoning to become audible and knowledge becomes publicly accountable. It is characterised by critical and constructive exchanges. Challenges are justified and alternative ideas are offered.</td>
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Activity G: Plan a lesson, teach it and reflect on the outcomes

Time needed:
• 15 minutes discussion before the lesson
• 1 hour for the lesson
• 15 minutes after the lesson

Plan a discussion lesson for your students.
As you do this, discuss the following issues:

• Which task will you use? It should have scope for discussion.
• How will you arrange the room?
• How will you group students?
• How will you introduce the purpose of discussion?
• How will you establish ground rules?

Some notes on these issues are given on Handout 8.

• How will you organize the phases of the session?
  When will students be working individually?
  When will they be in small groups?
  When will there be a whole class discussion?

Refer back to Handouts 6 and 7 to support your planning.

Encourage teachers to record and transcribe some student-student discussions, if possible.
After teachers have taught the lesson, meet again and discuss the following questions.

Compare the different lessons that have been experienced.
How did you:

• Organize the room and introduce the task?
• Orchestrate and sustain phases of the work? (E.g. ‘Think, pair, share’)
• Organize and share ideas as a whole class?

Give examples of helpful and unhelpful discussions that were evident.
Play any recorded extracts of student-student talk from your lesson and discuss the type of talk this illustrates.

• Did students listen to and build on each others’ reasoning?
• Can you think of occasions when your own intervention was helpful?
  When was it unhelpful?
**Handout 8: Planning a lesson**

### Plan to offer the task in a form that will encourage collaboration

Prepare shared tasks in a form that will encourage discussion. For example:
- Provide resources to share (e.g. one copy between three) and ask for outputs that are jointly produced.
- Provide big resources so that reasoning may be visible and shared, such as large sheets of paper, felt-tipped pens or ‘mini-whiteboards’.
- Require joint outcomes: e.g. a poster or a report. Make students share responsibility for this.

### Plan how you will arrange the room

Arrange tables and chairs so that students are facing each other while working together. When computers are used, then pair two students to a computer and give them space and resources to record their joint thinking (e.g. using mini-whiteboards). Encourage turn taking when using the computer.

### Plan how you will group students

Most students are more able to discuss in smaller groups than larger ones: pairs or threes is often most effective.
- Some teachers find a snowball approach helpful:
  - Students first tackle the task individually. They have time to think before they are asked to discuss.
  - Pairs are then formed and students are asked to try and reach agreement.
  - Pairs then join together so that a broader consensus might be reached.
  - Groups of four then report back to the whole class in a plenary discussion.

### Plan how you will introduce the purpose of discussing

Plan your introduction to pre-empt the questions:
- “Why do you want us to discuss?”
- “What do you want us to discuss?”

For example:
*This lesson is not about ‘me showing you a method and then you using it’. No, I want to see if you can find your own methods. There is more than one way of doing this! I want you to discuss your own ideas for starting on this problem.*

### Plan how you will establish ground rules

Introduce ground rules for students. New habits are not established overnight, but over a long time through consistent reinforcement.