An Introduction to the

Teaching for Robust Understanding (TRU) Framework

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An Introduction to the

Teaching for Robust Understanding (TRU) Framework

Overview

This document introduces the **T**eaching for **R**obust **U**nderstanding (**TRU**) framework. TRU provides a research-based response to the question,

> "What are the attributes of equitable and robust learning environments – environments in which all students are supported in becoming knowledgeable, flexible, and resourceful disciplinary thinkers?"

The answer, which resonates with what we know as teachers and researchers, appears in distilled form in Figure 1. The quality of a learning environment depends on the extent to which it provides opportunities for students along the following five dimensions:

(1) The richness of disciplinary concepts and practices ("the content") available for learning;
(2) Student sense-making and "productive struggle";
(3) Meaningful and equitable access to concepts and practices for *all* students;
(4) Means for constructing positive disciplinary identities through presenting, discussing and refining ideas; and
(5) The responsiveness of the environment to student thinking.

The Five Dimensions of Powerful Classrooms

The Content	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.	*The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Figure 1. The five dimensions of powerful classrooms.

What you'll find in this document

The Teaching for Robust Understanding (TRU) community[1] has developed a collection of tools for teachers, coaches, administrators to support the improvement of teaching, and tools and papers to support researchers in developing deeper understandings of teaching and how to enrich it. This document introduces the framework and some of its key tools. Most of the tools are available in mathematics-specific and domain-general versions. Complete documentation and all TRU tools will soon be available at http://map.mathshell.org/trumath.php and http://tru.berkeley.edu.

We begin with a discussion of the intentions behind TRU. This is followed by brief descriptions of each of its five dimensions. These, in turn, are followed by the annotated exemplification of the documents in the TRU document suite. The descriptions should provide readers enough of a sense of the framework, and each of the tools, to determine which tools are most useful for their own purposes.

Where did these ideas come from and how can they be useful?

There are huge literatures on teaching and learning, and on "good things" that should happen in classrooms. There is also a wide range of frameworks for observing classrooms. The motivation for TRU was to organize that knowledge so that it is easily understood, organized, and used[2]. The five focal dimensions of TRU have the following properties:

1. They are comprehensive. If a learning environment supports student learning along these dimensions, then the students who emerge from that environment will be knowledgeable, flexible, and resourceful thinkers and learners.
2. Each dimension can be the focus of coherent professional development. Departments, schools, and districts can organize themselves in ways to make systematic improvements.
3. Together, they provide a language and a framework for inquiring into instruction and improving it – not a set of "recipes" telling teachers what they should do.

TRU contains no "thou shalts," in that it does not prescribe what should happen in the classroom – there are many different ways for teachers to create powerful learning environments and no one "right way" to teach. The key idea is that TRU specifies the attributes of learning environments in which students flourish.

[1] Work on TRU began with projects at the University of California, Berkeley, Michigan State University, and the University of Nottingham. Our partners now include research teams at the SERP Institute and Mills College, and school districts such as Chicago, Oakland, and San Francisco, as well as networks of school districts and professional organizations across California and beyond. Our products are available to teachers, administrators, and researchers for non-commercial use at no cost.

[2] For the history and some of the details, see Schoenfeld, 2013, 2014, 2015; Schoenfeld, Floden, and the Algebra Teaching Study and Mathematics Assessment Project, submitted.

Classroom instruction, no matter how powerful, can always be enriched. It will be enhanced if, as a matter of routine and habit, teachers, coaches, and administrators take the five dimensions of TRU into account when planning, implementing, and reflecting on instruction[3].

Improving teaching is not easy, but knowing what to focus on can be a big help. Engaging in systematic and collaborative reflection on: disciplinary ideas; on ways to open up those ideas to students; on ways to provide *all* students opportunities for sense making; on providing opportunities for students to express their understandings and build on their own ideas and those of others; and to adjust instruction in the light of the understandings that students reveal, will result in the ongoing enhancement of instruction. Professional learning communities that focus on what counts will produce sustained improvement in teaching and in student understanding.

[3] Making a practice of reviewing what counts can result in significant improvements. For example, Gawande (2007, 2009) has shown that checklists that remind doctors and nurses of things they know they should be doing result in significant improvements in hospital recovery and mortality rates. If reminders to wash one's hands before interacting with patients can improve medical results, then it stands to reason that instruction can be enhanced by routinely asking (for example) where in a lesson students have opportunities to engage in sense making at an appropriate level of cognitive demand.

The Five Dimensions of TRU

Dimension 1: The Content

The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.

Students' understanding of a discipline is shaped in fundamental ways by their classroom experience of it. If, for example, a reading class is focused on decoding text, a history class on memorizing dates of major events, or a mathematics class on memorizing procedures, there is little chance that the students in that class will emerge from it with either an appreciation of the discipline or the understandings they need.

