TRU Math Conversation Guide
Module A: Contextual Algebraic Tasks

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A companion document, the *TRU Math Conversation Guide: A Tool for Teacher Learning and Growth,* supports reflection on the five general dimensions of productive mathematics classrooms. This *TRU Math Conversation Guide, Module A: Contextual Algebraic Tasks* is the first of a planned series of content-specific conversation guides aimed at supporting classroom engagement with centrally important mathematical ideas. The *TRU Math Conversation Guide and Modules* will all be accessible at http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/pd.php.

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As algebra teachers, researchers, and coaches, we have experienced the challenges of helping students transition from arithmetic to algebraic thinking and supporting *all* students in developing a robust understanding of algebra. The intent of this content-specific conversation guide is to promote conversations with teachers about ways to engage students in understanding algebra as a powerful tool for problem solving. In particular, we highlight topics that research recognizes as crucial for students to make sense of algebra, particularly as they work toward solving complex problems. These topics include: identifying important quantities, identifying the relationships between them, and representing these relationships in ways that make sense in the problem context.

What this tool is, and who it is for

As part of the set of TRU Math professional development tools, this conversation guide was developed with the same intentions as those presented in *TRU Math Conversation Guide: A Tool for Teacher Learning and Growth* (Baldinger & Louie, 2014). That is, this tool can be used to guide conversations between teachers and coaches, administrators, and colleagues, or as a guide for a teacher's own planning and reflection. If you haven't done so yet, we suggest that you read the introduction and suggestions at the beginning of the domain-general *Guide* for a better sense of how we intend this guide to be used. For example, as with the general *Guide*, conversations about developing students' algebraic thinking are ideally grounded both in the teacher's long-term learning agenda and in the specifics of a lesson that everyone in the conversation has participated in or observed.

This document is a content-specific companion to the general *Guide.* While the collections of general questions could be used in any mathematics classroom, we designed this guide to be used in algebra-related lessons, particularly when students are engaging in open-ended modeling or problem solving tasks. This guide is also a way to think about "important mathematical ideas" discussed in the general *Guide* (Baldinger & Louie, 2014, p. 11), as they specifically relate to algebra. As such, this part of the conversation guide can be thought of as an exemplar of a content-specific complement to the first dimension—"The Mathematics"—of the general *Guide.* Subsequent modules may elaborate on "The Mathematics" in other important content areas such as proof, ratio and proportion, thinking with data, and so on.

The collections of questions in this module may be used alongside questions in the content-general document, such as those related to access or uses of assessment, or alone to focus more closely on the development of robust algebraic understanding. In the following sections, we highlight important competencies in algebra and further elaborate on how the conversation guide might be used.

Developing robust understanding of algebra

We believe that all mathematics instruction should help students gain the ability to successfully and confidently engage in solving complex, open-ended tasks. In turn, these kinds of tasks can provide an arena for developing mathematical content and reasoning. That is, the development of algebraic concepts can occur through students' work as they make sense of contexts, choose representations, and work toward solving problems. For example, students can learn about and explore linear relationships through engaging with contextual situations involving constant changes in time and distance. We generally use the word *context* to refer to the problem scenario or "story" in which the mathematics is embedded.

This section of the conversation guide offers ways to generate discussions about opportunities for students to develop algebraic understanding and how to use algebraic tools and reasoning when working with open-ended, contextual situations. Algebra students have traditionally struggled with story problems, often because the "story" is more of a cover story that does little more than to disguise a problem to which students are expected to apply pre-determined algorithms. The tasks we focus on are those with situations requiring more student exploration, possibly involving diagrams and other representations. Important algebraic ideas can surface *through* students' work on the tasks (see the Appendix for an example).

This algebra-specific conversation guide was developed around five competencies identified by the Algebra Teaching Study as necessary to solving complex algebraic tasks. We call these process goals *robustness criteria* (RCs)[1] because each of them represents a necessary step in demonstrating a robust understanding of algebra, especially as required for successfully solving complex tasks. We briefly present these competencies below:

RC 1   Reading and interpreting text, and understanding the contexts described in problem statements.

RC 2   Identifying important quantities and the relationships between them.

RC 3   Using algebraic representations of relationships between quantities.

RC 4   Performing calculations and procedures with precision and checking the plausibility of results.

RC 5   Providing convincing explanations that give further insight into the depth of students' algebraic thinking.