Learning to "think like a historian," or like a scientist, or a practitioner of any discipline, means coming to grips with the concepts and practices of that discipline – approaching phenomena through a disciplinary lens, with a broad spectrum of knowledge and tools at one's disposal. Historians "place themselves in context" to understand the motivations and actions of historical figures. Writers having a sense of purpose and a sense of audience when writing, as well as relevant factual and grammatical knowledge. Scientists and mathematicians inquire into "what makes things tick," using reason, equations, representations, and models in the service of sense making. This combination of disciplinary orientations, knowledge (including concepts and tools), practices and habits of mind is what we refer to in shorthand as the "content" of the discipline. Students need to experience that content in its full richness if they are to become disciplinary thinkers.

Every major discipline has produced one or more sets of standards – statements regarding the essential understandings that students should develop. This is not the place to review such documents, but simply to note that if the activities in a classroom do not live up to the relevant disciplinary standards, it is hard to imagine that the students who emerge from that classroom will have a deep sense of the discipline or be able to use their knowledge effectively.

The tools section of this document provides descriptions of tools such as the TRU Conversation and Observation Guides, which offer ways to inquire into and reflect on the richness of the disciplinary content offered to students.

Rich content, however, is just a beginning. The primary idea behind TRU is that what counts in instruction is how students encounter the content – how they are or are not positioned to take advantage of the riches the discipline has to offer. We have all been in classes where, for example, the content was "over our heads" or we failed to connect to it for some reason; no matter how beautiful it may have been, we were lost. That is why dimensions 2 through 5 of the TRU framework – how students themselves experience the discipline – are so important.

Dimension 2: Cognitive Demand

The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."

If students are given work that is too easy, there is little for them to learn – and, they are likely to be bored or frustrated. If students are given work that is too distant from their current understandings and they can see no pathways to progress, then there is no pathway to learning; they are likely to be bored or frustrated as well. As Stein and Smith (1998) put it, "Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking." The challenge is to find tasks and classroom activities, in every discipline, that are framed in ways that provide students with meaningful opportunities for learning and that support their growth through active engagement with the content.

Researchers use the term "cognitive demand" to describe the level of difficulty, relative to what they know, of the work that students are asked to engage in. The goal is to find a middle ground, where students have opportunities to build on what they know and stretch their current understandings. In order to make sense of rich content, students need to engage in "productive struggle" (Stein and Smith, 1998; Hess, 2006). One broad schema for thinking about different levels of challenge is Webb's (1997, 2002) Depth-of-Knowledge (DOK) framework, which identifies four levels of DOK: Recall & Reproduction, Skills & Concepts, Strategic Thinking & Reasoning, Extended Thinking (see also Hess, 2013). At various times, students need to engage at all of these levels.

When students experience difficulty dealing with complex issues there is a tendency for teachers to reduce cognitive demand, and thus to deprive the students of opportunities for productive struggle and sense making (Henningsen and Stein, 1997). The challenge for instruction in all disciplines is to provide clarifications and other support (e.g., heuristic advice, raising issues, suggesting approaches) without telling students precisely what to do. This is by no means easy (but see Dimension 5, *formative assessment*).

There are many ways that teachers can initiate cognitively demanding activities in the classroom, and work to maintain appropriate levels of cognitive demand. For example,

- In designing and selecting tasks, teachers can avoid providing detailed step-by-step instructions for solving problems, repetitive exercises, or detailed "recipes" for completing tasks that allow little room for students to build on their current understandings.
- Teachers can actively support students in individual work, group work, and whole class discussions by asking clarifying questions and providing scaffolds, instead of moving directly to suggesting overly specific ways to go about assigned tasks.

- Teachers can employ a range of techniques to support students in “getting their ideas on the table” and working through them. See, for example, SERP (2016) on academically productive talk.
- Teachers can encourage students’ productive struggle in a general way by discussing ideas of malleable intelligence and a growth mindset (Dweck, 2007), making it clear that learning is not a matter of memorization, and that one gets better at any discipline by working hard at it.

See the “tools” section of this document for access to resources related to cognitive demand.

Here we note briefly connections to the other dimensions of TRU. As noted above, “productive struggle” is the mechanism for developing deep understanding of the content (Dimension 1). It is essential for all students (Dimension 3), not only for meaningful participation but so that students engage with the content in ways that they come to “own” it and develop positive disciplinary identities (Dimension 4). And, the best way to arrange for students to be working at the right levels of challenge is to make their thinking publicly accessible, so instruction can “meet them where they are” in order to support their moving forward (Dimension 5).

Dimension 3: Equitable Access to Content

The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.

Equitable classrooms provide *all* students access to meaningful disciplinary concepts and practices, supporting those students in developing their own understandings and building productive disciplinary identities.

This dimension, *Equitable Access to Content,* focuses on the question of whether, within the classroom, there is differential access to the content being addressed. There may be rich discussions or other productive activities taking place – but, who participates in those discussions or activities?

There is a long history of differential achievement by students from varied racial, ethnic, and economic backgrounds, which, it has been argued, can be tied to differential access to opportunities to learn (Oakes, Joseph, & Muir, 2001). While one obvious source of this differential access is tracking, which is outside of the scope of a classroom improvement efforts, another is the pattern of discourse within classrooms. Do all students have frequent opportunities to discuss important ideas? In *How Schools Shortchange Girls* (American Association of University Women, 1992), for example, research revealed a pattern of boys being called upon far more often than girls. Moreover, when girls were called upon, they were often asked questions that were less conceptually oriented than the questions that were asked of boys. What opportunities do English language learners have, or students from differing demographic or racial groups? Do multiple opportunities exist for students to engage with the content, to develop and display competence (Cohen 1994), and to build understanding based on the knowledge they bring with them into the classroom (see, e.g., Moll, Amanti, Neff, & Gonzalez, 1992)?

Research indicates that effective teachers encourage participation by all students in the intellectual community of the classroom (Boaler, 2008; Cohen & Lotan, 1997; Schoenfeld, 2002). They select and utilize tasks that enable all students to engage in challenging content, and they establish and reinforce expectations for various ways to participate in and contribute to classroom activities.

There are numerous ways in which students can be supported in access to disciplinary content and practices.

- In choosing and designing activities, and in launching activities, teachers can provide multiple access points to the relevant material, supporting the expectation that *all students* are able and expected to participate.

- Tasks that can be approached in multiple ways or from multiple perspectives, and in which approaches can be compared and contrasted, provide access to students who choose different pathways into the activity. In addition, they provide opportunities for making connections between student approaches.

- Teachers can encourage the generation and refinement of ideas rather than mainly critiquing or ignoring comments that are only partially correct.

- Teachers can support the use of multiple language registers by, for example, asking one student to restate another's contribution in more precise academic language, or, perhaps, in more informal language.

- During discussions, teachers can use a variety of strategies to encourage broad participation, for example: choosing to call only on students who have not yet spoken; allowing time to talk to a partner before responding publicly; and randomly selecting students to contribute.

- Teachers can use tasks with language and contexts that connect to students' lived experiences and provide windows into unfamiliar experiences, being mindful of power and privilege.

See the tools sections of this document for a deeper discussion and pointers to further resources.

Dimension 4: Agency, Ownership, and Identity

The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.

Dimension 4 focuses on the extent to which students have the opportunity to generate and share ideas, both in whole class and small group settings; the extent to which student contributions are encouraged, recognized and supported as part of regular classroom activity; and the extent to which student ideas are built upon as the classroom constructs its collective understandings.

People's dispositions and identities – e.g., "I am a reader," or "I'm just not a history person," – are derived from experiences with the discipline. Such dispositions and identities, often formed in the classroom, shape the ways in which people relate to the discipline for the rest of their lives. Many students develop counterproductive beliefs about themselves and a discipline, e.g., that they are "bad at science," or that history has nothing to do with contemporary events, or that only geniuses can create mathematics. But it need not be this way.

One fundamental aspect of disciplinary identity is *agency* – an individual's willingness to engage in the discipline, which comes from the perception that she or he make can progress on challenging issues by working away at them, and trust in the conclusions that he or she draws. Engle (2011) writes,

> Learners have intellectual agency when they … share what they actually think about the problem in focus rather than feeling the need to come up with a response that they may or may not believe in, but that matches what some other authority like a teacher or textbook would say is correct.

Ownership refers to the sense that one has control of disciplinary ideas, rather than parroting or memorizing those of others. It is the difference between saying "I've reasoned this through and I'm confident it makes sense" and relying on external authority.

A key issue is the extent to which a learning environment provides students with opportunities to develop these aspects of their disciplinary and personal identities. Effective teachers recognize and capitalize on the strengths of each student, finding ways to help individual students enter into the learning community when they do not easily enter it on their own (Boaler, 2008; Cohen & Lotan, 1997). There are multiple ways to do this. Teachers can create opportunities for public recognition of students' contributions to disciplinary discussions, help students work together in small groups, and attend to students who are struggling by building on the strengths in their thinking. For example, Resnick, O'Connor, and Michaels (2007) identify powerful talk moves by teachers such as revoicing (repeating, paraphrasing, or summarizing a student contribution for the whole class to react), asking students to restate others' reasoning, to build on what other students have said, and prompting for explanations.

Above and beyond teacher moves, however, is the very nature of the classroom environment. Do students feel safe making contributions to classroom conversations? Have norms been established for making contributions? For building on contributions from others? For critiquing contributions from others?

There is a large literature on "accountable talk," the kind of classroom discourse that supports students in responsibly and respectfully co-constructing ideas. For a large portfolio of resources, see, Institute for Learning (2016).

To give one example, a technique for shaping classroom discourse productively is the use of "sentence stems" aimed at promoting accountable talk:

- I disagree (or agree) with that, because ______
- I still have questions about ______
- This is the same, because ______
- I observed ______
- I'm confused by ______
- To expand on what ______ said ______

  (see http://www.ces.rcs.k12.tn.us/web_uploads/203_accountable_talk_toolkit_10-09.pdf)

Only in climates where students feel comfortable contributing to the development of disciplinary ideas will they have opportunities to develop a sense of academic and disciplinary agency, ownership of the ideas discussed, and positive disciplinary identities.