We do not consider the robustness criteria to be a sequence of steps for solving a problem, but they are all interrelated. For example, in order to model a contextual situation with a representation, students must first identify the important quantities and the relationships between them. This identification may, in turn, contribute to the justification of a result. Based on observations through the Algebra Teaching Study, we believe it is possible and important for algebra instruction to give students opportunities to develop these competencies, thereby helping them to develop a robust understanding of algebra.

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1 For more information regarding the RCs, see the *RCs Elaborated* document at http://ats.berkeley.edu/tools.

Structure of the Conversation Guide

Each part of this module begins with a core question to frame that portion of the conversation guide. The core questions represent the big ideas that we hope will be helpful as you read through the remaining questions and *think about* statements. Next, there are a series of questions corresponding with parts of the teaching cycle—planning, reflecting, and considering next steps—to provide a basis for conversations about lessons. Finally, the *think about* statements focus on different aspects of a lesson to help guide and provide ideas for those conversations. We encourage you to be flexible in your conversations about the *think abouts*! We anticipate there will be some conversations that address most of the *think about* statements, and some that may only focus on one or two. As with the RCs, they do not necessarily represent a sequence of questions to answer, but rather serve as resources to support reflection on what research highlights as promising teaching moves.

Suggestions for using Parts 1 and 2 of the Algebra Module

We used the robustness criteria to develop this conversation guide in two ways, which are reflected in each part. In Part 1, there is a series of *think abouts* that touch on each criterion in an introductory way. The second part consists of three sets of questions, each of which provide a concentrated focus on one or two specific algebraic competencies. The two parts can be used individually and together in a variety of ways.

Part 1 of this algebra-specific conversation guide (pp. 5-6) is designed around general questions incorporating all of the RCs, and might be used to consider an algebra lesson overall. In this sense, it looks at how work on individual tasks may be used to develop each of the algebraic competencies. We recommend that teachers initially start with conversations around these questions for one or more lessons. Through these general discussions about developing a robust understanding of algebra, teachers may choose to focus future discussions on a specific area of implementing open-ended contextual algebra tasks, such as making sense of the problem, representing relationships between quantities, and supporting students in articulating mathematical explanations and/or justifications. Part 2 of the algebra-specific conversation guide (pp. 7-10) provides the opportunity to concentrate more intensely on specific areas of robust understanding. Teachers may choose to spend an extended amount of time on these specific aspects of robust understanding, because it takes students time to develop them! The questions in Part 1 could still serve as a checkpoint to launch these more focused discussions.

Part 1: General Reflections on Developing Robust Understanding of Algebra

Robust Understanding of Contextual Algebraic Tasks

Core Question: How do algebraic ideas and skills that support students in making sense of and solving open-ended contextual algebraic tasks develop in this lesson?

The intent of this portion of the conversation guide is to lay the groundwork for important algebraic ideas that will surface from students' engagement with the task(s)—ideas that will develop over the course of the school year. Because of this focus, the questions and *think about* statements focus on a broad range of objectives that likely go beyond a single lesson, and address several algebraic competencies. So, rather than focusing on specific concepts, such as helping students develop ideas regarding the use of representations, this part of the conversation guide focuses on algebraic ideas that develop as a result of working on complex tasks during a lesson, and how the use of complex tasks supports this development.

Robust Understanding of Contextual Algebraic Tasks

Pre-observation	Reflecting After a Lesson	Planning Next Steps
How will important algebraic ideas and competencies relative to contextual algebraic tasks develop through students' engagement in this lesson?	How did students actually engage with algebraic ideas in this lesson? How did this lesson support students in using algebraic tools and reasoning to answer questions about a contextual situation?	How will students continue to engage with the algebraic ideas relative to contextual algebraic tasks that developed in this lesson?

Think about:

- **Mathematical goals**: The algebra learning objectives for the lesson.
- **The context**[2]: How the context relates to the algebraic concepts embedded in the problem, and how the context can and does support students in making sense of algebraic concepts.
- **Important quantities**: How students discuss the important quantities and the relationship between quantities. (For example, identifying givens and unknowns, identifying a linear relationship because of a constant rate of change, etc.)
- **The use of algebraic representations:** Opportunities for students to represent relationships between quantities. (For example, do students have opportunities to model with diagrams, graphs, tables, or variable equations? Are connections made between multiple representations? What features of the representation(s) do they focus on?)
- **Algebraic procedures:** Opportunities for students to choose which algebraic procedures can or should be used to solve the task, and how they are encouraged to attend to accuracy when completing the procedures.
- **Explanation and justification:** Opportunities for students to learn and practice justifying their solutions and/or explaining their algebraic thinking.
- **Making sense of results:** Ways in which students are prompted to connect solutions back to the problem situation.