See the tools sections of this document for further discussion.

Dimension 5: Formative Assessment

The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.

Formative assessment involves orchestrating classroom activities that reveal the current state of student understanding *during the learning process*. Revealing the ways in which students are making sense of the content as they learn provides the teacher and the students opportunities to build upon the understandings that students have developed, and to address emerging misunderstandings. Formative assessment may involve quizzes or tests, but it involves much more. It often includes informal information gathering, e.g., posing questions that may bring out into the open incorrect assumptions or ideas that need to be challenged, or that help students realize that they need to dig more deeply into the content. The use of formative assessment contrasts strongly with the use of summative assessment – the formal end-of-unit or end-of-year tests that can reveal what students know and can do, but provide that information too late for it to be useful in helping students develop deeper understandings as they are learning.

In formative assessment, the information gathered about student reasoning and understanding gathered plays a major role in shaping the classroom activities that follow (Black et al., 2003; Shepard, 2000). This may seem challenging at first – who knows what students will say, given the chance? – but it is essential in order to meet students where they are. Once one starts providing students opportunities for students to engage openly in the discipline, it becomes an easily sustained habit. There are large literatures on student misconceptions, or "alternative conceptions," that document the kinds of partial understandings students typically develop in specific content areas. Knowing about these typical patterns of student reasoning can help teachers to be prepared to deal with them[4]. For more detail in mathematics, see the Formative Assessment Lessons described in the tools section of this document.

Through deliberately attending to student reasoning and understanding, and then shaping instruction in response, teaching "becomes clearer, more focused, and more effective" (National Research Council, 2001, p.350). In addition, hearing student reasoning provides the information that allows teachers to adjust the level of cognitive demand, so that students are positioned to engage in meaningful sense making. That is, Dimension 5 (formative assessment) provides the support structure for Dimension 2 (cognitive demand).

Black and Wiliam's (1998 a,b) widely cited reviews document the substantial learning gains that result from teachers' use of formative assessment. When assessment becomes an integral and ongoing part

[4] Of course, teachers develop such knowledge (part of what is called "pedagogical content knowledge") over time. The point here is that the process can be accelerated if one goas about it self-consciously.

of the learning process, as opposed to an interruption of classroom activities, student thinking takes on a more central role in determining the direction and shape of classroom activities (Shepard, 2000; Shafer & Romberg, 1999; de Lange, 1999).

Of course, formative assessment is only useful if there is something interesting and important to assess – namely, meaningful disciplinary understandings and the ability to apply those understandings in powerful ways. Thus, Dimension 1 (the content) is implicated in establishing the disciplinary context for Dimension 5 (formative assessment).

In every discipline, multiple cycles of writing (pre-writes, outlines, drafts, revised drafts, etc.) provide students with opportunities to refine their ideas *and* to improve their writing. Sharing and critiquing ideas with other students places all students within a zone of productive thinking, as well as providing opportunities for the refinement and ownership of ideas. Thus, effective formative assessment (and the use of classroom structures to support student interactions) supports the right levels of cognitive demand for students (Dimension 2) and opens up opportunities for the development of student voice (Dimension 4). If supportive classroom norms are established, and the tasks have multiple entry points (which supports rich disciplinary conversations), then the major goal of equitable access (Dimension 3) is served as well.

It is important to note that the teacher is not necessarily responsible for addressing all of the issues that emerge when student thinking is solicited. When groups or the whole class work on rich tasks, students can serve as powerful resources for each other, in eliciting and building on each other's thinking. The Mathematics Assessment Project's Formative Assessment Lessons, described in the next section of this document, provide numerous examples of how this can be done, with tasks that invite student collaboration and critique.

Tools for the Teaching and Administrative Communities

The extended TRU community has developed multiple tools for understanding and supporting teaching. What follows is a sampler, providing descriptions of the following tools:

- The TRU Conversation Guides, in domain-general and mathematics-specific versions.
- The TRU Observation Guides, in domain-general and mathematics-specific versions.
- The Mathematics Assessment Project's Formative Assessment Lessons, Summative Assessment Tasks and Tests, and draft PD modules.
- The Mathematics Network of Improvement Communities (MathNIC) tools for building supportive instructional and administrative communities.

All of these documents are or will be available at http://map.mathshell.org/trumath.php and http://tru.berkeley.edu. In the short run, we point to temporary sites housing work in progress. Much of our descriptive language is taken from the documents themselves.

The TRU Domain-General and Mathematics-Specific Conversation Guides

The purpose of the TRU domain-general and mathematics-specific Conversation Guides (Baldinger, Louie, & the Algebra Teaching Study and Mathematics Assessment Project, 2016; Louie, Baldinger, & the Algebra Teaching Study and Mathematics Assessment Project, 2016) is to facilitate coherent and ongoing discussions in which teachers, administrators, coaches, and others learn together. We hope that the questions in the Conversation Guide will support educators with different experiences, different expertise, and different strengths to work together to develop a common vision, common priorities, and common language, in order to collaboratively improve instruction and better support students to develop robust understandings.

The Conversation Guides can be used to support many different kinds of conversations, including (but not limited to):

- Conversations to develop common vision and priorities across groups of educators (such as subject-matter departments, grade-level teams, or an entire school faculty)
- Conversations between teachers and administrators and instructional coaches around classroom observations (see also the TRU Observation Guides)
- Conversations between teachers around peer observations
- Conversations around video recordings of classroom teaching and learning
- Conversations about planning a particular unit or lesson
- Conversations about a particular instructional strategy or set of strategies (not necessarily content-specific)
- Ongoing individual reflection

There are two guides, one domain-general and one specific to mathematics[5]. Each guide describes the purposes of the guide (as above) and how it might be used – both for planning lessons and for reflection on them. The body of each guide focuses on reflective questions. Figure 2 reproduces the content page from the domain general conversation guide.

[5] We hope, over time, to work in partnership with teachers and researchhers in a range of disciplines to build “custom” conversation guides for those disciplines as well.

The Content

Core Questions: How do key disciplinary ideas and practices develop in this lesson/lesson sequence? How can we create more meaningful connections?

Students often experience schooling as a presentation of isolated facts, procedures and concepts that they are to rehearse, memorize, and apply. Our goal is to instead give students opportunities to experience coherent and meaningful disciplinary ideas and practices. This means identifying the important ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It means engaging students with centrally important disciplinary ideas in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding. It also means engaging students in authentic performances of important disciplinary practices (e.g., reasoning from evidence, communicating one's thinking to various audiences, etc.).

Planning

How will important disciplinary ideas and practices develop in this lesson and unit? How can we connect the ideas and practices that have surfaced in recent lessons to future lessons?

Reflecting

How have we seen students engage with important disciplinary ideas and practices? How has this engagement looked and sounded in specific cases?

Things to think about

- What are the content goals for the lesson?
- What connections exist (or could exist) between important ideas in this lesson and important ideas in past and future lessons?
- How do important disciplinary practices develop in this lesson/unit?
- How are facts and procedures in the lesson justified?
- How are facts and procedures in the lesson connected with important ideas and practices?
- How do we see/hear students engage with important ideas and practices during class?
- Which students get to engage deeply with important ideas and practices?
- How can we create opportunities for more students to engage more deeply with important ideas and practices?

Figure 2. The content page from the domain-general *Conversation Guide.*

The Conversation Guides have been widely adopted across the U.S. For one example of impact, see Brownell, Mahon, and Seward's (2016) description of the results in the Chicago public schools.

The TRU Math Conversation Guide Module A: Contextual Algebraic Tasks

Each of the Conversation Guides discussed above covers a huge amount of territory – the strategies for inquiring into one's own teaching in the Guides were designed to be relevant at all grade levels. Clearly, however, the specifics of instruction, and thus instructional goals, vary from year to year or topic to topic. Thus, ultimately, one would hope to create conversation guides at a much finer grain size than those discussed above. As a case in point the Algebra Teaching Study (ATS) team constructed one such guide, for addressing contextual algebraic tasks ("story problems") in introductory algebra.

The algebra-specific conversation guide (Wernet & Lepak, 2014) identifies five "robustness criteria" (aspects of understanding necessary for a robust understanding of algebra):

RC 1: Reading and interpreting text, and understanding the contexts described in problem statements.

RC 2: Identifying important quantities and the relationships between them.

RC 3: Using algebraic representations of relationships between quantities.

RC 4: Performing calculations and procedures with precision and checking the plausibility of results.

RC 5: Providing convincing explanations that give further insight into the depth of students' algebraic thinking.

The conversation guide offers specifics for each of these – e.g., for Robustness Criterion 3, *using algebraic representations of relationships between quantities*, the list of "things to think about" is given in Figure 3.

Representations of Relationships Between Quantities

Think about:

- Opportunities students have to generate and interpret algebraic representations.
- Opportunities students have to discuss how the algebraic representations show the relationship between important quantities in the task and how those relationships can be generalized to a family of functions. (For example, how change in a dependent variable is revealed in a table, graph, or equation.)
- Which aspects of representations (for example, specific points in a table, or more global patterns of change in a graph, or parameters in an equation) are important for students to attend to, and how students are supported to attend to them.
- Opportunities for students to consider the affordances of algebraic representations used and why certain representations are appropriate in solving the task, and opportunities to choose appropriate representations.
- How students explore connections between representations.
- How students explain their reasoning about their algebraic representations.
- Evidence indicating students effectively used representations to model and solve the problem, and how different students across the class engaged with these ideas.

Figure 3: "Things to think about" Related to RC 3, from the CG module on Contextual Algebraic Tasks

One can imagine constructing such guides for varied topics in mathematics (e.g., "reasoning and proof"), for writing in various genres in English Language Arts, and so on. Our hope is that, over time, analogous modules will be constructed for core disciplinary themes in all subject domains.