[2] *Context* here refers to the problem scenario or "story" in which the mathematics is embedded. Depending on the task, some aspects of the guide might not apply. The context-related parts are relevant only when the class is working on a contextual problem

Part 2: Focused Reflections on Specific Aspects of Robust Understanding

Part 2 of this algebra-specific conversation guide (pp. 7-10) provides the opportunity to concentrate more intensely on specific areas of robust algebraic understanding. The following sets of questions focus on making sense of algebra problems, representing relationships between quantities, and supporting students in articulating mathematical explanations and/or justifications. The questions in Part 1 could still serve as a checkpoint for launching these more focused discussions. In fact, we encourage teachers to think about the algebra learning goals in planning and reflecting on every lesson. Specifically, when using Part 2 of the conversation guide, it may be helpful to first consider the following questions, as appropriate:

- How can the problem context and/or emphasizing relationships between quantities support students' development of the important algebraic ideas in the lesson?

- How do different algebraic representations support students' development of the important algebraic ideas? Which representations are most appropriate in the lesson, and why?

- How should solutions be interpreted in a way that supports students' learning of the important algebraic ideas, and how can that learning be furthered through students' explanations and justifications?

Making Sense of Problem Contexts

Core Question: What opportunities do students have to explore important algebraic ideas as they relate to the context of the task(s)?

Thinking about what a task is asking and uncovering the mathematics embedded in it is an essential first step in solving any contextualized problem. This includes determining the important quantities, how quantities are related, and what mathematical ideas are needed to draw from before "jumping into" a task. To unpack a contextual task in this way requires students to understand both the contextual and mathematical language of the task and to highlight important given and needed information. Thus, instruction should focus on supporting students in making sense of the problem, concentrating on providing opportunities for students to think and talk about the context, the quantities, and the important mathematical ideas in a given task.

Making Sense of a Problem

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities will students be given to make sense of important algebraic ideas through unpacking the language and important quantities in this problem situation?	How did students engage with the language and quantities in the problem context to develop important algebraic ideas?	How can making sense of a problem context be built on in future lessons?

Think about:

- Opportunities for students to build on their prior learning, including both mathematical and nonmathematical knowledge.
- How the context relates to the algebraic concepts embedded in the problem, and how the context can and does support students in making sense of algebraic concepts.
- Opportunities for students to determine and discuss important quantities in the task (both given information and quantities to be determined), the relationship between those quantities, and how can they use the context to justify the relationships. (For example, identifying givens and unknowns, discussing how a change in one quantity causes change in another, identifying a linear relationship because of a constant rate of change, etc.)
- Opportunities students have to explain their algebraic reasoning about relationships between quantities and the problem context.
- Evidence indicating that students make sense of the language of the task, the context, and how the context relates to algebraic ideas; and how different students across the class engaged with these ideas.

Representations of Relationships Between Quantities

Core Question: How do students use algebraic representations to show and explore the relationships between quantities and solve problems in the task?

Using algebraic representations[3] to solve a contextual problem is a powerful way to uncover important algebraic ideas and generalizations. This use includes making observations about multiple representations, connecting how changes in one representation relate to changes in another, generalizing these changes to families of functions, and choosing the best representation(s) for a given situation. Supporting students in developing these competencies involves providing opportunities for students to create, interpret, connect, and use algebraic representations to solve open-ended problems.

Representations of Relationships Between Quantities

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities will students have to use algebraic representations and make sense of the relationship between quantities in this lesson?	How did students use algebraic representations and make sense of the relationships between quantities?	How can students use what they learned about representations and the relationship between quantities in future lessons?

Think about:

- Opportunities students have to generate and interpret algebraic representations.
- Opportunities students have to discuss how the algebraic representations show the relationship between important quantities in the task and how those relationships can be generalized to a family of functions. (For example, how change in a dependent variable is revealed in a table, graph, or equation.)
- Which aspects of representations (for example, specific points in a table, or more global patterns of change in a graph, or parameters in an equation) are important for students to attend to, and how students are supported to attend to them.
- Opportunities for students to consider the affordances of algebraic representations used and why certain representations are appropriate in solving the task, and opportunities to choose appropriate representations.
- How students explore connections between representations.
- How students explain their reasoning about their algebraic representations.
- Evidence indicating students effectively used representations to model and solve the problem, and how different students across the class engaged with these ideas.

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[3] We include variable equations, coordinate graphs, tables, and diagrams as *algebraic representations.*

Interpreting Solutions and Explaining Results

Core Question: What opportunities do students have to make sense of and justify their results?

Using algebraic procedures with precision and being able to justify the results are important mathematical practices for students, both for students to be able to communicate their understanding to others and for teachers to informally assess student thinking. This includes attending to the plausibility of solutions and explaining why an answer may or may not make sense with respect to a problem scenario. Additionally, students should be able to justify the strategies they used to solve the problem, drawing on mathematical resources to justify the results. Supporting students in developing these competencies involves providing explicit guidelines for explanations and justifications as well as giving students opportunities to reflect on—and provide a rationale for—their results within the context of the problem.

Interpreting Solutions and Explaining Results

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities will students have to develop algebraic understanding through explaining and justifying their reasoning and solutions?	How did students explain and justify their algebraic reasoning and solutions?	How can students use what they learned about explaining and justifying their algebraic reasoning and solutions in future lessons?

Think about:

- Opportunities students have to use algebraic procedures to solve the task, which procedures they use, and what opportunities exist to attend to precision in solving problems and presenting their work.
- Opportunities students have to explain their algebraic procedures and respond to each others' explanations.
- Opportunities students have to explain their algebraic reasoning and respond to other students' explanations (e.g., through an algebraic representation, qualitative relationship between quantities, or the problem context).
- How students' explanations and justifications are solicited, and the nature of the explanations/justifications requested. (For example, *can you explain your thinking*?; *why does this procedure make sense*?; *what evidence do you have to justify your results*?).
- How students are supported or instructed toward effective (complete, convincing) algebraic explanations.
- What opportunities exist for students to check for accuracy and/or connect their results to the problem context.
- Evidence indicating that students effectively made sense of and justified their results, and how different students across the class engaged with these ideas.

References

Baldinger, E., & Louie, N. *TRU Math conversation guide: A tool for teacher learning and growth*.
     Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley
   & College of Education, Michigan State University. Retrieved from:
     http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/pd.php.

Appendix: A sample task

Arranging Tables[4]

A company supplies tables for business meetings.
Each table is a rectangle, and can seat one person on its short edge, and
two people on its long edge, like the figures on the right.

The diagrams below show how these tables can be made into arrangements for different numbers of
people. The different arrangements are numbered, like the figures below.
No one sits inside the arrangements of tables.

Size 1
4 tables
12 people

Size 2
6 tables
16 people
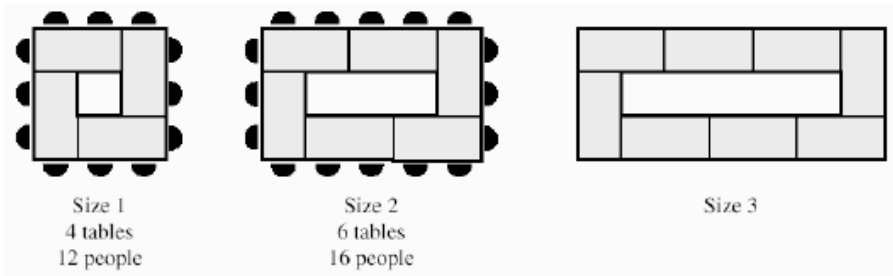
Size 3

(1) How many people can sit at a Size 3 arrangement?

(2) How many people can sit around a Size 13 arrangement? Explain how you know your answer is
correct.

(3) Write an equation for the number of people *p* who can sit at a Size *S* arrangement. Explain how the
parts of your equation relate to the table arrangements.

(4) James is trying to seat 75 people. What size arrangement will he need? Write an explanation that
would convince a classmate your answer is correct.

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[4] This task was adapted for the Algebra Teaching Study from a task developed by the Mathematics Assessment Resource
Service (available at http://www.noycefdn.org/resources.php). The full set of tasks used in the Algebra Teaching Study can
be found at http://ats.berkeley.edu/tools.html.