The TRU Domain-General and Mathematics-Specific Observation Guides

The TRU Observation Guides, also available in domain general and mathematics-specific versions (Schoenfeld, A. H., and the Teaching for Robust Understanding Project, 2016 a,b), were specifically designed to support planning for, conducting, and debriefing classroom observations.

The primary idea behind TRU is that what counts in instruction is how the student experiences the content. This is summarized in figure 4, which highlights the key aspects of instruction from the student point of view.

Observe the Lesson Through a Student's Eyes	
The Content	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored? In what ways am I kept engaged?
Agency, Ownership, and Identity	• What opportunities do I have to explain my ideas? In what ways are they built on? • How am I recognized as being capable and able to contribute?
Formative Assessment	• How is my thinking included in classroom discussions? • Does instruction respond to my ideas and help me think more deeply?

Figure 4. Observing a lesson from the student perspective

The observation guides adopt this perspective. As with the conversation guides, the intention is to support collaborative conversations. Prior to an observation, teacher and observer discuss the lesson plan and decide on the main points of focus for the observation. The observation might be comprehensive – it is possible for a practiced observer to take notes on all five dimensions. Alternatively, the teacher and observer might agree to focus on one or two areas the teacher wants to address in detail.

The form of the observation guide and its use are straightforward. Each observation sheet focuses on one dimension of the framework, and is one page long. That sheet summarizes key aspects of the dimension and goals for students, along with a sample list of "look fors" (signs that things are going well) for students and the teacher. There is room to tailor the observations to the specific lesson, and space for note-taking. Figure 5 shows the observation for cognitive demand taken from the domain-general Observation Guide.

<table>
<tr><th colspan="2">COGNITIVE DEMAND
The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to every student's deepening understanding of disciplinary content and practices. We seek "productive struggle."</th></tr>
<tr><td>Each student…
• Engages individually and collaboratively with challenging ideas
• Actively seeks to explore the limits of their current understandings
• Works to build productive disciplinary habits of mind
• Reasons and tests ideas in ways that connect to and build on what they know
• Explains what they have done so far before asking for help
• Continues to wrestle with an idea after the teacher leaves</td><td>Teachers…
• Position students as sense makers who can make sense of key conceptual ideas.
• Use or adapt materials and activities to offer challenges that students can use, individually or collectively, to deepen understandings
• Build and maintain classroom norms that support every student's engagement with those materials and activities
• Monitor student challenge, adjusting tasks, activities, and discussions so that all students are engaged in productive struggle
• Supports students without removing the challenge from what they are working on</td></tr>
<tr><td colspan="2">• Other focal points for observation:</td></tr>
<tr><td colspan="2">What opportunities do students have to make sense of disciplinary content and practices? How are they supported in sense making so that they are not lost – yet real challenge has been maintained, so that they have opportunities to grapple with important ideas?</td></tr>
<tr><td colspan="2">Goal: All students have opportunities to make their own sense of important ideas, developing deeper understandings, connections, and applications by building on what they know.</td></tr>
</table>

Figure 5. The "cognitive demand" sheet from the Observation Guide for Mathematics

The first version of the observation guide was created by the San Francisco Unified School district for its own use. The TRU team modified the document, with SFUSD's permission; the general version will be used for observations in all of U.C. Berkeley's teacher preparation programs.

Mathematics-Specific Tools from the Mathematics Assessment Project

The Mathematics Assessment Project, an ongoing partnership between the University of Nottingham and U.C. Berkeley, has produced a wide range of instructional support materials in synergy with the TRU project. The MAP website, http://map.mathshell.org/, houses the materials described in this section, in addition to housing TRU tools and papers.

Formative Assessment Lessons in Mathematics

Learning to conduct formative assessment – to provide opportunities for students to express their understandings, and to react in the moment in ways that "meets the students where they are" – is a significant challenge, especially for teachers whose primary experience is with the "demonstrate and practice" model of instruction. To support teachers in this endeavor, the Mathematics Assessment Project produced 100 2-3 hour Formative Assessment Lessons (FALs). The lessons were designed to have the following properties:

- The lessons focus on key mathematical concepts and practices in grades 6 through 10, with 20 lessons at each grade level.
- Each lesson can be "inserted" into the regular grade level curriculum, so that for particular topics they help teachers discover what their students have learned, and what challenges they face. They provide ways to address those challenges.
- The lessons – with lesson plans that extend to 20 pages to support the use of a pre-assessment and 2-3 hours of instruction – are aimed at helping teachers to:
    - Uncover some misconceptions by using the pre-assessment, and have time to think through the ways in which the main content of the lesson addresses them;
    - Be prepared for the main lesson with a list of "common issues" that the lesson will likely uncover, and ways to respond to those issues without simply re-teaching the content (e.g., by using questions that cause the students to consider a particular example that challenges their statement, or to look at a specific simpler case to see how things work);
    - Launch the main lesson in ways that (often contradictory!) student ideas are made public, so it becomes apparent to all that there are issues to resolve;
    - Lead a number of activities in which students build on each other's ideas (in making posters for presentation, etc.) as supported by the teacher; and
    - Bring the lesson to closure, with activities that expand on and solidify student learning.
- Perhaps most ambitious, the FALs are designed to support teachers in changing their pedagogy – the goal being that, having been scaffolded in teaching this new way with very carefully guided lessons, the teachers might open up their practice so that their "regular" lessons are taught differently. The FALs scaffold teaching in a way that is entirely consistent with the five dimensions of TRU.

The FALs are available from the MAP website, http://map.mathshell.org/. To date, there have been more than 6,000,000 FAL downloads. The Gates Foundation, which funded the project, also funded professional development projects, known as the Mathematics Design Collaborative (MDC) to support their implementation. An independent evaluation on the impact of MDC on 9th grade algebra students in Kentucky (Herman, Epstein, Leon, La Torre Matrundola, Reber, & Choi, 2014) describes the results:

> Participating teachers were expected to implement between four and six [FALs], meaning that students were engaged only 8-12 days of the school year. (p. 10)
>
> We used recently developed methodology to convert the observed effect size for MDC into a gross indicator of the number of months of learning represented. Relative to typical growth in mathematics from ninth to tenth grade, the effect size for MDC represents 4.6 months of schooling. (p. 9)

That is, the average student learning gains resulting from 8-12 days of instruction using FALs was 4.6 months. A report by Research for Action (2015) indicates why: the vast majority of teachers who used the FALs learned new strategies that they implemented in regular instruction.

Summative Assessment Tasks and Tests

Summative assessments need to be aligned with the desired outcomes of instruction. If “off the shelf” tests focus on skills and procedures, they will fail to demonstrate students’ problem solving skills and other impacts of robust instruction. This is at best is demoralizing; worse, since testing often drives instruction, using such tests may have a significant negative impact on instruction. Recognizing this, the Mathematics Assessment Project designed a series of “novice,” apprentice,” and “expert” tasks dealing with grade 6-10 mathematics. Novice tasks explicitly test particular items of content knowledge. Expert tasks are far less structured, requiring strategic problem solving skills in addition to content knowledge: in these tasks students face significant sense making demands and have great latitude with regard to the choice of methods. Apprentice tasks lie in-between the two. Their problem statements provide enough scaffolding so that students, while still required to do problem solving, are oriented in the direction of productive approaches. A “balanced diet” of Novice, Apprentice and Expert tasks is needed to properly assess the Mathematical Practices.

A collection of 94 summative tasks can be found at http://map.mathshell.org/tasks.php. Each task includes a scoring rubric, a set of pre-scored sample student work, and the same set of work without the scores. (Scoring practice for this type of task can be an effective professional development activity.) These tasks can be used whenever it seems appropriate, during the year. The MAP team has also compiled a number of end-of-year tests, with accompanying scoring rubrics. Those are available at http://map.mathshell.org/tests.php.

Draft PD modules

The Mathematics Assessment Project has developed five draft Professional Development Modules designed to help teachers with the practical and pedagogical challenges presented by the Formative Assessment Lessons.

Module 1 introduces the model of formative assessment used in the lessons. Modules 2 & 3 look at the two types of FALs (concept-oriented and problem solving) in detail. Modules 4 & 5 explore two crucial pedagogical features of the lessons: asking probing questions and collaborative learning.

The modules are activity-based, built around a collection of example classroom activities. The aim is to engage groups of teachers in constructive discussions about their own practices and how these could change. They then plan and teach a lesson using these ideas in their own classroom, and meet again to reflect on their experiences.

These are draft materials. The videos feature students in England working on projects with similar goals. The Mathematics Assessment Project hopes, ultimately, to be able to make videos showing the actual materials in US Middle and High schools. See http://map.mathshell.org/pd.php.

The Mathematics Network of Improvement Communities (MathNIC) Tools for Building Supportive Instructional and Administrative Communities

In support of both TRU and the FALs, The Gates Foundation funded a "Mathematics Network of Improvement Communities" (MathNIC). The idea was to bring together school districts and other partners to identify challenges they faced in creating and sustaining robust learning environments, and to craft tools to address those challenges. Alpha versions (one revision round) of tools created by this still ongoing project can be found at http://mathnic.mathshell.org.uk/wordpress/. The site offers an overview of the tools, as well as downloads. Caveat emptor: these are still being refined.

Papers and Tools for the Research Community

Papers

A series of papers describes the research underlying the tools and perspectives described in this document. Major project papers include:

Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM, the International Journal of Mathematics Education, 45: 6-7-621.* DOI 10.1007/s11858-012-0483-1.

This paper describes the genesis of the TRU framework. It explores the dialectic between theorizing teachers' decision-making and producing a workable, theoretically grounded scheme for classroom observations. One would think that a comprehensive theory of decision-making would provide the bases for a classroom observation scheme. It turns out, however, that, although the theoretical and practical enterprise are in many ways overlapping, the theoretical underpinnings for the observation scheme are sufficiently different (narrower in some ways and broader in others) and the constraints of almost real-time implementation so strong that the resulting analytic scheme is in many ways radically different from the theoretical framing that gave rise to it. This essay characterizes and reflects on the evolution of the observational scheme. It provides details of some of the failed attempts along the way, in order to document the complexities of constructing such schemes.

Schoenfeld, A. H. (2014, November). What makes for powerful classrooms, and how can we support teachers in creating them? *Educational Researcher, 43(8),* 404-412. DOI: 10.3102/0013189X1455

This article, and my career as an educational researcher, are grounded in two fundamental assumptions: (1) that research and practice can and should live in productive synergy, with each enhancing the other; and (2) that research focused on teaching and learning in a particular discipline can, if carefully framed, yield insights that have implications across a broad spectrum of disciplines. This article begins by describing in brief two bodies of work that exemplify these two fundamental assumptions. I then elaborate on a third example, the development of a new set of tools for understanding and supporting powerful mathematics classroom instruction (and by extension, powerful instruction across a wide range of disciplines) – the TRU framework. In doing so, this paper situates the corpus of work on TRU in a much larger R&D framework.

Schoenfeld, A.H. (2015). Thoughts on scale. *ZDM, the international journal of mathematics education*, *47*, 161-169. DOI: 10.1007/s11858-014-0662-3.

This essay reflects on the challenges of thinking about scale – of making sense of phenomena such as continuous professional development (CPD) at the system level, while holding on to detail at the finer grain size(s) of implementation. The stimuli for my reflections are three diverse studies of attempts at scale – an attempt to use ideas related to professional development in two different countries, the story of how research did or did not frame a nationwide attempt at undergirding CPD, and a fine-grained study of the quality of a dozen mentors' implementation of CPD. The challenge is to "see the forest for the trees," to be able to situate such diverse studies within a larger framework. The bulk of this article is devoted to offering such a framework, the Teaching for Robust Understanding (TRU) framework, which characterizes five fundamentally important

dimensions of powerful learning environments. At the most fine-grained level, TRU applies to classrooms, establishing goals for instruction. But, more generally, it applies to all learning environments, and thus characterizes important aspects of CPD. The paper addresses issues related to the kinds of systemic coherence necessary to make progress on professional development at scale.

Schoenfeld, A. H., Floden, R. B., and the Algebra Teaching Study and Mathematics Assessment Project. (2016) On Classroom Observations. Manuscript submitted for publication.

This article proposes desiderata for frameworks and rubrics used for observations of classroom practice, noting when particular criteria are more important for specific purposes (research, professional development, or evaluation) than others. It characterizes similarities, differences, and affordances of three observational frameworks: Framework for Teaching, Mathematical Quality of Instruction, and Teaching for Robust Understanding of Mathematics. It describes the ways that each framework assesses selected instances of mathematics instruction, documenting the ways in which the three frameworks agree and differ. Specifically, the frameworks do not agree on what counts as high quality instruction. These differences are consequential, given that such frameworks are widely used for professional development and for teacher evaluations.

Swan, M., & Burkhardt, G. H. (2014) Lesson Design for Formative Assessment. Educational Designer, Volume 2, Issue 7, available from http://www.educationaldesigner.org/ed/volume2/issue7/article24/index.htm

The potential power of formative assessment to enhance student learning is clear from research. This, however, demands a different learning culture and a broader range of teaching approaches than are found in most mathematics classrooms. Earlier efforts to introduce formative assessment for learning have focused on teacher professional development. Here we describe a major project that explores how this change may be stimulated and supported by teaching materials that embody the principles of formative assessment. We describe the design challenges we faced, the previous research and development experience we drew upon, and the principles that directed our designs. We illustrate these elements with examples of the products themselves, some outcomes and lessons learned.

Tools

The fundamental claim underlying the TRU Framework is that a classroom's performance on the five dimensions of TRU is positively related to its students' emergence as knowledgeable, flexible, and resourceful thinkers and problem solvers. On the one hand, the dimensions of TRU were derived from an exhaustive review of the literature and the examination of a large number of videotapes of classroom instruction. Thus, there is a literature backing. On the other hand, the opening sentence of this paragraph makes an empirical claim – one that should, then, be tested empirically. Doing so requires a mechanism for assigning scores to instances of instruction. In service of that goal the TRU team created the TRU scoring rubric. Using the rubric, one can assign scores on a five point scale (1, 1.5, 2, 2.5, 3) for each of the five dimensions, for a range of classroom configurations: whole class, small groups, individual student work, and student presentations. Weighted averages can be computed to assign scores for each dimension. Scores can then be correlated with student performance on robust measures of student thinking and problem solving (e.g., the MAP tasks and tests).

The summary scoring rubric is given in Figure 6.

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content (as specified in the Common Core Standards)	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence (see glossary).	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but the teacher is the primary driver of conversations and arbiter of correctness. In class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for building a coherent view of mathematics.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Figure 6. The TRU Summary Rubric

The scoring manual was derived using tapes for the TRU collective had proprietary access, so cannot be distributed widely. We are currently updating the scoring manual, though the categories and scores are robust. Interested researchers can contact Alan Schoenfeld at alans@berkeley.edu to discuss uses of the rubric for scoring purposes.

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